

Contiguous Refinement of Discrete Octafilial Selfing Homozygosities for Diploid (2n=10) Thale-Cress *Arabidopsis thaliana* Employing QCA vs. ECA (QUANTUM Cellular Automata) vs. Elementary Cellular Automata model

Praharshit Sharma¹, Santhosh Thakur²,
*Corresponding Author: **Journal Targeted**

Rajiv S. Chauhan*

Assignment-5 due Monday, 9th March 2026 Scientific Writing course.

Background= In 2nd Class post-Republic day subsequent Monday, respected Dean Sir had heavily focused on **PBG: Plant Breeding and Genetics** concept of “Selfing” and Octafilial Generations **F0, F1, F2, F3, F4, F5, F6, F7, F8** being inspired by Gregor Mendel’s famous Pea-plants Crossing. In this Direction, **ECA (Elementary Cellular Automata)** schemata broadly do neatly justify **Discrete Octafilial Homozygosity computations** as outlined below, We here aim inspired by **QuBit-QuDit Extrapolations to Capture Contiguity** of Homozygosity values from **F0 = 100%** through **F8 = 99.61%** Subsequent to **Myself: Praharshit Sharma’s Society for Plant Biotechnology & Biochemistry (BITS Goa) Presentation** and Greatly Extending my own Work.



Kindly render a Careful Purview demonstrated Overleaf for the Seamless Transition from Discrete Octafilials F0-F8 through Contiguous Homozygosities

For the F_8 generation, the theoretical proportion of homozygosity is **99.61%**.

1. Define the variables

The calculation of homozygosity in self-pollinating organisms (selfing) depends on the number of generations (n). For an "octa-filial" line, we identify the generation as F_8 , meaning $n = 8$.

2. Apply the homozygosity formula

The proportion of homozygous loci (H) in a population undergoing successive generations of selfing is determined by the formula:

$$H = 1 - \left(\frac{1}{2}\right)^n$$

This formula represents the reduction of heterozygosity (which halves each generation) subtracted from the total population (1).

3. Calculate the F_8 value

Substituting $n = 8$ into the equation:

$$H = 1 - \left(\frac{1}{2}\right)^8$$

$$H = 1 - \frac{1}{256}$$

$$H = 1 - 0.00390625$$

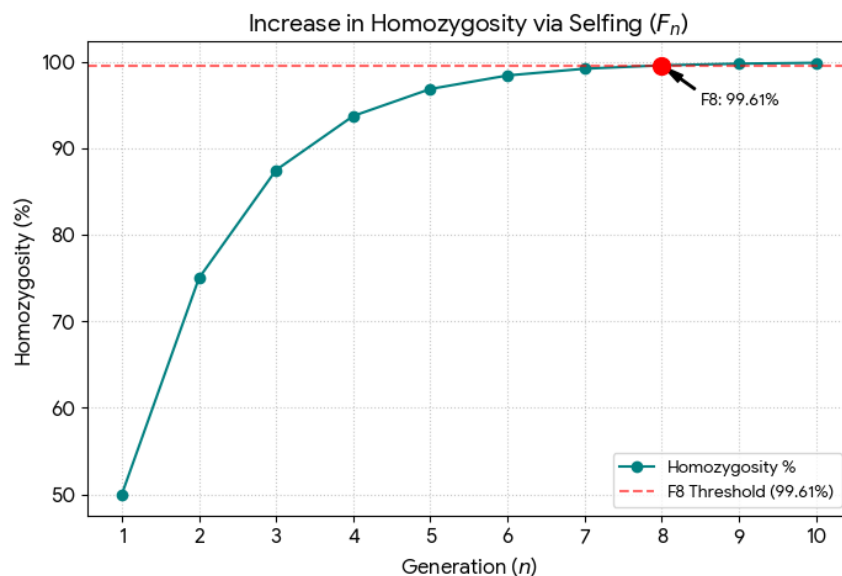
$$H = 0.99609375$$

4. Determine the percentage

To express this as a percentage, multiply the result by 100:

$$0.99609375 \times 100 \approx 99.61\%$$

This indicates that after 8 generations of selfing, only 0.39% of the genome remains heterozygous.



For Simplicity of Initial Draft Manuscript, we Exclude for time-being F9-F10 Selfing Generations and rigorously Tabulate below as to How **ECA: Elementary Cellular Automata** framework Rules Corroborate with F0-F8 **Homo_zygosity** values which are but 1^s Complements of **Hetero_zygosities**.

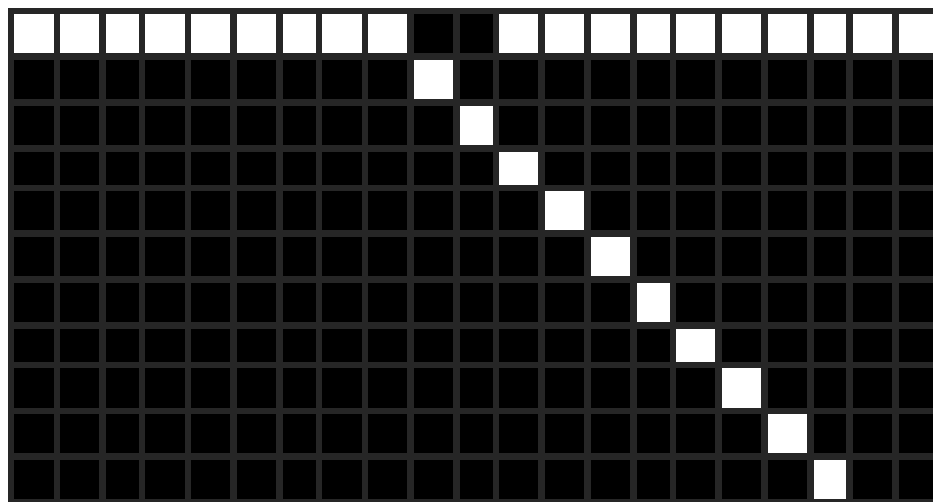
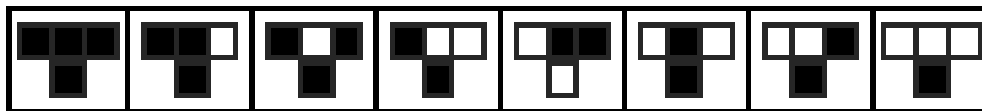
Key References: ECA are Comprised of $(2^{(2^3)})=256$ Rules, for our Diploid Bi-allelic Case of Arabidopsis, Black=1=Major Allele, White=0=Minor Allele.

<https://mathworld.wolfram.com/ElementaryCellularAutomaton.html>

<https://mathworld.wolfram.com/CellularAutomaton.html>

F5 Corresponds to Rule-247 , which is Explained as for All Octafilials below:

https://atlas.wolfram.com/01/01/247/01_01_1_247.html#01_01_3_247



TABULATION of Reciprocal of Specific Rule-Numbers-Octafilial Formula

Filial Homozygosity	$(x+1)/(255+1) \rightarrow \text{rule-x}$	Filial Homozygosity	$(x+1)/(255+1) \rightarrow \text{rule x}$
F0 = $(1/2^0)= 100\%$	255 – ECA Rule	F6=$1-(1/2^6)= 98.4375\%$	251 – ECA Rule
F1= $1-(1/2^1)= 50\%$	127 – ECA Rule	F7=$1-(1/2^7)= 99.21875\%$	253 – ECA Rule
F2= $1-(1/2^2)= 75\%$	191 – ECA Rule	F8=$1-(1/2^8)= 99.609375\%$	254 – ECA Rule
F3=$1-(1/2^3)= 87.5\%$	223 – ECA Rule	F9 (Equation (6) Page :: 1847 cf. Link F10, F11- → https://www.ias.ac.in/article/fulltext/reso/028/12/1845-1863	
F4=$1-(1/2^4)= 93.75\%$	239 – ECA Rule		
F5=$1-(1/2^5)= 96.875\%$	247 – ECA Rule		

However , there is a Caveat here. While **ket-Zero** $\langle 0|$ and **ket-One** $\langle 1|$ as per Quantum Logic in terms of Qubit logic involve “Sum of Squares Normalization” of Alpha and Beta [<https://en.wikipedia.org/wiki/Qubit>]

F5 Octafilial for Thale -cress Diploid Self-crossing (Similar Argument exhibited below can be applied for F0-F4 and F6-F8 also on Lines of F5 case):-

Decimal to Binary Equivalence yields us a “Weighted Geometric Series” with Common ratio- ($r = 2$):=

$$247_{10} = 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

As can be seen for “Qutrits” (that Potentially Accommodates F9 also) with Hilbert Space= $(3^2) = 9 = (8+1) = 1 + (2^3)$, the Remnant “1” for F9 > F8.

$$\begin{aligned} |\psi\rangle &= \sum_{l=0}^{3^2-1} a_l |j_l^1 j_l^2\rangle = \sum_{l=0}^8 a_l |j_l^1 j_l^2\rangle \\ &= a_0|00\rangle + a_1|01\rangle + a_2|02\rangle + a_3|10\rangle \\ &\quad + a_4|11\rangle + a_5|12\rangle + a_6|20\rangle + a_7|21\rangle + a_8|22\rangle, \quad (6) \end{aligned}$$

$$\text{with } \sum_{l=0}^8 |a_l|^2 = 1 \text{ and dimension of its Hilbert space is } 3^2.$$

We Follow “Directly-Proportionate” Logic for Converting Binary Weights into Square Amplitudes! Subsequently Represent Them as Squares using Intelligible Weight Factors -- →

In quantum information (qudit logic), a normalized state requires:

$$\sum_i |a_i|^2 = 1$$

So interpret the binary coefficients as **square amplitudes**:

$$a_i^2 \propto 2^i$$

Normalization factor:

$$Z = \sum_{i=0}^7 2^i = 255$$

Therefore

$$a_i = \sqrt{\frac{2^i}{255}}$$

Now the state vector becomes

$$|\psi\rangle = \left(\sqrt{\frac{1}{255}}, \sqrt{\frac{2}{255}}, \sqrt{\frac{4}{255}}, \dots, \sqrt{\frac{128}{255}} \right)$$

and

$$\sum_i a_i^2 = 1$$

Thus the **powers-of-2 weights become squared amplitudes**.

Each coefficient can now be visualized as a square area:

$$S_i = \frac{2^i}{255}$$

So the rule weight decomposes into **eight normalized squares**.

Example:

bit	weight	square area
2^7	128	$128/255$
2^6	64	$64/255$
2^5	32	$32/255$
2^4	16	$16/255$
2^3	0	0
2^2	4	$4/255$
2^1	2	$2/255$
2^0	1	$1/255$

These areas sum to 1, satisfying the **quantum normalization condition**.

Idea and Conceptualization: Praharshit Sharma, SE25PLSC012
Guidance and Supervision: Dr Santosh Thakur, PI and PhD Guide
PBG Conceptual Consultation: Dean Sir, Prof Rajinder S. Chauhan Sir

Further promising Extension of My Seminal work aims to “Intrapolate” Discrete F0, F1, F2, F3, F4, F5, F6, F7, F8 (and Beyond F9 such as Genetically “Pure” Filials in the Context of Octafilials) by QCA: QUANTUM Cellular Automata Approaches to interrogate Contiguity of Fililas and Grand Inference is the Beauty that in Near-Perfect Saturation, Graph(1) → Page(1)

F8-F0 Zone (99.609375% to 100%) Rule-254 has “All Black Cells as an Ever-Growing Triangular Pattern” (All Major Alleles=1 as in our Case Thale-cress). Now we Parametrize “Genetic-Selection” Compliant-Narrow Sense Heritability (h-Squared) based upon Fundamental Logic 255-ECA/QCA Rule being occupied by ALL Black/1 Cells across All Generations, Sum of Consecutive ODD Numbers=Always Perfect Square Number → 1 (Simple Initial Condition), Black-cell , $(1^2) = 1$ therefore, $h=1$

2nd Row of ECA Space-Time Evolution (Space = Horizontal Axis of ECA) and (Time = Vertical Axis of ECA), $1+(3) = 4 = (2^2)$ therefore, $h=2$

3rd Row of ECA Space-Time Evolution

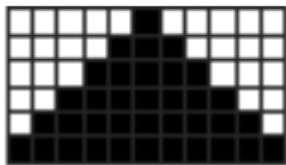
$1+(3)+(5) = 9 = (3^2)$ therefore, $h=3$ so on and so Forth, See Rule-254 Evolution →

<https://atlas.wolfram.com/01/01/254/>

Elementary Cellular Automata

Rule 254

Rule properties



Example



Rule icon

Courtesy of Google Gemini= (Below) Account on PBG (h^2) F0-F8 Regressions

To compute narrow-sense heritability (h^2) for successive generations from F_0 to F_8 , one must understand how additive genetic variance (V_A) evolves in a breeding population, typically through a process like selfing (self-pollination). [🔗](#)

1. General Formula for h^2

The fundamental definition of narrow-sense heritability is the proportion of total phenotypic variance (V_P) that is attributable to additive genetic variance (V_A):

$$h^2 = \frac{V_A}{V_P}$$

where $V_P = V_A + V_D + V_I + V_E$. [YouTube +1](#)

2. Evolution of V_A through Filial Generations

In a selfing population (common in plant breeding), the additive genetic variance (V_A) increases as heterozygosity decreases. If we assume the starting population (F_0 or F_2 depending on the model) has an additive variance of V_A , the additive variance in the n -th filial generation (F_n) is estimated as:

$$V_{A(F_n)} = (1 + F) V_A$$

where F is the **inbreeding coefficient**. For successive generations of selfing: [🔗](#)

- $F_1: F = 0$
- $F_2: F = 0.5$
- $F_3: F = 0.75$
- $F_n: F = 1 - \left(\frac{1}{2}\right)^{n-1}$ [🔗](#)

As n increases, F approaches 1, and the additive genetic variance among lines eventually doubles ($2V_A$) as they become pure lines. [SESRIC +1](#)

3. Computation Across Octafilial Generations (F_0 to F_8)

Narrow-sense heritability is often calculated between generations using **parent-offspring regression** (b). For selfing species, the relationship is:

$$h^2 = \frac{2}{1 + F_{parent}} \times b$$

Using the standard correction factors for specific transitions: [ARCC Journals +1](#)

Generation	Inbreeding (F)	Computation Method / Correction
F_0 (Parents)	0	Starting point; h^2 is defined by the base population variance.
F_1	0	Usually 0 for traits where parents are pure lines (no genetic variance).
F_2	0.5	Calculated as $V_A/V_{P(F_2)}$; often the first generation with segregating variance.
F_3	0.75	Calculated from F_2 - F_3 regression: $h^2 = \frac{2}{3} b$.
F_4	0.875	Calculated from F_3 - F_4 regression: $h^2 = \frac{4}{7} b$.
F_5	0.9375	Calculated from F_4 - F_5 regression: $h^2 = \frac{8}{15} b$.
F_6	0.96875	Regression of F_6 on F_5 .
F_7	0.984375	Regression of F_7 on F_6 .
F_8	0.9921875	Approaches pure line status; h^2 reflects fixed additive variance.

Typically, Narrow Sense PBG Heritability for Discrete Octafilial Homozygosity-Heterozygosity Regressions are computed for a specific trait when the Breeder is provided with variance data to perform a numerical computations to deduce the same. Here, we Beautifully touch upon 2 Fundamental Aspects:

(1) h is a Natural Number for Sum of Consecutive Odd Number of Major Alleles as in Case of Diploid ($2n=10$) *Arabidopsis thaliana* , Thale-cress.

(2) QCA (Quantum Cellular Automata) Aims to Capture Contiguous Octafilial h^2

