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Model for damage evolution through dynamic stiffness degradation in three wood species subjected to very high cycle fatigue

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Abstract

The main objective of this research project is to characterize the evolution of fatigue damage produced by a flexural excitation device in three types of wooden beams of different species, namely *Pinus pseudostrobus*, *Quercus scytophylla*, and *Cedrela odorata*. A stiffness degradation model is proposed that considers an acceleration of damage as it increases. The methodology used for this purpose is based on laboratory measurement of the first natural frequency of the wooden beams using the impact and sensing technique and vibration detection using an accelerometer for subsequent computer data processing. The first natural frequency is measured before fatigue testing begins. The theory of fatigue degradation dictates that a material loses stiffness over the course of working cycles. Measurements of the first natural frequency of the wooden beams are taken after more than 19 million cycles, verifying this behavior in the laboratory. The results obtained show a decrease in natural frequency, which translates into a loss of stiffness. The order of 10^7 cycles was reached for all beams, and a reduction of 19.06% in the stiffness of *Cedrela odorata*, 16.83% for *Pinus pseudostrobus* and 11.20% for *Quercus scytophylla* was observed. Based on the experimental data and using a simple exponential degradation model as a basis, a new model for damage evolution is proposed that considers degradation accelerating as damage increases and approaches failure. The experimental data are compared with the data obtained using the proposed model, obtaining a correlation coefficient (R^2) of 0.998 for *Pinus pseudostrobus*, 0.950 for *Quercus scytophylla*, and 0.835 for *Cedrela odorata*. The model fits the experimental data very well for *Pinus pseudostrobus* and *Quercus scytophylla* but not as well for *Cedrela odorata*.

Keywords Stiffness degradation, Very high cycle fatigue, Damage evolution model, Fatigue life prediction, S–N curves, Modal analysis, *Pinus pseudostrobus*, *Quercus scytophylla*, *Cedrela odorata*

Introduction

Fatigue is defined as the degradation of a material's structural integrity as a result of external conditions that vary over time. These external conditions usually take the form of fluctuating mechanical loading and stress. The structural integrity of the material is commonly measured in terms of mechanical properties such as stiffness and strength. Loss of stiffness in a material can lead to instability in a structural component, and loss of strength is directly associated with

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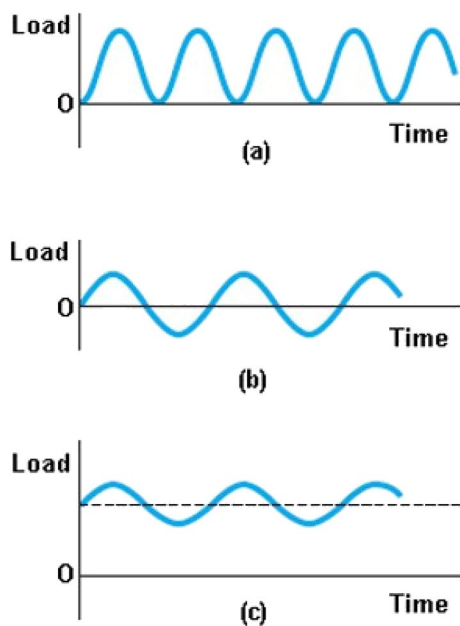


Fig. 1 Common patterns of cyclic loads, (a) shows a load that is applied, removed, and applied again in the same direction, (b) shows an alternating load that reverses direction during each load cycle, and (c) illustrates a fluctuating load that varies from an average value. [1]

component failure. Such failure can occur at a small fraction of a material's static strength. Cyclic loading causes damage, reducing strength until the material can no longer withstand even the service load [1].

Some common patterns of repeated loads are shown in Fig. 1. The first graph (a) shows a load that is applied, removed, and applied again, always acting in the same direction. The second graph (b) shows an alternating load that reverses direction during each load cycle, and the third graph (c) illustrates a fluctuating load that varies from an average value. Repeated loads are generally associated with machinery, motors, turbines, generators, shafts, impellers, aircraft parts, automobile parts, and the like. Some of these structures undergo millions (and even billions) of load cycles during their service life [1].

In the fatigue analysis carried out in this research work, a load such as that shown in Fig. 1c is used, i.e., a load that fluctuates around a mean value.

Very high cycle fatigue (VHCF) is defined as a type of fatigue that occurs from small elastic deformations under a large number of load cycles before failure, in

the order of 10^7 cycles. It is characterized by combined mean and alternating stresses, which mainly involve elastic deformation and generally result in failure without significant macroscopic plastic deformation [2].

The goal of modal analysis is to determine, either numerically or experimentally, the natural frequencies and vibration modes of a structure [3]. The experimental study of structural dynamics has always made an important contribution to our efforts to understand and control the many vibration phenomena encountered in practice [3]. If energy is applied to a system, it will vibrate at its natural frequency, and the level of vibration will depend on the strength of the energy source and the absorption inherent in the system [4].

The Fast Fourier Transform (FFT) proposes a continuous or periodic signal that can be represented as the sum of a series of appropriately chosen sine waves, allowing any periodic and irregular time series to be represented as a sum of regular sinusoids of different frequencies, amplitudes, and relative phases. The FFT is based on three points:

1. Enable time domain signals to be processed by sorting the frequency domain
2. Periodic signals in the time domain are transformed into sums or Fourier series of sinusoidal signals of different frequencies, and
3. Non-periodic signals in time are transformed into a set of signals whose frequencies cover a significant spectrum [5].

For a given discrete signal of length n , $x(n)$, its Fourier transform in the frequency domain results in $X(k)$:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-i2\pi kn/N} \quad (1)$$

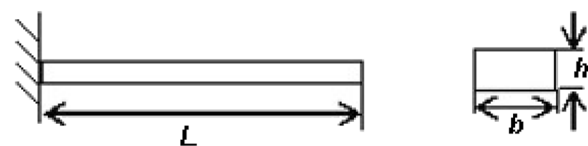


Fig. 2 Cantilever beam

Table 1 Physical properties and dimensions of wooden beams

Wood	Young's modulus (MPa)	Density (kg/m ³)	L (m)	b (m)	h (m)
<i>Pinus pseudostrabus</i>	8947	742.15	0.8000	0.1810	0.0115
<i>Quercus scytophylla</i>	21,551	656.52	0.8000	0.1810	0.0130
<i>Cedrela odorata</i>	10,604	547.10	0.8000	0.1810	0.0130

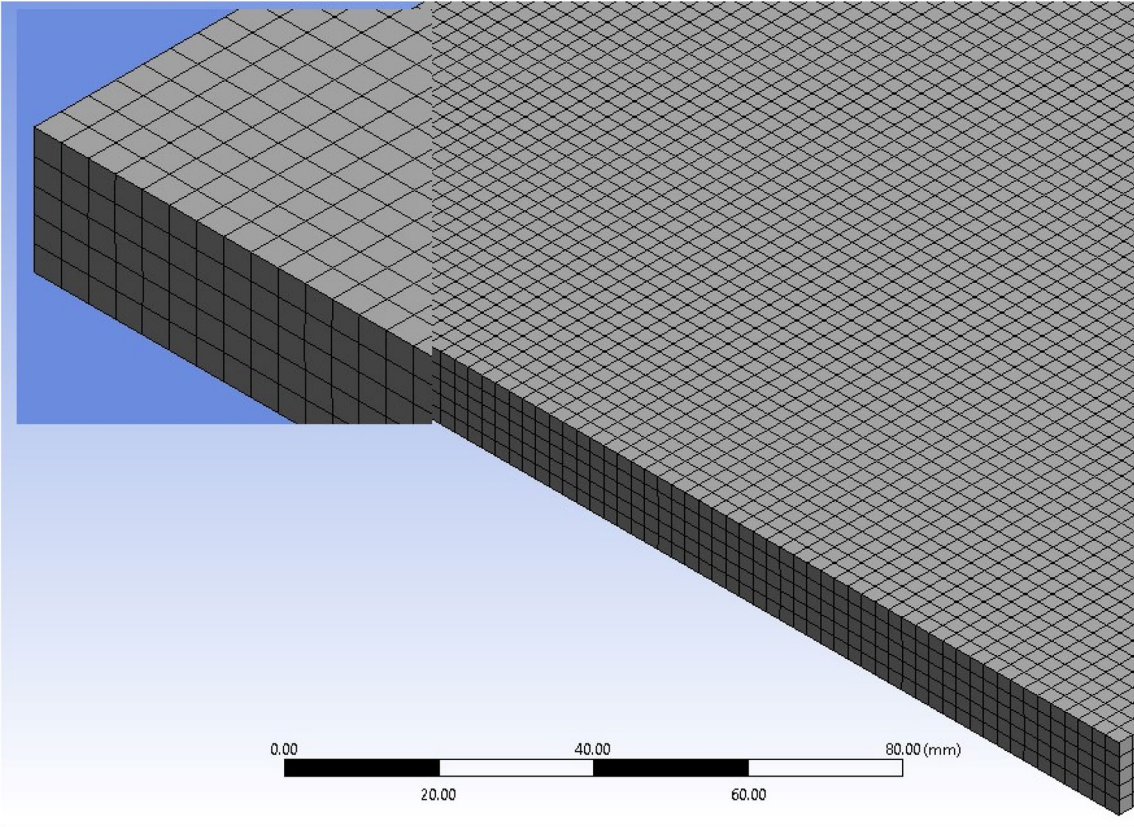


Fig. 3 Mesh model

Table 2 Mesh metrics

Parameter	<i>Pinus pseudostrobus</i>	<i>Quercus scytophylla</i>	<i>Cedrela odorata</i>
Element type	Hex20	Hex20	Hex20
Number of elements	122,994	116,925	116,705
Number of nodes	556,183	542,584	541,352
Element size	2.5 mm	2.5 mm	2.5 mm
Average element quality	0.99244	0.99106	0.99222
Average aspect ratio	1.1655	1.0997	1.0666

where $X(k)$ is the signal in the frequency domain, k is the frequency index, $x(n)$ is the signal in the time domain, i is the imaginary part, and n is the index in the time domain [6].

With regard to fatigue studies in wood, Hacker and Ansell conducted research on changes in properties and the accumulation of fatigue damage in laminated wood-epoxy composite elements, concluding that the application of a reversible load is the most destructive mode of fatigue [7].

For their part, Clorius et al. present experimental research on fatigue in wood subjected to stress perpendicular to the grain. The study was designed with particular consideration of the influence of load frequency. The research reveals an interaction between the number of load oscillations and the cumulative time under load until failure. This interaction corresponds to frequency-dependent fatigue [8].

Lipovec Zupanc et al. investigated the effects of load frequency and magnitude on the durability of *Picea abies* wood. Frequencies of 5 and 10 Hz were compared at three load levels of 70%, 80%, and 90% of the static

Table 3 Electrical characteristics of the ADXL335 sensor

Parameter	Value
Degrees of freedom	3
Input voltage	3.3–5 V
Operating voltage	2–3.6 V
Operating current	0.140 mA
Interface	I ² C–SPI (5 MHz)
Adjustable sensitivity	2g/4g/8g/16g

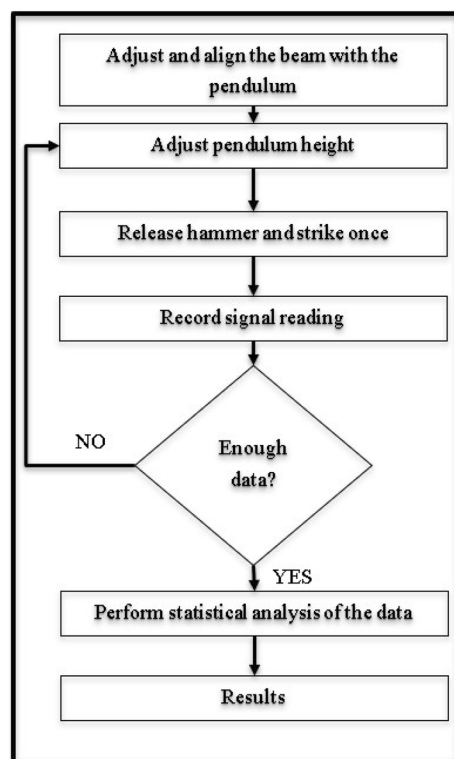


Fig. 4 Methodology for modal tests

breaking force. The research results show that the magnitude of the load significantly influences the dynamic strength at the same fatigue frequency [9].

In their article, Klemenc and Fajdiga present an approach to modeling the fatigue resistance properties

of wood. The model is based on the Weibull conditional probability density function, which is biparametric, with a constant shape parameter and a scale parameter that is a function of wood density, load direction, and processing batch. The new model demonstrated that it is capable of adequately replicating the S–N curves of Norwegian spruce with a dispersion, despite the relatively small amount of experimental data from different production batches loaded in different directions and with significant variation in density [10].

On the other hand, Yang et al. explores essential knowledge about the behavior of wood under fatigue, focusing on S–N curves, stress–strain behavior, and failure mechanisms [11].

Methods/experimental

Three beams of three different types of wood were analyzed: pine (*Pinus pseudostrobus*), white oak (*Quercus scytophylla*), and cedar (*Cedrela odorata*). These were visually inspected to ensure that they had as little surface damage as possible. The cedar beam had some damage, but it was minimal. The pine comes from the Neovolcanic Axis that crosses the northern part of the state of Michoacán, Mexico, an area with a temperate subhumid climate and an average annual temperature of 20 °C. The white oak comes from native forests in the northern Appalachians in the United States of America, where a temperate humid climate prevails with an average annual temperature of 10 °C. The cedar was extracted from plantations in Africa, specifically Gabon, where the prevailing climate is warm with an average temperature ranging between 25 and 27 °C.

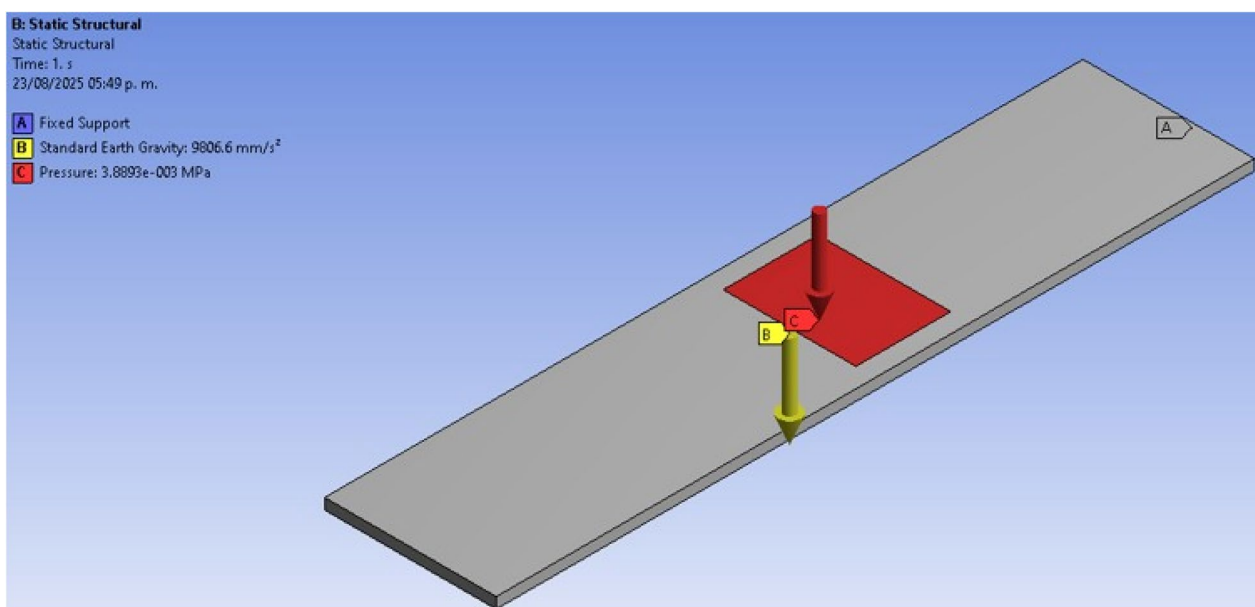


Fig. 5 Boundary conditions for the first simulation

Table 4 Characteristics of the excitation device

Characteristic	Value
Maximum rotation speed	1183 RPM
Cycles per hour	75,600
Eccentric mass	0.10 kg
Eccentric distance	0.07 m
Force produced	107.42 N

The density was calculated using the ratio of mass to volume, as dictated by ISO 13061-2014-2 [12]. Table 1 shows the dimensions of the beams, as well as the modulus of elasticity of *Pinus pseudostrobus* [13], the modulus of elasticity of *Quercus scytophylla* and *Cedrela odorata* [14], and their densities, values that will be used in the finite element simulation, as well as Poisson’s ratio, which varies depending on the material, but for a modal simulation in wood, it is recommended to use a value of 0.3 [15]. Figure 2 shows the shape of the beams.

Modal analysis using FEM simulation

The finite element method (FEM) is a general numerical method for approximating solutions to partial differential equations that is widely used in various engineering and physics problems. It is based on the idea of constructing a complex object from simple blocks, or dividing a complex object into smaller, more manageable pieces [16].

FEM simulation is used to calculate the first natural frequency of each wooden beam, thus obtaining the way

Table 5 Goodman-corrected stresses

Wood	MOR (MPa)	$\sigma_{a,eq}$ (MPa)
<i>Pinus pseudostrobus</i>	57.27 [19]	3.96
<i>Quercus scytophylla</i>	87.1811 [19]	4.53
<i>Cedrela odorata</i>	49.0333 [19]	4.74

in which the beam vibrates at its first natural frequency, but also to obtain a reference value when taking measurements in the laboratory with the accelerometer, for which finite element software was used. First, the geometry must be modeled using computer-aided design (CAD) and the properties of the material to be used defined. In this case, the different types of wood. To calculate the natural frequencies of a system, a modal analysis must be performed, which only requires geometric data, boundary conditions, meshing, modulus of elasticity, and material density. In this case, it was modeled as an isotropic material. Figure 3 shows the mesh of the model for the *Cedrela odorata* beam. Table 2 shows the mesh metrics for the three wooden beams.

Once the mesh geometry has been created, the boundary conditions are set. For this study, only a fixed support on one side of the beam is required. The next step is to run the simulation and obtain the first natural frequency.

Modal analysis in the laboratory

To carry out the laboratory tests, the ADXL335 3-axis accelerometer (see Table 3 for its characteristics) was connected to an Arduino Mega2560 card to receive the

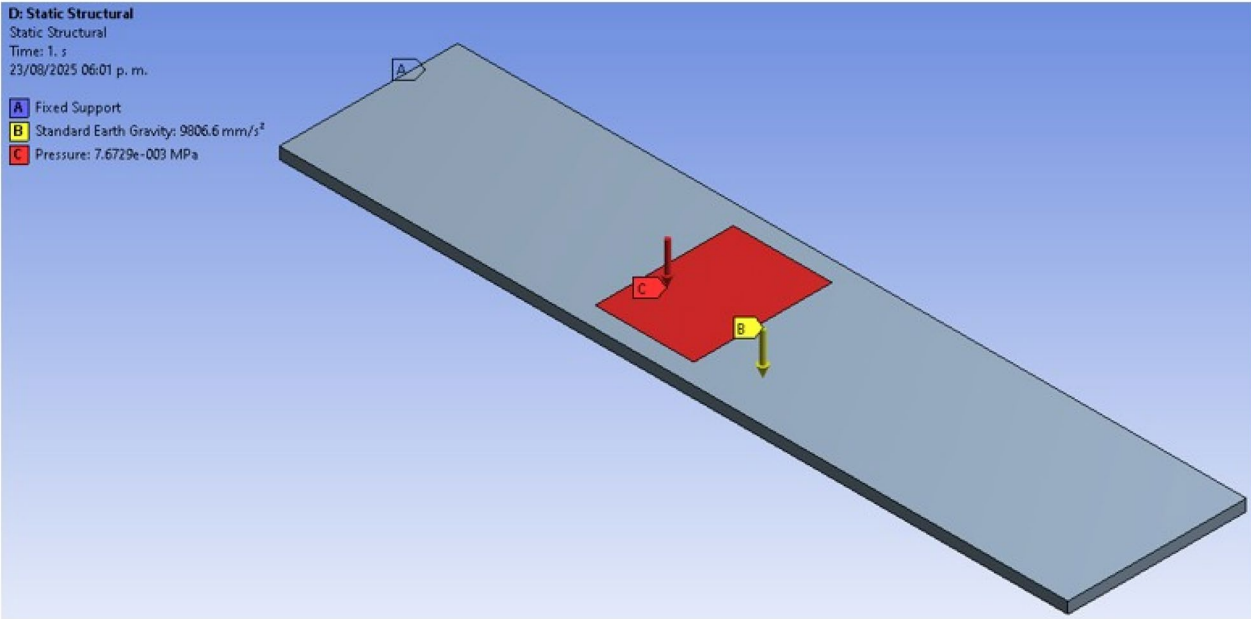


Fig. 6 Boundary conditions for the second simulation

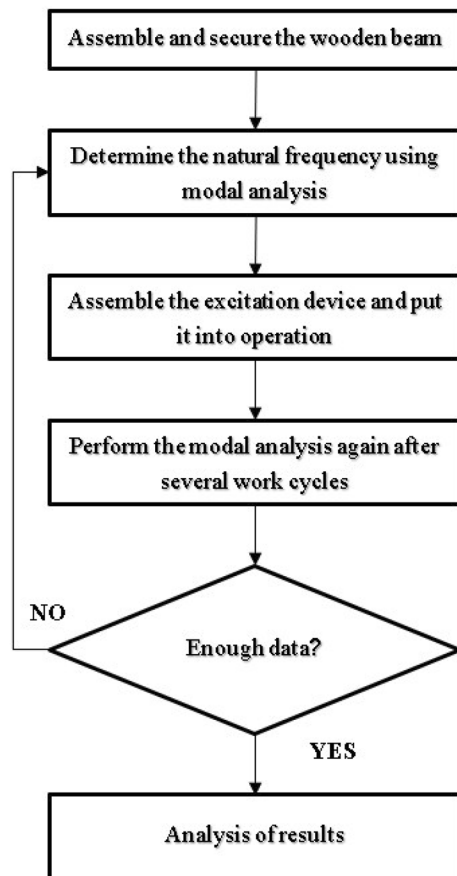


Fig. 7 Methodology used in fatigue testing

signals and send them to the computer for processing with LabView 2015. The 45-day trial version available from National Instrument on its website was used to program the signal processing [17]. It is necessary to apply the FFT to the signal collected by the accelerometer, as it sends it in the time domain, in order to visualize it in the frequency domain and thus determine the first natural frequency in Hz of the beam, which is of interest.

The methodology for performing modal testing in the laboratory is shown in Fig. 4. A pendulum with a rubber hammer is used to excite the structure, ensuring that the same point is always struck with the same force.

Twenty impact tests are performed, obtaining 20 measurements, from which an average is calculated for the first natural frequency. The humidity and ambient temperature variables are recorded at the time of performing the impact and sensing tests.

Excitation device characteristics

A motor with an eccentric designed and manufactured in-house is used, which has the following operating characteristics.

To characterize the average stress to which the beams are subjected, first, the normal stresses (XX, YY, ZZ) and shear stresses (XY, YZ, XZ) experienced by the beam are calculated using numerical simulation based on the weight of the excitation device itself, which has a mass of 5.55 kg (placed in the center for cedar and pine, and at 3/4 of the embedment for oak), which is converted into pressure using the area of the base plate that holds the device to the beam, which measures 100 × 140 mm. Figure 5 shows the boundary conditions for this simulation, which are a fixed support, the pressure from the weight of the device, and standard Earth gravity. Once the results for normal and shear stresses have been obtained, they are exported to be preloaded for a second simulation.

For the second simulation, the force generated by the excitation device is used as the boundary condition (see Table 4), as well as the previously calculated stress data. This stress data is preloaded into the setup of the second structural simulation, which also includes the fixed support condition and standard Earth gravity (Fig. 6). Thus, obtaining the maximum principal stress in the embedded part of the beam. The simulation is repeated, but now with the pressure of the force exerted upwards, since the device pulls the beam downwards but also upwards for each work cycle, resulting in alternating stress. With these two stresses, the average stress to which the beam is being subjected during fatigue testing is calculated, as well as the alternating stress and the R of the load.

$$\sigma_{\text{mean}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \quad (2)$$

$$\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \quad (3)$$

$$R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \quad (4)$$

where σ_{mean} is the average stress, in MPa, σ_{max} is the maximum stress, in MPa, σ_{min} is the minimum stress, in MPa, σ_a is the alternating stress, in MPa, R is the load factor, dimensionless.

In addition, using the modulus of rupture in bending (MOR), a correction is made to the average stress by Goodman [18], using the following equation:

Table 6 Results for the first natural frequency

Wood	First natural frequency (Hz)
<i>Pinus pseudostrobus</i>	10.215
<i>Quercus scytophylla</i>	19.057
<i>Cedrela odorata</i>	14.639

$$\sigma_{a,eq} = \frac{\sigma_a}{1 - \frac{\sigma_{mean}}{MOR}} \quad (5)$$

where $\sigma_{a,eq}$ is the equivalent alternating stress according to Goodman, in MPa, σ_a is the alternating stress, in MPa, σ_{mean} is the average stress, in MPa, MOR is the modulus of rupture in bending, in MPa.

The methodology described above is used to characterize the mean, alternating, and Goodman-corrected stresses in the three wooden beams. Table 5 summarizes the corrected stresses for each beam.

Fatigue testing

The methodology for fatigue testing consists of obtaining the first natural frequency through modal analysis in the laboratory before starting the fatigue tests. The excitation device is then assembled and put into operation. After a certain number of hours (work cycles), the excitation device is removed and the modal analysis is performed again, recording a decrease in the first natural frequency, which translates into a decrease

Table 7 Experimental first natural frequency

Wood	First natural frequency (Hz)
<i>Pinus pseudostrobus</i>	15.05 ± 0.32 (4.92*)
<i>Quercus scytophylla</i>	22.97 ± 0.28 (2.86*)
<i>Cedrela odorata</i>	20.21 ± 0.19 (2.01*)

* Coefficient of variation

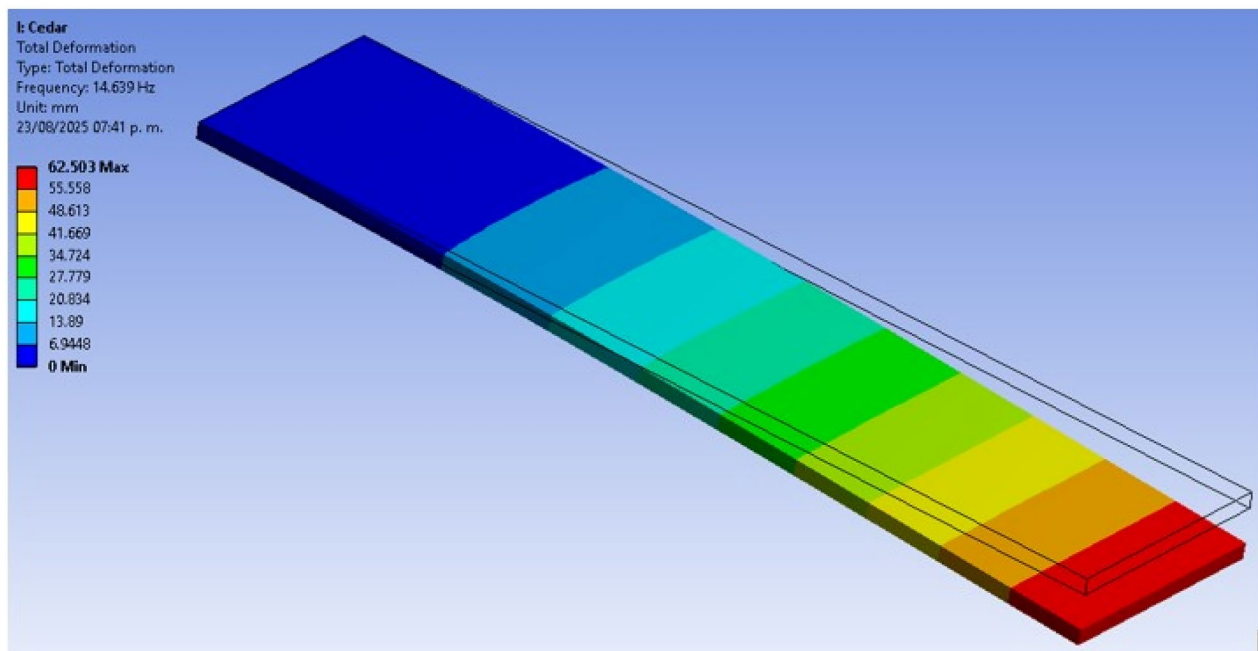
in the stiffness of the beam. This process is continued until sufficient data is obtained to create an S–N diagram (since failure is not reached), where the reduction in stiffness is compared to the number of work cycles. Figure 7 shows the methodology used in the form of a flowchart.

Relative stiffness (S) is used to quantify the degradation of a material subjected to N work cycles.

$$S_{(N)} = \left(\frac{f_N}{f_0} \right)^2 \quad (6)$$

where $S_{(N)}$ is the relative stiffness as a function of the N number of cycles, dimensionless, $f_{(N)}$ is the natural frequency after N cycles, in Hz, f_0 is the initial natural frequency, before fatigue, in Hz.

Shaoxiong et al. [20], propose calculating the damage as shown below:

**Fig. 8** First natural frequency for *Cedrela odorata*

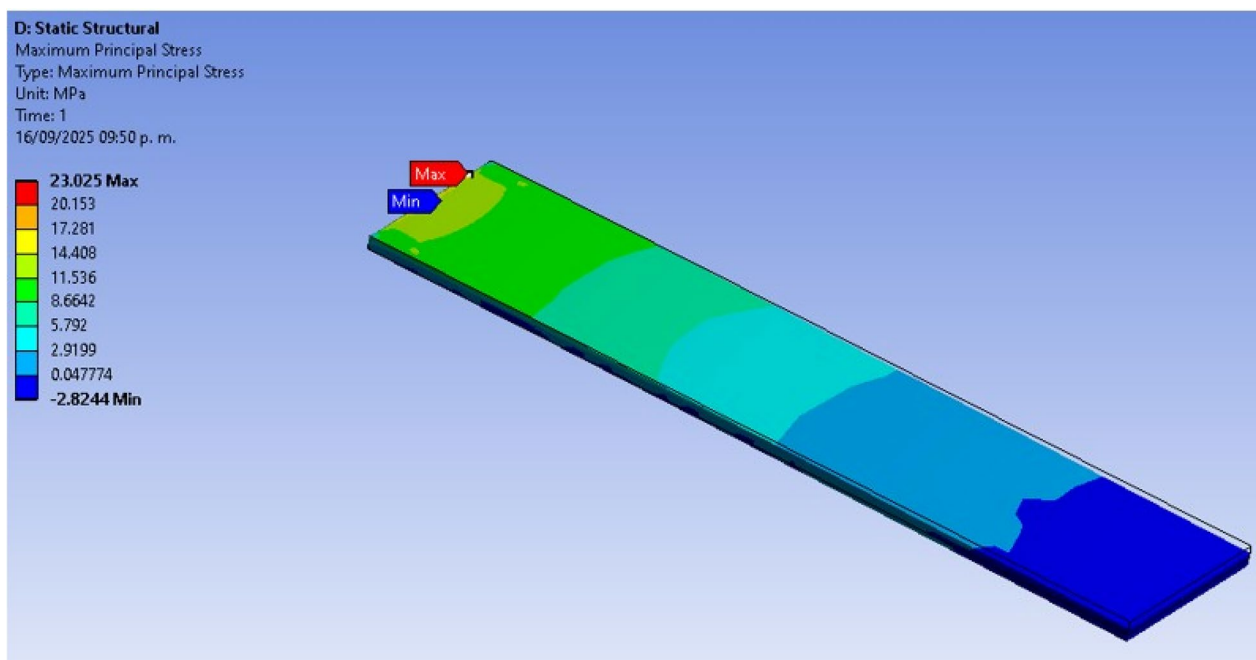


Fig. 9 Maximum stress for the *Quercus scytophylla* beam (with downward load)

$$D = 1 - S_N \quad (7)$$

Although they propose this for fiberglass with epoxy resin, it can also be applied to wood because it is also an orthotropic material and its properties do not change along its longitudinal axis.

Proposed model for stiffness degradation

A stiffness degradation model is proposed based on exponential degradation (the degradation rate is constant), which accelerates with the same damage, making it a dynamic model (the degradation rate increases as the material deteriorates). The evolution of damage D is described by the differential equation:

$$\frac{dD}{dN} = k_0 x^m (1 - D)(1 + \lambda D) \quad (8)$$

where D is the accumulated damage, N is the number of cycles, k_0 is the base degradation rate per cycle, $x = \frac{\sigma_{a,eq}}{MOR}$, introduces the effect of average stress and material (MOR) in a dimensionless form, m is the sensitivity to the load level, λ is the acceleration of degradation as D increases.

The term $(1 - D)$ forces the equation to zero when the damage reaches 1 (failure), and the term $(1 + \lambda D)$ accelerates the degradation as it approaches failure.

In terms of normalized stiffness $S = 1 - D$, and assuming that for $N = 0$: $S_0 = 1$, the solution is:

$$S_N = \frac{(1 + \lambda)e^{-KN}}{\lambda + (1 + \lambda)e^{-KN}} \quad (9)$$

With $K = k_0(1 + \lambda)x^m$, in cycle⁻¹.

It is important to make a few observations:

When $\lambda \rightarrow 0$, simple exponential degradation is recovered $S_N = e^{-KN}$.

When $N \rightarrow \infty$: $S \rightarrow 0$, always taking into account that $N \geq 1$.

Equation (9) is adjusted using the least squares method to obtain K and λ based on the experimental data, calculating the respective R^2 for each wooden beam. It should be noted that the adjustment is made for the data from which fatigue damage was recorded, i.e., the first point is left out, since there $N = 0$ and $S_N = 1$.

Finally, an estimate can be made of the number of cycles at which the beams will fail, using a failure threshold. In this study, the failure criterion adopted is a threshold of $S_{crit} = 0.70$, i.e., a loss of 30% of the

Table 8 Summary of the stresses acting on the beams

Wood	σ_{max} (MPa)	σ_{min} (MPa)	σ_m (MPa)	σ_a (MPa)	R (-)
<i>Pinus pseudostrobus</i>	25.548	20.829	23.188	2.359	0.81
<i>Quercus scytophylla</i>	23.025	15.978	19.505	3.523	0.69
<i>Cedrela odorata</i>	21.938	16.137	19.037	2.900	0.73

Table 9 Fatigue test results for *Pinus pseudostrobus*

<i>Pinus pseudostrobus</i>						
Time (h)	Number of cycles (N)	Frequency (Hz)	Experimental relative stiffness Sexp (–)	Damage (–)	Humidity (%)	Ambient temperature (°C)
0	0	15.050 ± 0.32 (4.92*)	1	0	55	22.6
100	7,200,000	14.525 ± 0.31 (4.92*)	0.931449432	0.068550568	51	23.6
172	12,384,000	14.224 ± 0.29 (4.76*)	0.893370382	0.106629618	50	24.1
268	19,296,000	13.725 ± 0.36 (6.08*)	0.831671284	0.168328716	50	23.4

* Coefficient of variation

Table 10 Fatigue test results for *Quercus scytophylla*

<i>Quercus scytophylla</i>						
Time (h)	Number of cycles (N)	Frequency (Hz)	Experimental relative stiffness Sexp (–)	Damage (–)	Humidity (%)	Ambient temperature (°C)
0	0	22.975 ± 0.28 (2.86*)	1	0	55	23.2
116	8,352,000	22.205 ± 0.29 (3.04*)	0.937883705	0.06211629	43	24.8
188	13,536,000	21.875 ± 0.25 (2.66*)	0.906536058	0.09346394	51	22.7
262	18,864,000	21.650 ± 0.23 (2.49*)	0.887983224	0.11201678	57	20.4

* Coefficient of variation

Table 11 Fatigue test results for *Cedrela odorata*

<i>Cedrela odorata</i>						
Time (h)	Number of cycles (N)	Frequency (Hz)	Experimental relative stiffness Sexp (–)	Damage (–)	Humidity (%)	Ambient temperature (°C)
0	0	20.175 ± 0.17 (2.01*)	1	0	65	23.5
106	7,632,000	18.825 ± 0.14 (1.78*)	0.870648554	0.12935145	62	22.5
154	11,088,000	18.400 ± 0.20 (2.58*)	0.831780164	0.16821984	69	24
270	19,440,000	18.15 ± 0.14 (1.80*)	0.809330993	0.19066901	60	22

* Coefficient of variation

initial stiffness (or associated natural frequency). This choice is a practical convention frequently used in fatigue degradation studies where stiffness reduction is

used as an indicator of accumulated damage (in studies on asphalt mixtures [21–24], and composite materials [25–27], reductions of 25–50% have been used as an

Table 12 Coefficients for each beam

Wood	K (cycle ⁻¹)	λ (-)	R^2
<i>Pinus pseudostrobus</i>	8.129393×10^{-8}	0.044269	0.998
<i>Quercus scytophylla</i>	5.692122×10^{-8}	0.046205	0.950
<i>Cedrela odorata</i>	3.306659×10^{-8}	0.144869	0.835

Table 13 Results for stiffness calculated by the proposed model

<i>Pinus pseudostrobus</i>		<i>Quercus scytophylla</i>		<i>Cedrela odorata</i>	
N (cycles)	S_{model} (-)	N (cycles)	S_{model} (-)	N (cycles)	S_{model} (-)
7,200,000	0.929266	8,352,000	0.933667	7,632,000	0.859947
12,384,000	0.896045	13,536,000	0.912883	11,088,000	0.845609
19,296,000	0.830916	18,864,000	0.885551	19,440,000	0.806027

operational failure criterion). Given that there is no single universally accepted threshold in the literature on wood, $S_{\text{Crit}} = 0.70$ is adopted as a consistent and conservative criterion for comparing species and obtaining relative lifespans.

To calculate the number of cycles to failure, it is assumed that the fatigue life N_f is reached when the stiffness falls to a critical value S_{crit} , thus arriving at the following equation:

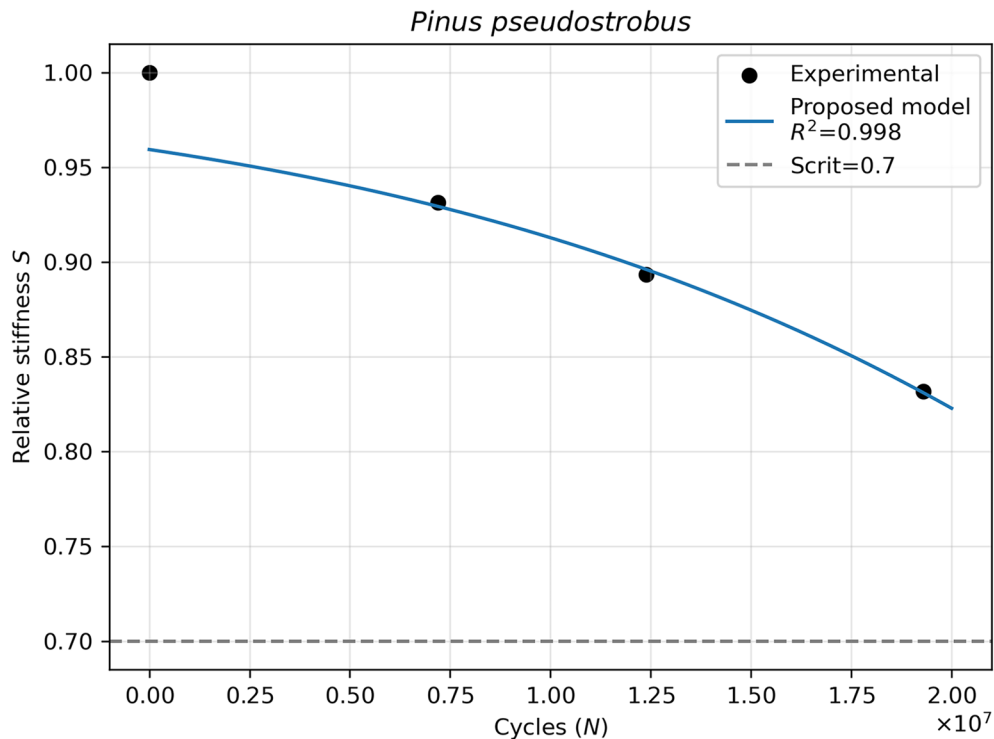
$$N_f = -\frac{1}{K} \ln \left(\frac{S_{\text{crit}} \lambda}{(1 + \lambda)(1 - S_{\text{crit}})} \right) \quad (10)$$

Results and discussion

Modal analysis using FEM simulation

Table 6 shows the results for the first natural frequency obtained by numerical simulation for the three types of wood.

Numerical simulation is extremely important when conducting laboratory tests to determine what values to expect when performing modal testing, but more importantly, it provides us with the way in which the beam vibrates at this frequency (known as modal shape) and allows us to reproduce it in the laboratory in a more optimal way. In this case, for the three wooden beams, it is a frequency that flexes the beam along its transverse axis, as shown in Fig. 8 for *Cedrela odorata*.

**Fig. 10** Correlation between the experimental data and those obtained by the proposed model for *Pinus pseudostrobus*

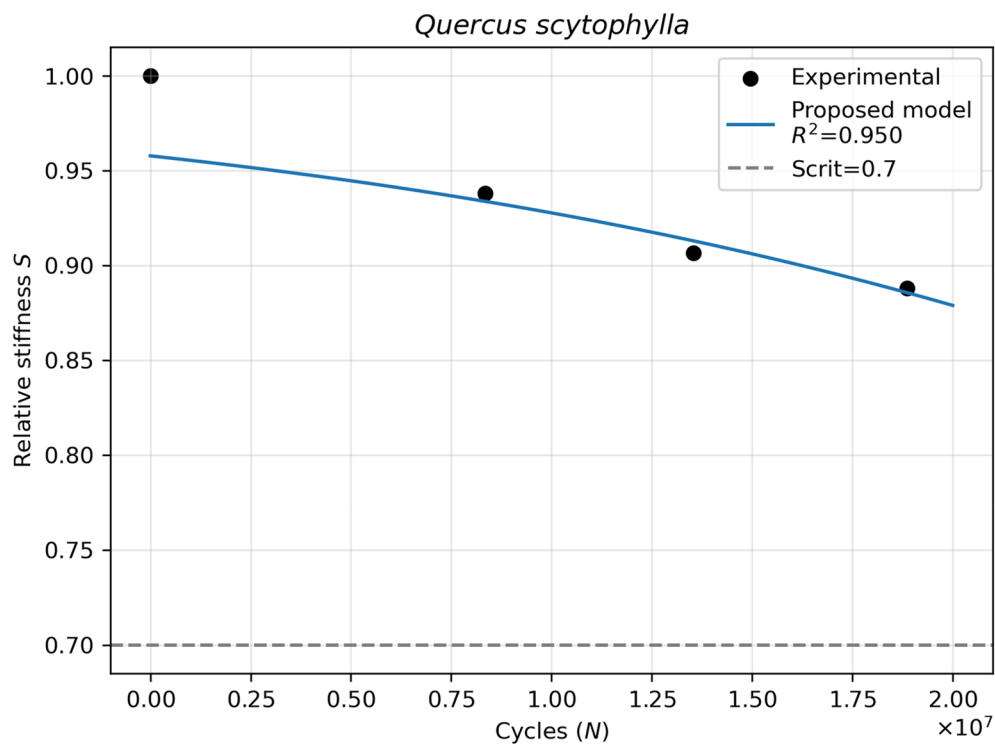


Fig. 11 Correlation between experimental data and data obtained by the proposed model for *Quercus scytophylla*

Table 14 Number of cycles to failure

Wood	S_{crit} (-)	N_f (cycles)
<i>Pinus pseudostrobus</i>	0.70	2.84×10^7
<i>Quercus scytophylla</i>	0.70	3.99×10^7
<i>Cedrela odorata</i>	0.70	3.68×10^7

Modal analysis in the laboratory

Now that we know the value of the first natural frequency through numerical simulation and its modal shape, we proceed to reproduce this vibration in the laboratory in order to obtain the first frequency experimentally. Using the methodology shown in Fig. 4, modal tests are performed on the wooden beams, obtaining the results shown in Table 7.

Stresses acting on the beams

The average and alternating stress to which the beams are subjected during fatigue testing is obtained by numerical simulation. Figure 9 shows the result for the maximum principal stress in the *Quercus scytophylla* beam. Table 8 summarizes the results for each beam.

Fatigue tests

Fatigue testing is initiated using the methodology described in Fig. 7, with humidity and temperature recorded in each modal analysis. The first hours recorded after the start of testing are the working hours during which there was already a considerable change in the natural frequency of the beam. A total of three modal tests were performed for each beam (not counting the initial one). In each modal test, 20 repetitions were performed, thus obtaining an average for the natural frequency. The coefficient of variation was also calculated, which tells us whether there was repeatability in an experiment or how homogeneous the measurements are [28].

The results obtained experimentally for each beam are shown in Table 9 for *Pinus pseudostrobus*, Table 10 for *Quercus scytophylla* and Table 11 for *Cedrela odorata*, bearing in mind that Eq. (6) is used for S_{exp} and Eq. (7) for damage.

Results for the proposed model

As mentioned above, a least squares adjustment was performed to obtain the values of K and λ , as well as their respective R^2 using the experimental data of N and S_{exp} as a basis, where damage due to fatigue already exists, i.e.,

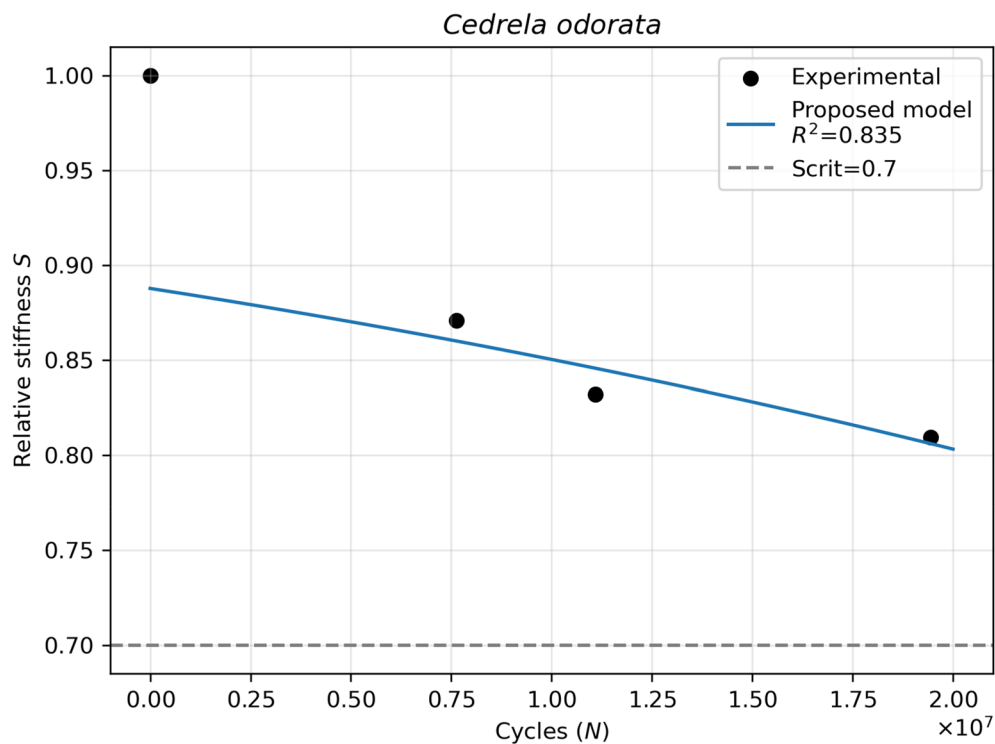


Fig. 12 Correlation between experimental data and data obtained by the proposed model for *Cedrela odorata*

the starting point was not considered since the fatigue test had not yet begun. Table 12 below shows the results obtained for each beam.

Once the coefficients have been obtained, Eq. (9) is applied to obtain the $S(N)$ predicted by the model. Table 13 shows the results obtained for each number of cycles N , bearing in mind that $N \geq 1$.

The results for *Pinus pseudostrobus* are shown in Fig. 10, in Fig. 11 for *Quercus scytophylla* and in Fig. 12 for *Cedrela odorata* in graph form to better illustrate the correlation between the relative stiffness obtained experimentally and that calculated by the proposed model.

Finally, the fatigue life is calculated using Eq. (10), as shown in Table 14 for the three beams (Table 14).

Conclusions

A new model for very high cycle fatigue (10^7) was proposed for stiffness degradation based on the measurement of the first natural frequency for wooden beams and validated using experimental data. A good correlation was observed between the data obtained experimentally and those predicted by the model from 7 million cycles onwards, obtaining a correlation factor (R^2) of 0.998 for the *Pinus pseudostrobus* beam, 0.950 for the *Quercus scytophylla* beam, and the lowest for the *Cedrela odorata* beam at 0.835.

The adjusted parameters K and λ describe a smooth initial degradation that accelerates towards the end, which can be seen most clearly in *Pinus pseudostrobus*.

The model reproduces the observed degradation with good accuracy and allows the life to be interpolated/extrapolated from a stiffness threshold, thus calculating the fatigue life of the three beams with a 30% stiffness loss failure criterion, i.e., when the relative stiffness falls to 70% ($S_{\text{crit}} = 0.70$).

A decrease in experimental relative stiffness was observed compared to the first value obtained of 16.83% after 19,296,000 cycles for *Pinus pseudostrobus* at an average stress of 23.1885 MPa; 19.06% for *Cedrela odorata* after 19,440,000 cycles at an average stress of 19.0375 MPa; and finally, 11.20% after 18,864,000 cycles for *Quercus scytophylla* at an average stress of 15.5015 MPa.

As can be seen, all beams reached more than 10^7 working cycles. Despite this, the damage obtained is relatively low, with a maximum value of 0.19 for *Cedrela odorata*, and the lowest value obtained for *Quercus scytophylla* with just 0.08. This indicates that the weight of the eccentric in the excitation device must be increased in order to produce considerable damage with a lower number of working cycles. Another limitation observed during the course of the experiment is the accuracy of the

accelerometer, although the idea arose from the need for an economical method of performing modal analysis in the laboratory on structures such as beams, floor slabs, wind turbine blades, etc.

As far as model validation is concerned, further testing at different load levels is necessary to corroborate its behavior, as well as to carry out tests to failure in order to obtain more critical data.

Abbreviations

CAD	Computer-aided design
FEM	Finite element method
FFT	Fast Fourier transformation
MOR	Modulus of rupture
VHCF	Very high cycle fatigue

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Author contributions

Adrian Santibañez conceived and designed the experiments, performed the experiments, analyzed the data, performed the statistical analysis, prepared the figures, performed the computer simulations, and wrote the article. Victor López conceived and designed the experiments, reviewed drafts of the article, and approved the final draft. Juan Felipe Soriano reviewed drafts of the article, and approved the final draft. Georgina Carbajal reviewed drafts of the article, and approved the final draft. Tizoc López conceived and designed the experiments, performed the experiments, performed the programming in LabView, reviewed drafts of the article, and approved the final draft. Carlos Arana prepared the tables and reviewed drafts of the article.

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Availability of data and materials

The datasets used and analyzed in the current study are available from the corresponding author upon reasonable request.

Declarations

Ethics approval and consent to participate

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Consent for publication

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Competing interests

The authors declare that they have no competing interests.

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