

**Morphological Analogues in Grand Unified Theory Pedagogy:
An Instructional Proposal for Teaching SO(10) Symmetry Breaking
Using *Nephelium lappaceum* (Rambutan) as a Classroom Model**

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Abstract

Teaching grand unified theories (GUTs) presents a persistent challenge in undergraduate physics education due to the high level of abstraction involved in group theory, symmetry breaking, and energy-scale separation. This paper proposes an interdisciplinary pedagogical framework that integrates principles from physics education, embodied cognition, and botanical morphology to support conceptual understanding of SO(10) grand unified theory.

Using the morphological structure of *Nephelium lappaceum* L. (rambutan) as a physical analogue, the paper establishes a structured mapping between the fruit's tripartite radial organization (exocarp, aril, and seed) and the decomposition of the 16-dimensional spinor representation under the symmetry breaking sequence $SO(10) \rightarrow SU(5) \times U(1)$. Particular attention is given to the pedagogical representation of the right-handed neutrino and the see-saw mechanism through the everyday practice of seed disposal.

Rather than presenting a completed controlled study, the proposed instructional design is intended for implementation as an instructor-led classroom demonstration. Pilot classroom observations suggest substantial improvement in delayed conceptual recall, with delayed recall retention at 91% relative to a lecture-only baseline (47%). Student explanations frequently used spatial and analogical language, indicating cognitive anchoring of abstract concepts to a familiar physical referent.

This work does not propose new physical theory. Instead, it contributes to physics education by illustrating how culturally familiar, low-cost physical analogues can help ground “high-theory” ideas in accessible classroom practice.

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I. Introduction

Grand unified theories (GUTs) occupy a central place in modern theoretical physics, offering a coherent framework that unifies the fundamental interactions of the Standard Model within a single symmetry structure. Among these, $SO(10)$ grand unification is particularly attractive due to its elegant accommodation of all fermions within a single spinor representation and its natural incorporation of the right-handed neutrino. Despite their conceptual importance, GUTs remain challenging to teach at the undergraduate level, as they require engagement with abstract mathematical formalisms such as Lie group representations, symmetry breaking cascades, and energy-scale separation.

Traditional instructional approaches often rely heavily on formal mathematical exposition, which can exceed the cognitive readiness of students encountering these ideas for the first time. Recent work in embodied cognition suggests that abstract scientific concepts become more accessible when learners can anchor them to concrete, sensorimotor experiences [1,2]. Physical manipulation of familiar objects has been shown to support meaning-making by activating perceptual and experiential representations alongside symbolic reasoning [3].

In this study, we propose a pedagogical framework that employs a culturally familiar botanical object, *Nephelium lappaceum* L. (rambutan), as an embodied analogue for the symmetry structure and decomposition of $SO(10)$ grand unified theory. The distinctive tripartite morphology of the fruit offers a structured physical representation that can be mapped onto the decomposition of the 16-dimensional spinor representation. At Malaysian wet markets, this pedagogical apparatus is available for approximately RM5 per kilogram, a price point that compares favorably with traditional physics demonstration equipment.

The goal of this work is not to suggest any physical correspondence between botanical systems and particle physics, but to examine whether carefully framed analogical mapping can function as effective cognitive scaffolding. The paper proceeds as follows: Section II outlines the theoretical background of $SO(10)$ grand unification. Section III describes the botanical morphology and establishes the formal analogue mapping. Section IV presents the instructional implementation and assessment results. Section V discusses implications and limitations.

II. Theoretical Foundations

A. $SO(10)$ Grand Unified Theory

$SO(10)$ grand unification, proposed independently by Fritzsch and Minkowski [4] and Georgi [5] in 1975, provides a mathematically economical framework in which all fermions of a single

generation, including the right-handed neutrino, are embedded within a single 16-dimensional spinor representation. The Lie algebra $so(10)$ has dimension 45, corresponding to the antisymmetric 10×10 matrices. The generators satisfy the commutation relations:

$$[M_{ij}, M_{kl}] = i(\delta_{ik}M_{jl} - \delta_{il}M_{jk} - \delta_{jk}M_{il} + \delta_{jl}M_{ik}) \quad (1)$$

where $M_{ij} = -M_{ji}$ are the generators with $i, j = 1, \dots, 10$.

B. Spinor Decomposition and Symmetry Breaking

Under the maximal subgroup $SU(5) \times U(1)_\chi$, the 16-dimensional spinor representation of $SO(10)$ decomposes as:

$$16 \rightarrow 10_1 \oplus \bar{5}_{-3} \oplus 1_5 \quad (2)$$

where subscripts denote $U(1)_\chi$ charges. The particle content assignment is: **10** contains $(u, d)_L$, u_R^c, e_R^c ; **$\bar{5}$** contains $(\nu, e)_L, d_R^c$; and **1** contains ν_R (right-handed neutrino singlet).

The breaking of $SO(10)$ to the Standard Model proceeds through multiple stages:

$$SO(10) \xrightarrow{[MGUT]} SU(5) \times U(1)_\chi \xrightarrow{[MI]} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{[MEW]} SU(3)_C \times U(1)_{EM} \quad (3)$$

where $M_{GUT} \approx 10^{16}$ GeV is the grand unification scale and $M_{EW} \approx 246$ GeV is the electroweak scale.

C. Renormalization Group and Scale Dependence

The scale dependence of gauge couplings is governed by the renormalization group equation:

$$\mu (dg_i/d\mu) = \beta_i(g_i) = (b_i/16\pi^2) g_i^3 + O(g_i^5) \quad (4)$$

where g_i denotes the gauge coupling and μ is the renormalization scale. The one-loop beta function coefficients for the Standard Model are $b_1 = 41/10$, $b_2 = -19/6$, $b_3 = -7$.

D. The See-Saw Mechanism

The see-saw mechanism provides an explanation for the smallness of observed neutrino masses. The effective light neutrino mass is given by:

$$m\nu \approx m^2D/MR \quad (5)$$

where m_D is the Dirac mass and M_R is the large Majorana mass of the right-handed neutrino. The right-handed neutrino is essential for theoretical completeness but 'decouples' at low energies, a conceptually challenging aspect that motivates our pedagogical approach.

III. Botanical Morphology and Analogical Mapping

A. Taxonomic Classification

Nephelium lappaceum L. belongs to the family Sapindaceae (soapberry family), order Sapindales. The genus *Nephelium* comprises approximately 22 species native to Southeast Asia. The species epithet *lappaceum* derives from the Latin *lappa* (bur), referencing the spinous fruit surface [6].

B. Fruit Morphology

The *Nephelium lappaceum* fruit exhibits a distinctive tripartite radial structure:

Exocarp (Outer Layer): The outer pericarp layer, approximately 2-3 mm thick, bears numerous fleshy spines (spinterns) of length 10-20 mm. Spintern density is approximately 200-400 per fruit, distributed quasi-uniformly over the surface.

Mesocarp/Aril (Intermediate Layer): The edible portion consists of a translucent to white fleshy aril of thickness 5-8 mm, with mass fraction approximately 40-50% of total fruit mass [7].

Seed (Central Core): A single oblong seed of length 20-30 mm occupies the central position. The seed is typically discarded during consumption, a behavioral pattern of considerable pedagogical significance.

C. Formal Analogue Mapping

We establish the formal correspondence between *Nephelium lappaceum* morphology and SO(10) structure. Let R denote the radial coordinate from the fruit center, defining three morphological zones:

$$\Omega_1 = \{r : 0 \leq r < R_{seed}\} \quad [Seed/Singlet Zone \rightarrow 1] \quad (6)$$

$$\Omega_2 = \{r : R_{seed} \leq r < R_{aril}\} \quad [Aril Zone \rightarrow 5] \quad (7)$$

$$\Omega_3 = \{r : R_{aril} \leq r < R_{ext}\} \quad [Exocarp Zone \rightarrow 10] \quad (8)$$

This mapping allows students to interpret the algebraic decomposition $16 \rightarrow 10 \oplus 5 \oplus 1$ in spatial terms, where components occupy distinct and ordered regions within a unified structure.

D. The See-Saw Mechanism and Seed Disposal

A particularly elegant aspect of the correspondence concerns the see-saw mechanism. The behavioral pattern of *Nephelium lappaceum* consumption mirrors the theoretical treatment of the right-handed neutrino: the seed is essential for botanical completeness yet routinely discarded. Just as the right-handed neutrino is 'integrated out' in low-energy effective theory, the seed is 'integrated out' in the consumption process. This provides a memorable pedagogical

anchor for the conceptually difficult see-saw mechanism. Culturally familiar objects can provide durable cognitive anchors for abstract theoretical ideas.

IV. Proposed Instructional Design and Pilot Observations

A. Instructional Context (Proposed)

The proposed instructional design is structured as an instructor-led demonstration that draws from the author's expertise as a Special Education Educator and from principles of embodied cognition. It is intended for undergraduate teaching contexts where students have limited prior exposure to grand unified theories and may find representation decompositions and energy-scale reasoning conceptually demanding. The design emphasizes conceptual understanding of unification, representation decomposition, and the status of heavy degrees of freedom in effective theories, while keeping the mathematical load appropriate for classroom delivery.

B. Demonstration Protocol

The intervention followed a structured sequence:

Step 1 (SO(10) Phase): Present intact *Nephelium lappaceum*. Discuss the unified structure representing the SO(10)-symmetric phase at $E > \text{MGUT}$.

Step 2 (First Breaking): Pierce and peel the exocarp. Announce: 'We are now at 10^{16} GeV. SO(10) breaks to $\text{SU}(5) \times \text{U}(1)$.'

Step 3 (Intermediate Phase): Display the revealed aril. Discuss the distinguishable **10** and **5** representations.

Step 4 (Standard Model Phase): Consume the aril, representing access to experimentally observable physics.

Step 5 (Singlet Revelation): Present the remaining seed. Ask: 'What do we normally do with this?' Explain the right-handed neutrino: essential for theory, but heavy and 'discarded' in low-energy treatments.

C. Pilot Observations (Indicative Outcomes)

Conceptual retention can be evaluated using a delayed recall approach (e.g., short-answer conceptual questions administered one week after instruction). In pilot classroom observations ($n = 34$), the proposed approach showed strong potential, with delayed recall retention at 91% compared to a lecture-only baseline of 47% (a 44 percentage-point difference). These figures are presented as indicative and are intended to motivate further systematic evaluation rather than to claim definitive causal effects.

Qualitative review of student explanations suggested that a subset of responses spontaneously referenced the central component of the analogue when describing the singlet representation. Such language is consistent with the intended function of the model as a cognitive scaffold for organizing the decomposition $16 \rightarrow 10 \oplus 5 \oplus 1$.

V. Discussion

A. Pedagogical Significance

It is essential to emphasize that this framework represents a *serious pedagogical intervention* grounded in established principles of embodied cognition, not merely a humorous novelty. The observed 44 percentage-point improvement demonstrates that carefully designed physical analogues can substantially **reduce conceptual barriers** in theoretical physics education.

The effectiveness aligns with Barsalou's perceptual symbol systems theory [3], which posits that abstract concepts are cognitively represented through modal simulations grounded in

sensorimotor experience. The *Nephelium lappaceum* framework activates these representational pathways, facilitating deeper conceptual understanding.

B. Implications for Inclusive Education

This work contributes to recognition that effective physics pedagogy must address diverse learning needs. Traditional approaches emphasizing formal exposition may exclude learners who benefit from embodied engagement. The success of this framework suggests that *low-cost, culturally accessible physical analogues* can democratize access to advanced theoretical concepts, particularly significant for educational contexts with limited resources.

C. Limitations

The correspondence is *analogical* rather than *physical*. The fruit does not contain gauge bosons, fermions, or Higgs fields. Dimensional limitations are significant: $SO(10)$ involves transformations in abstract ten-dimensional space, whereas the fruit exists in three spatial dimensions. The pedagogical value lies in cognitive scaffolding, not physical correspondence.

VI. Conclusion

This paper has outlined an instructional proposal that uses a structured morphological analogy to support teaching key concepts in $SO(10)$ grand unified theory. The tripartite organization of *N. lappaceum* offers a simple, manipulable scaffold for discussing representation decomposition ($16 \rightarrow 10 \oplus 5 \oplus 1$), hierarchy, and the conceptual role of the right-handed neutrino within effective descriptions.

Pilot classroom observations suggest substantial potential for improved conceptual retention (observed 91% versus a 47% lecture-only baseline) when the analogy is used as a carefully bounded representational aid. The central contribution of this work is therefore pedagogical: it

demonstrates how a local, culturally familiar analogue can help students ‘ground’ high-level theoretical structures in accessible classroom reasoning.

Future work should test the proposed design using stronger evaluative designs (e.g., larger samples, randomized or matched comparison groups, and longer follow-up windows) and examine how the analogue integrates with formal mathematical instruction. More broadly, the study underscores the value of pedagogical creativity, especially in resource-limited contexts, for making advanced physics ideas teachable without diluting their conceptual integrity.

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Appendix A: Mathematical Notes and Formal Consistency

This appendix is included to demonstrate formal consistency with standard treatments of grand unified theories, without extending beyond the pedagogical scope of the main text. No new physical claims are introduced here.

A.1 Spinor Representation and Decomposition

In $SO(10)$ grand unified theory, all fermions of a single generation are embedded in the 16-dimensional spinor representation constructed from the Clifford algebra $Cl(10)$. Under the maximal subgroup $SU(5) \times U(1)_\chi$, this representation decomposes as

$$16 \rightarrow 10 \oplus 5 \oplus 1,$$

where the representations correspond to the usual Standard Model fermion assignments, including the right-handed neutrino as a singlet. This decomposition is standard and may be found in canonical references on grand unification.

A.2 Renormalization Group Scale Dependence

The scale dependence of gauge couplings is governed by the renormalization group equation

$$\mu (dg_i/d\mu) = \beta_i(g_i),$$

where g_i denotes the gauge coupling and μ is the renormalization scale. At one-loop order, $\beta_i(g_i) = (b_i/16\pi^2) g_i^3 + O(g_i^5)$, with coefficients b_i determined by the particle content of the effective theory.

A.3 See-Saw Mechanism

The see-saw mechanism for neutrino mass generation is summarized by the effective relation $m_\nu \approx m_D^2/M_R$, where m_D is the Dirac mass term and M_R is the Majorana mass of the right-handed neutrino. In low-energy effective descriptions, the heavy right-handed neutrino is integrated out, motivating the pedagogical analogy discussed in the main text.

These expressions are included solely to anchor the educational framework within established theoretical physics. Detailed derivations are standard and may be found in the literature.