Environmental and Resource Economics (2025) 88:1297–1337
https://doi.org/10.1007/s10640-025-00966-2

When Climate Change Turns Good Plant Bad–A Dynamic Multispecies Model of Reindeer Herding in a Changing Arctic

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Accepted: 3 February 2025 / Published online: 3 March 2025
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Abstract
In many Nordic countries, climate-induced encroachment of the native and allelopathic *Empetrum nigrum* (crowberry) on other palatable vegetation decreases pasture quality for the culturally important reindeer herds. Unfortunately, current pasture management plans do not include pasture quality indicators. To argue for the importance of pasture quality and provide adaptation guidance, we develop a three-species bioeconomic model (reindeer-vegetation-crowberry) with three controls (reindeer slaughter, supplementary feeding of reindeer, and crowberry control). Our model indicates that without human intervention, vegetation biomass and reindeer herd size will decline due to crowberry's allelopathic effects, resulting in 42% loss in reindeer stock and 40% (100 million NOK) profit loss. Human interventions, specifically supplementary feeding and crowberry control, reduces the profit loss by half. To incentivize these interventions, which are partially public goods, the paper proposes a conditional subsidy program that only compensates herders for crowberry control when the shadow value of crowberry becomes negative. This approach can be generalized to other regions experiencing similar ecological changes, in which habitat quality is a crucial yet often overlooked factor.

Keywords Allelopathic Encroachment · Bioeconomic Modelling · Reindeer Husbandry · Supplementary Feeding · *Empetrum nigrum*

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1 Introduction

The Arctic's terrestrial ecosystems are changing (Ims et al. 2013), leading to impacts on the culturally important reindeer husbandry of indigenous Sami people in northern Europe (Bråthen et al. 2018; Heggenes et al. 2017; Tuomi et al. 2024). Due to climate and environmental changes, an evergreen dwarf shrub, *Empetrum nigrum* (crowberry), is encroaching on more palatable vegetation via excretion of several allelopathic substances (e.g., batatasin-III) through its leaf litter and humus, in turn suppressing growth of other plants (González et al. 2021; Pilsbacher et al. 2020). Despite being a native species, crowberry is increasingly considered a pest because this allelopathic encroachment reduces pasture biodiversity (Bråthen et al. 2024; Bråthen and Ravolainen 2015) and contributes to grazing habitat degradation (Bråthen et al. 2024; Tybirk et al. 2000). A recent study in Northern Norway indicates that pasture land in which crowberry encroaches has increased from 76 to 83% over the last two decades (Tuomi et al. 2024). Private control of crowberry by individual herders is hampered by the fact that costly control actions are a public good while government agencies do not focus on pasture quality. This paper develops and applies a three-species bioeconomic model to identify optimal adaptation strategies in response to a previously overlooked crisis of the reindeer grazing pasture– the deteriorating pasture quality caused by the encroachment of the native pest crowberry.

Reindeer husbandry is a traditional lifestyle of the Sami people (Johnsen et al. 2022; Reinert et al. 2009), and the pasture plays a decisive part in income generated from reindeer herding (Krebs 2002; Tveraa et al. 2013). In Norway, the traditional Sami herding areas are systematically organized into 89 distinct herding districts, grouped into 6 regions (Ministry of Agriculture and Food 2017), with more than 200 000 reindeer across the country (Fig. 1). Herded using a semi-domesticated approach, the reindeer are allowed to graze freely in a free-range pasture. Sustainable development focused on managing the population to prevent the "tragedy of the commons" on grazing pastures has gained emphasis in recent years (Ministry of Agriculture and Food 2017). A suitable maximum limit for stock size is established for each herding district by the Directorate of Agriculture, based on pasture quantity. Although the concern for decreasing pasture quality has been mentioned in many reports (Eira et al. 2020; Oskal et al. 2022; Wengen 2023), the governmental management framework has neither adopted any clear-cut quantitative indicator for measuring pasture quality, nor estimated how biodiversity deterioration negatively impacts the husbandry (Ministry of Agriculture and Food 2008; Reinert et al. 2009; Tuomi et al. 2024). With the additional pressure from climate change, the presumed driver of crowberry encroachment, homogenization of the pasture by crowberry will decrease the maximum herd size that can be supported on the pasture.

The herders must adapt to the new ecological and climate change consequences. In addition to the usual control of reindeer dynamics– slaughter– we investigate the efficiency of the two adaptation measures– crowberry burning and supplementary feeding. Habitat restoration is a common human intervention in species conservation (Ando and Langpap 2018; Bulte and Horan 2003). Controlling crowberry, either by burning, plowing, or plucking, can be an adaptative measure. Based on previous literature on the effect of fire to control *Empetrum* on *Empetrum*-dominated understorey vegetation (Nilsson and Wardle 2005; Wilson et al. 2003), we incorporate crowberry burning in our model to investigate the impacts on the reindeer stock and grazing pasture of removing the native pest.

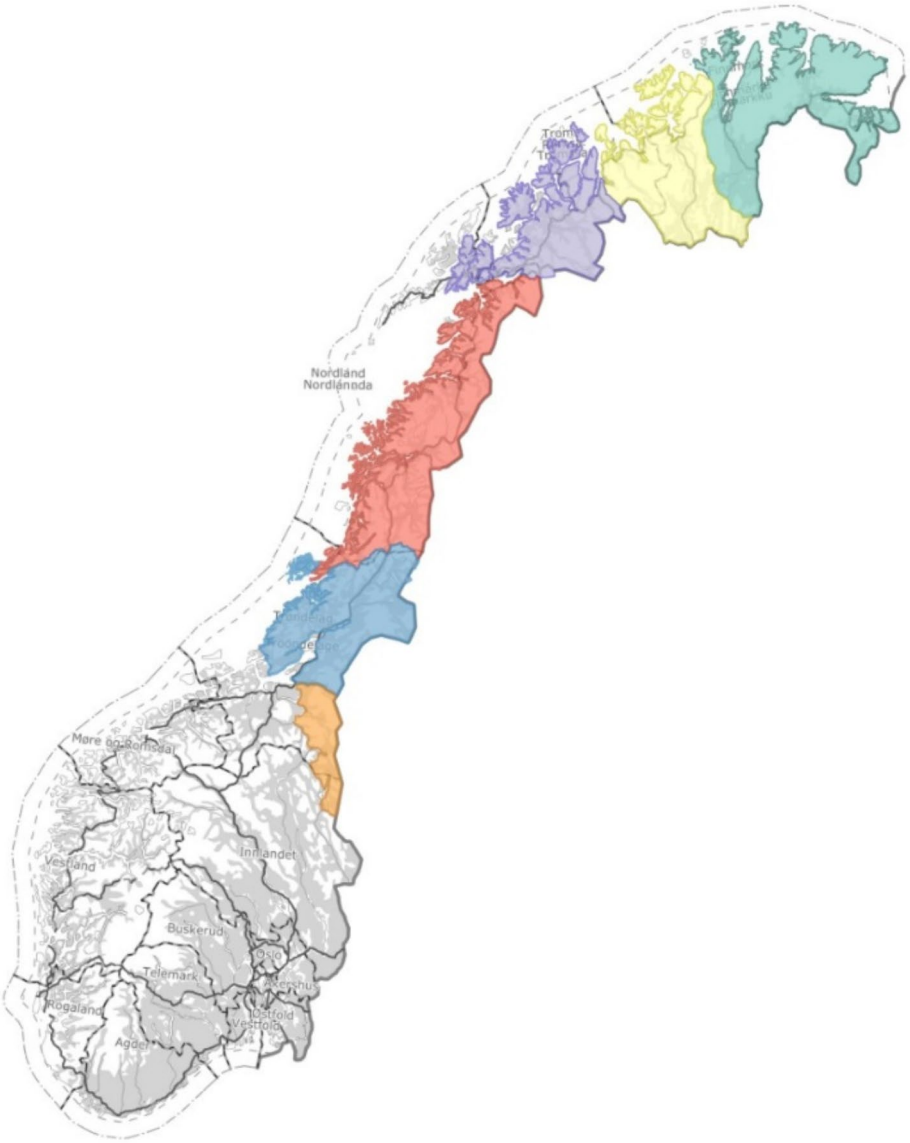


Fig. 1 Total defined grazing pasture of the Norwegian reindeer husbandry. Names of the regions from north to south: East Finnmark (green), West Finnmark (yellow), Troms (purple), Nordland (red), North -Trøndelag (blue), and South Trøndelag and Hedmark (orange).
Source: Agriculture Directorate (2022b)

Rather than restoring the habitat, many herders have been adapting to the new environmental situation by providing supplementary feeding (Helle and Jaakkola 2008; Horstkotte et al. 2020).[1] Despite ongoing supplementary feeding, there exist no economic studies of this intervention in the Norwegian reindeer husbandry setting. Supplementary feeding in the Finnish reindeer setting was modelled extensively in the studies of Pekkarinen et al. (2015, 2021). In the Finnish setting, supplementary feeding is a common practice, regardless of the pasture condition. Thus, energy from supplementary feeding does limit the vegetation intake of reindeer from the pasture in the Finnish setting. Contrary to this, supplementary feeding is conducted in Norway only during harsh winters when the animals cannot find food naturally. Norwegian reindeer energy intake from vegetation, then, is not limited by the manual feeding practice.

There exist very few economic studies of how the changes in pasture quality impact reindeer husbandry. Some papers have discussed the impact of climate change on the socio-ecological system in Northern Europe, yet with limited economic analysis (Rees et al. 2008; Reinert et al. 2009). Both Rees et al. (2008) and Reinert et al. (2009) emphasize how weather variation, especially snow quality, will put pressure on the grazing pasture and reindeer stock size. These papers are limited to qualitative analyses of subsidies or governmental management without considering adaptive strategies to mitigate climate change impacts. Other studies of reindeer husbandry show that both ecological and economic dynamics matter (Johannesen et al. 2019; Moxnes et al. 2001; Pekkarinen 2018). Existing models have incorporated how climate change influences reindeer husbandry without acknowledging the important role of the grazing pasture (Helgesen and Johannesen 2023). Those studies that do consider grazing focus mainly on deteriorating winter pasture quality (Pekkarinen et al. 2021, 2022).[2] None of these above-mentioned studies have discussed the problem of native pests in reindeer husbandry.

Our three-species predator-prey bioeconomic model focuses on how both ecological change in the tundra ecosystem (allelopathic species encroachment) and human interventions (slaughtering, restoring grazing pasture, and feeding the reindeer stock) impact the herd size and grazing pasture. To investigate the explicit ecological and economic impacts of crowberry encroachment on reindeer husbandry, we assume the grazing pasture consists of two plant types: (1) a combination of biodiverse nutrient-rich plants that are treated as a single species (in which lichen is included), and (2) crowberry, a native pest with low palatability. The allelopathic impact of crowberry will increase its competitiveness over the palatable vegetation leading to decreases in the palatable vegetation and the reindeer stocks, while the two adaptation measures will dampen or reverse the impacts of crowberry. A resource manager charged with managing the reindeer husbandry chooses levels of slaughter, feeding and crowberry treatment to maximize the profit from slaughtering and the cultural values of the reindeer herd over time.

Our paper provides both methodological and empirical contributions. We provide the first model of the economic implications of native species encroachment on grazing pas-

[1] Feeding modifies the natural habitat and can detrimentally alter the herding customs of the Sami (Helle and Jaakkola 2008; Horstkotte et al. 2020). Though supplementary feeding is unfavourable to the Sami, the Norwegian herders are increasingly forced to provide dry feed in harsh winter conditions due to climate change.

[2] Current literature on reindeer husbandry dynamics focuses on lichen, a class of fungal-algal symbiotic organism, as an important factor (Pekkarinen 2018; Tahvonen et al. 2014). However, since reindeer only graze on lichen in the winter (Aagnes et al. 1996; Reinert et al. 2009), modelling solely lichen provides an incomplete picture of pasture quality.

ture dynamics. We introduce a novel approach by dynamically quantifying the interactive carrying capacities of two competing plant species. The results of the model highlight a previously under-recognized implication of climate change– native pest encroachment that deteriorates grazing habitat. While other papers have modeled reindeer grazing pasture dynamics in the context of invasive species and general weather effects, ours is the first to account for allelopathic relationships in a bioeconomic model for the reindeer husbandry. Our study examines whether management strategies aimed at controlling a native pest for ecological reasons can also be economically motivated. This dual motivation, if present, could offer an additional rationale for improving pasture quality through such actions.

By applying the model to the Norwegian reindeer husbandry, we also provide three key insights into the management of this economically and culturally valuable natural resource. First, we provide the first measures of the economic implications of crowberry encroachment. Second, we use our model to provide recommendations for the use of two climate change adaptation measures that can be utilized by reindeer managers to alleviate these impacts. We show that there are ecological and economic benefits when both adaptation measures are used together suggesting that supplementary feeding could be considered as a long-term solution. Third, we propose a conditional subsidy program that only compensates herders for crowberry control when the shadow value of crowberry becomes negative. These results provide support for adopting a new bioeconomic metric for pasture quality that goes beyond simply measuring crowberry biomass.

We present our model in the next section. The model consists of three species-specific state variables and three controls. We then apply our model to the Norwegian husbandry. In the numerical application section, we consider three scenarios: (1) pre-encroachment (pre-EC; optimal slaughter with no allelopathic impact), (2) post-encroachment (post-EC; optimal slaughter with allelopathic encroachment but not adaptation), and (3) adaptive post-EC, which considers optimal slaughter in conjunction with reindeer feeding and crowberry control. The paper ends with a discussion of the implications of our findings and a conclusion section.

2 Model of Reindeer Management Under Crowberry Encroachment

Our three-species system consists of one herbivore, the reindeer stock denoted R (number of reindeer), and two plant species, vegetation stock $\Delta\ \boldsymbol{R}^{*}$ and crowberry stock K (kilogram area^{-1}).[3] A resource manager must choose how many reindeer to slaughter to maximize profit and cultural values created by the reindeer herd. The slaughtering decision is dynamic since reindeer reproduction, like any other renewable resource, creates an opportunity cost of slaughtering reindeer today. This opportunity cost tempers the incentive to slaughter the entire herd at a given time. The resource manager must also choose the intensity of two adaptation measures in response to climate-induced encroachment of crowberry: (1) supplementary feeding of the reindeer herd and (2) direct control of crowberry. In Norway, the husbandry is under the management of the Norwegian Directorate for Agriculture (Agriculture Directorate 2022a). To avoid overgrazing, the government regulates the

[3] To focus on the allelopathic interactions between the three species, we ignore age-sex structure. Including age-sex structure, while adding detail, would significantly increase the model's complexity without substantially contributing to the primary objectives of our analysis.

herd size. Hence, we consider the Norwegian government as the resource manager in our optimal control problem.

3 Ecological Dynamics

We adopt the Leslie model (Leslie 1948) which differs from the Lotka-Volterra model (Lotka 1920, 1925; Volterra 1928) in how the predator population carrying capacity depends on prey density. While growth of the predator in Lotka-Volterra models depends on the functional response between predator and prey (be it Holling type 1, 2, or 3, (see Koen-Alonso 2007), the Leslie model emphasizes the upper bound of the predator stock, i.e. its carrying capacity, is dependent on prey density (Leslie 1948). The Leslie model helps us to avoid model instability and capture more realistic interactions between the three species.

Within the non-spatial predator-prey literature, we find no studies where two species are modelled to share a common carrying capacity, in this case determined by the physical area. Given a fixed total areal carrying capacity of the pasture, crowberry encroachment must lower the carrying capacity of other palatable vegetation due to competition. We incorporate these implicit resource constraints by connecting carrying capacities of both species, thereby incorporating a spatial perspective in a non-spatial dynamic model[4]. The general ecological model is then:

$$\dot{V} = \alpha_V V\left(1 - \frac{V+\varphi K}{M}\right) - g_V V R \tag{1}$$

$$\dot{K} = \alpha_K K\left(1 - \frac{K}{M-V}\right) - g_K K R \tag{2}$$

$$\dot{R} = \alpha_R R\left(1 - \frac{R}{d_V V + d_K K}\right) \tag{3}$$

in which V, K, and R are the biomass of vegetation, crowberry, and reindeer respectively[5]. The required initial conditions for three species are: $V(t_0) > 0; K(t_0) > 0; R(t_0) > 0$. To satisfy biological feasibility, all the parameters $\alpha_V, \alpha_K, \alpha_R, \varphi, g_V, g_K, d_V, d_K \in \mathbb{R}^+$. All three species' growth follows a logistic function in which M is the pasture carrying capacity while α_V, α_K, and α_R are the intrinsic growth rates of the vegetation, crowberry and reindeer species, respectively. As crowberry grows alongside vegetation, its carrying capacity can be described as $M - V$. Grazing interactions between reindeer and the two plant stocks are described by the terms $g_V V R$ and $g_K K R$, in which g_V and g_K are the grazing coefficients. Carrying capacity of reindeer is determined by the available pasture, i.e. $d_V V + d_K K$, in which d_V and d_K are the impact rates of vegetation and crowberry upon reindeer carrying capacity, respectively. As reindeer only graze crowberry in early spring, we assume $g_K < g_V$ and $d_K < d_V$. The allelopathic impact of crowberry

[4] For spatial optimal control bioeconomic models with prey-predator interactions, see Demir and Lenhart (2021) or Lenhart and Workman (2007).

[5] The term in the denominator of mimics the spatial constraints of the two species. The same adjustment to the carrying capacity does not appear in Eq. (1), as crowberry impacts vegetation by the allelopathic rate following Li and Feng (2010). Furthermore, applying in the denominator of would needlessly complicate the system.

is captured by the term φK where φ is the allelopathic coefficient, as applied in allelopathic models mentioned by Li and Feng (2010). Appendix A shows the analytical equilibria achieved from solving the three Eqs. (1), (2), and (3).
Climate change initiates a gradual shift in the allelopathic coefficient from $\varphi = 0$ (no crowberry encroachment) to $\varphi > 0$ (crowberry encroachment) (Bråthen et al. 2024; Pilsbacher et al. 2020). One approach to modeling this transition is the fast-slow dynamics framework (Crépin 2007). This framework explores how fast changing variables, such as populations, interact with slow, and often less noticeable processes, to shift and possibly destabilize an ecosystem's steady states. Applying that framework to our model suggests the system dynamics described in Eqs. (1)–(3) (the fast system) converge to a steady state conditional on a value of φ that gradually changes over time due to climate change (the slow system). Provided that φ continues to increase due to climate change, the result is a corner solution equilibrium where crowberry encroachment forces other native vegetation off the pasture ($V = 0$). Unfortunately, the slow dynamics in this system are poorly understood because of a lack of long time series data on crowberry encroachment. Therefore, our modeling approach allows for an analysis of the system's fast dynamics as new observations of φ emerge (see Appendix B for more detailed analysis of the ecological model).

Analytically, there are seven equilibria, including two axial equilibria[6], three corner solutions and two interior equilibria (Appendix A). As we demonstrate later in the paper, a unique stable interior equilibrium exists given our chosen data. As climate change gradually increaes φ from 0, the interior equilibrium shifts to the corner solution with $V = 0$. This transition is marked by a threshold condition for φ, beyond which the system's stability switches, indicating a critical allelopathy threshold. While climate change often precludes steady state analysis, we believe this new corner solution with $V = 0$ marks a new post climate change equilibrium since the impact of climate change in our model (encroachment on vegetation) has been effectively maxed out. While climate change may continue to impact our system in other ways, the particular climate change pathway we study in this paper (allelopathic encroachment) has come to an end resulting in a new steady state.[7]

3.1 Management Actions

Crowberry encroachment impacts pasture quantity and quality, leading to the recent practice of supplementary feeding (Horstkotte et al. 2020; Turunen et al. 2016; Tveraa et al. 2013). The resource manager may turn to an alternative, such as crowberry removal by burning, plowing, or plucking. We expand the ecological model to include three controls: reindeer slaughter, supplementary feeding, and crowberry treatment.

Facing allelopathic encroachment of crowberry, the resource manager decides to either incorporate supplementary feeding F or apply crowberry treatment T, or both. Reindeer

[6] These axial equilibria represent the states where the system's dynamics are aligned along a principal axis.

[7] Climate change may also manifest as changes in climate variability causing possible changes in management to increase system resilience. In cases where environmental stochasticity is an important consideration, dynamic programming can be used to identify adaptive strategies to unpredictable weather variation (Conrad and Smith 2012; Horan et al. 2023; Sims et al. 2018). Adaptive strategies afforded by a dynamic programming approach may also be useful when considering the uncertainties associated with the slow dynamics of allelopathic encroachment.

slaughter enters the reindeer equation of change by the control S and supplementary feeding enters via the term βFR in which β is the feeding coefficient[8]. Equation (3) becomes

$$\dot{R} = \alpha_R R\left(1 - \frac{R}{d_V V + d_K K}\right) + \beta FR - S. \tag{4}$$

$$\dot{K} = \alpha_K K\left(1 - \frac{K}{M - V}\right) - g_K KR - \varepsilon TK. \tag{5}$$

Similarly, crowberry treatment T enters the model via the term εTK in which ε is the treatment coefficient:[9]

The reindeer herd generates revenue through the sale of slaughtered reindeer (at price p). Maintenance and feeding the herd is costly as is any effort directed at controlling crowberry encroachment. Reindeer herding, particularly for the indigenous Sami people, is not only an economic activity but also a bearer of profound cultural and social values (Ministry of Agriculture and Food 2017). This includes different types of non-use values, such as cultural, bequest and existence values, of the husbandry to the Sami, symbolizing a deep connection to their heritage and identity, as well as the cultural value to the nation, representing a living example of sustainable and traditional land use. Additionally, the size of a reindeer herd often carries social status implications within the community (Johannesen and Skonhoft 2011), representing non-use value of the species. We, therefore, include the non-use value of reindeer in the objective functional by adding a term linearly dependent on the reindeer population.

4 Optimal Responses to Crowberry Encroachment

Given a discount rate δ, the manager chooses to slaughter S reindeer, feeding reindeer with F tonnes of dry feed and using T days to burn crowberry. The discounted net benefit of reindeer management is maximized in infinite time:

$$max_{S,F,T} \int_0^\infty \left(pS - c_1 R - c_2 F^2 - c_3 T^2 + \gamma R\right) e^{-\delta t} dt$$

in which p is the price per slaughter reindeer, c_1 is the maintenance cost per reindeer[10] and γ is the unit non-use value per reindeer.[11] The feeding cost function $c_2 F^2$ is quadratic in feeding effort F. A quadratic cost function captures the increasing marginal cost of feed-

[8] We placed this term outside the logistic growth function of reindeer as no studies have yet examined the relationship between reindeer carrying capacity and supplementary feeding.

[9] Equation 5 assumes the effectiveness of crowberry treatment remains constant over time. Like many invasive species, the long-run effect of crowberry control strategies is unclear and more data to quantify their effectiveness is needed.

[10] The reason why we have a linear maintenance cost function is the slaughter cost is really minimal. The case of linear maintenance cost also arises in literature of livestock management (Aadland 2004; Skonhoft et al. 2010).

[11] In our model, the term reflects the non-use value per reindeer, represented in a linear function as a simple illustrative choice. However, it is conceivable that the non-use value per reindeer declines as herd size increases, implying a concave rather than linear relationship. Given the lack of data needed to parameterize the curvature of a non-use value function, we opted for a linear relationship.

ing, which arises not only from the scarcity of dry feed– a significant component of feeding costs– but also from the rising opportunity cost of manual feeding labor. We also assume that the additional controlling cost function c_3T^2 is quadratic in treatment effort T in which c_3 is the controlling cost effort. Controlling invasive or pest plant species like crowberry via picking, soil preparation, or burning is labor intensive. These characteristics of plant control suggest cost should be convex in effort and largely independent of the population of crowberry (Jardine and Sanchirico 2018; Kotani et al. 2011).[12]

The current value Hamiltonian is

$$\begin{aligned} H = pS + (\gamma - c_1)R - c_2F^2 - c_3T^2 + \lambda_R\left[\alpha_R R\left(1 - \tfrac{R}{d_V V + d_K K}\right) - S + \beta FR\right] \\ + \lambda_V\left[\alpha_V V\left(1 - \tfrac{V+\varphi K}{M}\right) - g_V VR\right] + \lambda_K\left[\alpha_K K\left(1 - \tfrac{K}{M-V}\right) - g_K KR - \varepsilon TK\right] \end{aligned} \tag{6}$$

subject to (1), (4) and (5) and the initial conditions V_0, R_0, K_0 where $V, R, S, K, F, T \in \mathbb{R}^+$. λ_R, λ_V, and λ_K are the shadow prices of reindeer, vegetation, and crowberry respectively. $\frac{\partial^2 H}{\partial S^2} = 0$ while $\frac{\partial^2 H}{\partial F^2} = -2c_2 < 0$ and $\frac{\partial^2 H}{\partial T^2} = -2c_3 < 0$, which satisfy the concavity condition of the maximization problem.

The first-order-condition (FOC) of this Hamiltonian is $\frac{\partial H}{\partial S} = p - \lambda_R = 0$ which provides $\lambda_R{}^* = p$ for the optimal S^*. Since the resource manager is a price taker, we attain a singular control variable S^* (see Appendix C for proof). The singular solution, a direct result of the linear cost function c_1R, reflects administration and maintenance costs which tend to scale linearly with herd size but do not significantly vary with harvest rates (Norwegian Government 2021). The FOC for F gives $\frac{\partial H}{\partial F} = -2c_2F + p\beta R = 0$ and thus $F^* = \frac{p\beta}{2c_2}R^*$. The intuition is that the more reindeer there are, the more feeding should take place. For a given number of reindeer, price per slaughtered reindeer, the feeding coefficient, and optimal reindeer stock will incentivize more feeding effort while the feeding cost will dis-incentivize the effort. The FOC for T is: $\frac{\partial H}{\partial T} = -2c_3T - \varepsilon\lambda_K K = 0$, thus $T^* = -\frac{\varepsilon K^*\lambda_K{}^*}{2c_3}$. As all the parameters and variables are positive real numbers, $T^* > 0$ only if the shadow price of crowberry is negative.[13] This implies that when adding one unit of crowberry into the system, the net benefit decreases by an amount of λ_K.

Three intuitive results can be drawn from the three adjoint equations:

$$\dot{\lambda_R} = \delta\lambda_R + c_1 - \gamma - \lambda_R\left(\alpha_R\left(1 - \tfrac{R}{d_V V + d_K K}\right) - \tfrac{\alpha_R R}{d_V V + d_K K} + \beta F\right) + \lambda_K g_K K + \lambda_V g_V V \tag{7}$$

$$\dot{\lambda_V} = \delta\lambda_V + \tfrac{\alpha_K K^2 \lambda_K}{(M-V)^2} - \tfrac{\alpha_R d_V R^2 \lambda_R}{(d_V V + d_K K)^2} - \lambda_V\left(-\tfrac{\alpha_V V}{M} + \alpha_V\left(1 - \tfrac{V+\varphi K}{M}\right) - g_V R\right) \tag{8}$$

$$\dot{\lambda_K} = \delta\lambda_K - \tfrac{\alpha_R d_K R^2 \lambda_R}{(d_V V + d_K K)^2} - \lambda_K\left(\alpha_K\left(1 - \tfrac{K}{M-V}\right) - \tfrac{\alpha_K K}{M-V} - g_K R - \varepsilon T\right) + \tfrac{\varphi\alpha_V V\lambda_V}{M} \tag{9}$$

First, with allelopathic encroachment of crowberry, induced by climate and environmental change, the sign of λ_K may switch from positive (pre-encroachment) to negative (post-

[12] It is possible that the cost of both adaptation measures may stochastically evolve with changing weather. For instance, the cost of feeding can increase sharply during harsh winter conditions, when the demand for dry feed escalates and herders dedicate more time to manually feeding their herd, thereby forgoing other valuable activities (Horstkotte et al. 2020; NIBIO 2020). We leave this extension for future work.

[13] If, i.e. if it is optimal to not burn crowberry, then the sign of follows Eq. (11) and switches sign when.

encroachment), making crowberry a pest (Rondeau 2001). At the dynamic equilibrium, $\dot{\lambda}_V = \dot{\lambda}_K = \dot{\lambda}_R = 0$. Solving Eq. (9) for λ_K, we get

$$\lambda_K{}^* = \frac{\frac{\alpha_R d_K \lambda_R^* R^{*2}}{(d_K K^* + d_V V^*)^2} - \frac{\alpha_V V^* \lambda_V^* \varphi}{M}}{g_K R^* + \varepsilon T^* + \delta - \alpha_K \left(1 - \frac{2K^*}{M - V^*}\right)}. \tag{10}$$

Without allelopathic encroachment, $\varphi = 0$ makes the numerator positive in all cases. For the denominator to be positive, we need the following condition $g_2 R^* + \varepsilon T^* + \delta > \alpha_K \left(1 - \frac{2K^*}{M - V^*}\right)$ when $1 - \frac{2K^*}{M - V^*} > 0$, which implies that the optimal crowberry stock lies on the LHS of its' logistic growth curve, $K^* < \frac{M - V^*}{2}$. Conversely, with $K^* > \frac{M - V^*}{2}$, the denominator in (10) will always be positive. Therefore, for the sign of $\lambda_K{}^*$ to be negative under the presence of crowberry encroachment, given the denominator is positive, we need

$$\frac{\alpha_R d_K \lambda_R^* R^{*2}}{(d_K K^* + d_V V^*)^2} - \frac{\alpha_V V^* \lambda_V^* \varphi}{M} < 0 \leftrightarrow \varphi > \frac{\alpha_R d_K M \lambda_R^* R^{*2}}{(d_K K^* + d_V V^*)^2 \left(\alpha_V V^* \lambda_V^*\right)}. \tag{11}$$

Hence, if the allelopathic coefficient surpasses the threshold of $\bar{\phi} = \frac{\alpha_R d_K M \lambda_R^* R^{*2}}{(d_K K^* + d_V V^*)^2 \left(\alpha_V V^* \lambda_V^*\right)}$, the native species becomes a pest from the perspective of the resource manager. This switch in the sign of λ_K is later confirmed in the numerical results (Table 1).

Second, vegetation becomes more valuable when crowberry becomes a pest. Solving the adjoint equation of V, we get

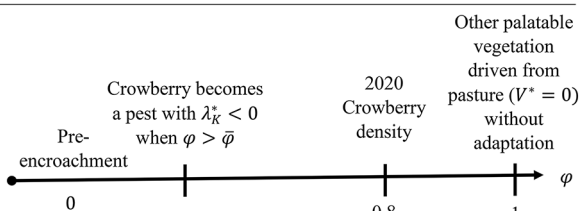
$$\lambda_V{}^* = \frac{M\left(\frac{\alpha_R d_K p R^{*2}}{(d_K K^* + d_V V^*)^2} + \left(\alpha_K - g_K R^* - \frac{2\alpha_K K^*}{M - V^*} - \delta - \varepsilon T^*\right)\lambda_K{}^*\right)}{\alpha_V V^* \varphi}. \tag{12}$$

Since all parameters and variables are positive real numbers, the denominator is positive. Early in the encroachment process when $\varphi < \bar{\varphi}$ and $\lambda_K{}^* > 0$, an increase in the shadow value of crowberry should decrease the shadow value of vegetation due to the competition between the species. This intuitive result will arise provided the second parenthetical term in the numerator is negative (i.e., the resource manager's discount rate is sufficiently low: $\delta < \alpha_K - g_K R^* - \frac{2\alpha_K K^*}{M - V^*} - \varepsilon T^*$). As encroachment continues and $\lambda_K{}^*$ switches from positive to negative, the shadow value of vegetation continues to increase, signaling its increased value as the only beneficial source of forage. This tradeoffs between the shadow values of the two forage species becomes more pronounced the lower the resource manager's discount rate.

Third, the presence of a native pest that deteriorates pasture quality creates an additional economic benefit of supplementary reindeer feeding. The presence of a native pest that deteriorates pasture quality creates an additional economic benefit of supplementary reindeer feeding. Solving each of the three adjoint equations for δ, we obtain three rate of return equations that characterize the optimal steady state (Clark 2010):

$$\delta = \left[\alpha_R \left(1 - \frac{2R^*}{d_K K^* + d_V V^*}\right) + \beta F^*\right] - \frac{c_1 - \gamma - \frac{2 g_K c_3 T^*}{\varepsilon} + \lambda_V^* g_V V^*}{p} \tag{13}$$

Table 1 Saddle point equilibrium for the fast dynamics of the bioeconomic model as the allelopathy parameter gradually changes due to climate change



Variable		Pre-EC	Post-EC		Adaptive Post-EC
V^*	Optimal vegetation population	6.73 x 10^{10}	4.07 x 10^{10}	0	5.4 x 10^{10}
R^*	Optimal reindeer stock	283 572	163 722	8220	251 301
S^*	Optimal slaughter volume	156 897	92 081	4395	148 000
K^*	Optimal crowberry population	1.93 x 10^{10}	4.6 x 10^{10}	8.7 x 10^{10}	2.3 x 10^{10}
F^*	Optimal feeding effort	--	--	--	63
T^*	Optimal burning effort	--	--	--	73.1
$\lambda_R{}^*$	Optimal reindeer's shadow price	3760	3760	3760	3760
$\lambda_V{}^*$	Optimal vegetation's shadow price	0.005	0.013	0.06	0.011
$\lambda_K{}^*$	Optimal crowberry's shadow price	0.0003	- 0.012	0.0002	- 0.016
$\bar{\varphi}$	Allelopathic threshold	0.024	0.011	--[17]	0.014
Profit (mill NOK)		249.7	149.8	6.7	220.4

$$\delta = \left[\alpha_V \left(1 - \frac{2V^* + \varphi K^*}{M}\right) - g_V R^*\right] + \frac{\frac{\alpha_R d_V R^{*2} p}{(d_K K^* + d_V V^*)^2} + \frac{2c_3 \alpha_K K^* T^*}{\varepsilon (M - V^*)^2}}{\lambda_V{}^*} \tag{14}$$

$$\delta = \left[\alpha_K \left(1 - \frac{2K^*}{M - V^*}\right) - g_K R^* - \varepsilon T^*\right] + \frac{\frac{\alpha_R d_K R^{*2} p}{(d_K K^* + d_V V^*)^2} - \frac{\varphi \alpha_V V^* \lambda_V^*}{M}}{\left(-\frac{2c_3 T^*}{\varepsilon K^*}\right)}. \tag{15}$$

The right-hand sides (RHS) of these three equations indicate the marginal return generated by holding more reindeer, vegetation, or crowberry stocks. Regarding the optimal management for reindeer, Eq. (13) emphasizes how supplementary feeding will increase reindeer's marginal growth. The reindeer's marginal return is increased with the marginal grazing rate of reindeer on crowberry. λ_V^* will also change as it is a function of the state variables (Eq. (10)). Therefore, the magnitude of F^* and λ_V^*, will determine the slaughter decision. While higher maintenance cost c_1 leads to an optimal smaller herd, higher crowberry controlling cost c_3 and reindeer non-use value help increase R^*. This is in line with the study of Johannessen et al. (2011) in which higher non-use value will increase the herd size and lower the slaughter rate. Moreover, when the discount rate δ increases, the manager will keep a lower stock level for other alternative investments, and vice versa. If the marginal return from reindeer is lower than the discount rate, the resource manager will slaughter more reindeer and invest the proceeds elsewhere in the economy with a higher rate of return. In the opposite case, the resource manager will conserve more reindeer, postponing harvest,

since reindeer conservation earns a higher rate of return than elsewhere in the economy. Sensitivity analysis of the numerical solutions later confirms these findings (see Appendix D).

Derived from (13), when the maintenance cost exceeds a critical threshold

$$c_1 > \gamma + (\alpha_R + \beta F^* - \delta) p + \frac{2c_3 T^*}{\varepsilon} \left[g_K + \frac{g_V M \left(\alpha_K + \frac{2\alpha_K K^*}{V - M} - \delta - \varepsilon T^* \right)}{\alpha_V \varepsilon \varphi K^*} \right] \tag{16}$$

Equation (13) can no longer hold with equality for any positive reindeer stock.

The required rate of return from vegetation includes vegetation marginal growth minus the marginal grazing rate from reindeer $g_V R^*$ plus the marginal growth of crowberry and reindeer constrained by their carrying capacities. As λ_V is the marginal increase in the objective function with respect to vegetation, an increase in its shadow price will decrease its required rate of return. This in turn stimulates the resource manager to let reindeer graze more on the pasture as it will decrease vegetation's marginal production, all other factors unchanged.

Regarding the marginal return of crowberry, Eq. (15) implies that crowberry's expected rate of return decreases by marginal constrained growth of reindeer and marginal treatment. Overall, while feeding effort will shift the optimal reindeer stock towards the RHS of the logistic growth curve as we have higher R^*, controlling effort will push the optimal crowberry population K^* towards the LHS of its logistic growth curve.

These dynamic relationships uncover an unintended benefit of feeding effort. Appendix E and F give details of an application of the implicit function theorem to the three equilibrium conditions (13), (14), and (15). While we need specific conditions to know the signs of several implicit functions, we have two certain results; $\frac{\partial R^*}{\partial F^*} > 0$ and $\frac{\partial K^*}{\partial R^*} < 0$ (see appendix E), from which we can deduce $\frac{\partial K^*}{\partial F^*} = \frac{\partial K^*}{\partial R^*} \frac{\partial R^*}{\partial F^*} < 0$. This emphasizes that feeding reindeer indirectly controls the crowberry population, holding all other variables unchanged, since feeding increases the reindeer herd and the reindeer herd grazes on crowberry.

Our model allows for a variety of complex dynamics including various types of cycles. Due to the complex interactions between the multiple state and control variables, the system dynamics and stability will depend on the values of the ecological and economic parameters. In the next section we investigate the dynamics and stability of the ecological system and bioeconomic equilibria using a numerical application of the model to Norwegian reindeer husbandry.

5 Numerical Application

We apply our model to the grazing pasture of the Norwegian reindeer husbandry which is located in the northern part of the country (Fig. 1). We outline two cases to illustrate how climate and environmental changes facilitate crowberry encroachment: pre-encroachment with no allelopathic impact (pre-EC, $\varphi = 0$) and post-encroachment with allelopathic impact (post-EC. $\varphi > 0$). Ecological parameters are presented in Appendix A and chosen to (1) be consistent with existing ecological studies, and (2) emphasize how encroachment will impact the reindeer herd and grazing pasture.

Intrinsic growth rates of vegetation α_V and reindeer α_R were adopted from papers modelling the same ecosystem (Moxnes 1993; Skonhoft et al. 2010). α_V is 0.5 for pre-EC and 0.6 for post-EC cases, due to the arctic greening phenomenon (Myers-Smith et al. 2020; Tuomi et al. 2024), and α_R is equal to 0.7 in both cases (Moxnes 1993; Skonhoft et al. 2010). Nutrition-rich plants develop palatable leaves and grow faster than nutrition-poor plants, forming unpalatable leaves for energy conservation purposes (Freschet et al. 2010; Wright et al. 2004). Since crowberry grows more slowly than other palatable plants (Hortipedia 2022; Zverev et al. 2008), α_K was set as one-fifth of α_V in the pre-encroachment case and one-third of α_V in the post-encroachment case. Based on a study of plant biomass and species richness in northern Norway, we computed the carrying capacity of vegetation and crowberry biomass as 1.2 kg per m^2 multiplied by estimates of the current total area of the grazing pasture (73 500 km^2)[14,15] (Bråthen and Lortie 2015).

We tuned the grazing rates g_V and g_K and the comparative coefficients of vegetation d_V and crowberry d_K to reflect differences in reindeer dietary preferences as it only grazes *E. nigrum* in the end of winter when other palatable plants have not regrown (Danell et al. 1994) and at the onset of the growing season when leaf buds are palatable (Iversen et al. 2014). One reindeer consumes between 1 and 10 tonnes of biomass per year, equivalent to $g_V V$ amount of vegetation biomass (Bakka et al. 2021; White and Trudell 1980). We choose the value of the parameter g_V to be 4×10^{-7} as the magnitude of V is 10^{10}. g_K is then chosen to be 1/50 of g_V because reindeer only consume this evergreen species in the last winter week when other vegetations have not regrown. Collectively, these four parameter values produce a reindeer stock size that is close to the current annual population of more than 200 000 reindeer (Agriculture Directorate 2022b; Norwegian Government 2021). We set the marginal impact of crowberry encroachment via seedling recruitment and development (Bråthen and Lortie 2015; González et al. 2015; Pilsbacher et al. 2020) to $\varphi = 0$ in the pre-encroachment case. In the post-EC scenario we fit $\varphi = 0.8$ from the available data of changes in crowberry density from 2003 to 2020 from Tuomi et al. (2024). Initial condition of $K(t_0)$ is 30% of M, thus $V(t_0) = 0.7M$. The value for $R(t_0)$ is set at 250 000 following recent governmental data (Agriculture Directorate 2009).

Economic parameters are reported in Appendix A and are chosen to be consistent with current economic conditions in Norway. By dividing the reported total product-based income and total cost by the total number of reindeer in 2020, we computed the unit maintenance cost per reindeer c_1 and unit price per slaughtered reindeer p (Norwegian Government 2021). Government subsidies and economic compensation for lost reindeer from predation and accidents (traffic, etc.) are not included. Based on communications with herders, the average cost of dry feed is 6 NOK per kilogram. We measure our feeding efforts in thou-

[14] The total defined grazing pasture of the Norwegian reindeer husbandry is approximately 40% of the whole country area, which accounts for 147 000 km^2 (Agriculture Directorate 2022b). However, this defined area does not take into account the current human-induced and environmental stress on the pasture (Riseth and Tømmervik 2017; Stoessel et al. 2022; van Rooij et al. 2023). These stresses have accumulated through decades, shrinking the defined pasture area (Ministry of Agriculture and Food 2017; NORUT 2018). One report of the Kvalsund municipality in Finnmark county– where the husbandry plays a culturally significant role– calculated that the actual grazing area is only 54% of the defined area (Eira et al. 2020). Therefore, to acknowledge different stress factors, we assume that the actual grazing area is only 50% of the defined area.

[15] Further analysis on pasture size variations and their impact on allelopathic encroachment intensity revealed that the magnitude of crowberry's negative effects on vegetation remains consistent across different pasture sizes (Bråthen et al. 2018). Additionally, when adjusting for pasture scale by reducing carrying capacities, the required effort for crowberry treatment correspondingly decreases in a proportional manner.

sands of tonnes of dry feed on the national scale, resulting in a unit cost c_2 of 6000 NOK per tonne. Since burning crowberry is still being tested, the cost of an effective burning program is unclear. Thus, we decided to use 2000 NOK for the burning effort, as it approximates the salary earned in one normal working day, given the effort unit is labour day. Both effort parameter values, ε and β, are the marginal percentage of change of crowberry and reindeer by burning and feeding respectively. We use Mathematica 13.2 to calculate the results and draw the figures. Finally, to focus more on the allelopathic impact of crowberry, we set the non-use value per reindeer $\gamma = 0$ for the calculations in the result section. Alternatively, $\gamma > 0$ will lead to a larger herd size and greater impacts from crowberry encroachment. As a robustness check, we set γ equal to the subsidized amount per reindeer (Norwegian Government 2021) in Appendix G. Incorporating this value of reindeer existence results in a more realistic slaughter rate, larger herd size, and higher feeding and control efforts in the Adaptive Post-EC scenario (Appendix G).

6 Results

Regarding the ecological model, there exist seven equilibria whose analytical solutions are presented in Appendix B. Given the chosen set of parameter values, the system reaches either a stable interior equilibrium or a stable corner solution. With $\varphi < 1 + d_K M \left(\frac{g_K}{\alpha_K} - \frac{g_V}{\alpha_V}\right) = 0.98$, the ecological system converges to a stable interior equilibrium where the three state variables co-exist (Appendix B, Fig. 2). If $\varphi > 0.98$, the system converges to a corner solution where V goes to null, thus highlighting the possible elimination of palatable vegetation under extreme encroachment scenarios (Appendix B). The bifurcation analysis of the allelopathic coefficient φ illustrates the gradual shift of the steady-state solution (Fig. 3).

Regarding the bioeconomic model where the reindeer are slaughtered, three scenarios are calculated, which include the pre-EC, post-EC, and adaptive post-EC scenarios in which feeding and crowberry control are performed in conjunction with slaughter.[16] Allelopathic crowberry encroachment is only captured in the last two scenarios. In each scenario there exists only one equilibrium where all the state and control variables are positive reals (Table 1.). For each of these three equilibria, there is one positive and two negative eigenvalues which indicates saddle points. This implies that, for given parameter values, there is a single saddle path that must be taken to reach the saddle point. The optimally controlled system cannot be expected to return to the saddle point equilibrium if environmental fluctuations knock the system off the saddle path (e.g., exogenous weather event that results in reindeer mortality and decreases R). Stability of all equilibria and sensitivity analysis are provided in Appendix D. In general, the equilibria are insensitive to a 10% increase of each parameter with the exception of the reindeer and crowberry intrinsic growth rates α_R and α_K.

In the pre-EC scenario, dynamic optimized results lead to a reindeer population of approximately 280 000 reindeer. When the objective functional is optimized with regard to S in all scenarios, the slaughter volume (roughly 60% of the optimal reindeer population), is slightly higher than government estimates of the average slaughter rate (Agriculture Directorate 2009; Norwegian Government 2021) which is expected due to the exclusion

[16] The extreme post-EC case from appendix B is not included in the profit calculation of the bioeconomic model because the extreme value of has not been recorded in any herding district.

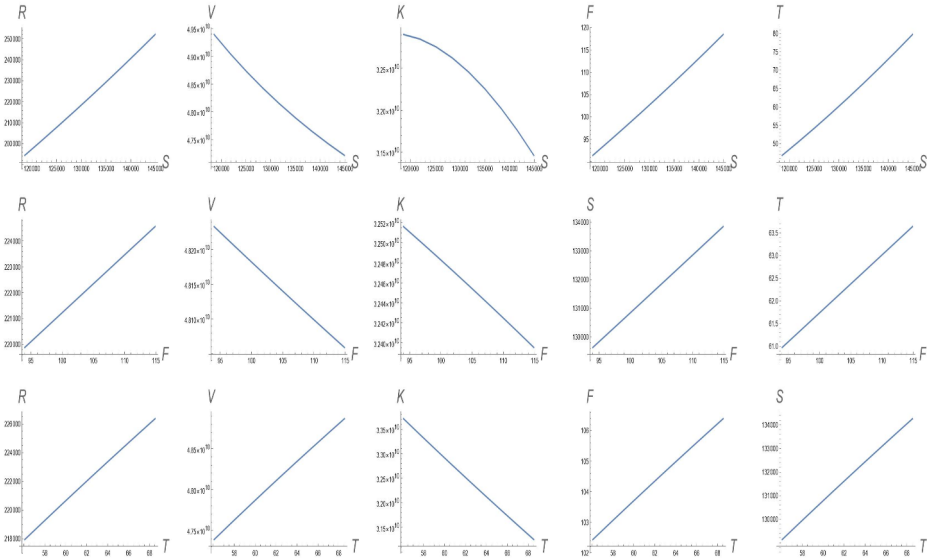


Fig. 2 Comparative statics between three control variables and other variables. Comparative statics illustrating the impacts of varying control measures on equilibrium levels of reindeer population, vegetation biomass, and crowberry density under adaptive post-encroachment management strategies. Each row demonstrates how changes in one control variable significantly influence other state and control variables

of reindeer non-use value γ. Furthermore, the present stock size of 220 000 reindeer falls within the median range of the pre-EC and post-EC scenarios, indicating that the local socio-ecosystem is gradually moving towards a new equilibrium with crowberry encroachment. In the post-EC scenario, crowberry encroachment causes a 42% decrease in V^*, R^*, and S^* and a 140% increase in K^*. This change in pasture quality results in a 40% decrease (100 million NOK) in profits.

To mitigate this profit loss, the resource manager has two options: (1) supporting the reindeer stock by feeding and (2) restoring the habitat by controlling crowberry. In the adaptive post-EC scenario, the optimal efforts from the two adaptive measures dampen the negative impact of crowberry, as the two state variables V^* and R^* increase by 33% and 53%, while K^* decreases by 50%. The combination of these two adaptation measures reduces the profit loss from crowberry encroachment from 100 million NOK to 29 million NOK.

Due to the simple functional form of profit, $pS - c_1 R$, which is linear in S and R, the optimal shadow price of reindeer $\lambda_R{}^*$ equals the price per slaughtered reindeer p for all scenarios (Table 1). The vegetation shadow price $\lambda_V{}^*$ is positive in all cases, implying the economic gains of adding one kilogram of vegetation to the pasture. Among all scenarios, $\lambda_V{}^*$ in the post-EC scenario shows the highest value of 0.013, implying its scarcity value is highest when crowberry encroachment is not controlled. Furthermore, the crowberry shadow price $\lambda_K{}^*$ shows a change in sign between the pre-EC and the other two post-EC scenarios. The reason is that in the post-EC case, the introduction of the allelopathic impacts causes the cost of an additional kilogram of crowberry to exceed the minimal grazing benefits that crowberry provides. Recall that when $\varphi > \bar{\varphi} = \frac{\alpha_R d_K M \lambda_R^* R^{*2}}{(d_K K^* + d_V V^*)^2 (\alpha_V V^* \lambda_V^*)}$ (Eq. (11), in which $\bar{\varphi}$ is the switching threshold), the sign of $\lambda_K{}^*$ will turn from positive to negative.

Table 1 provides $\bar{\varphi}$ in the three scenarios and confirms this inequality condition. Given available data that indicates φ is approximately 0.8 (Tuomi et al. 2024), crowberry should currently be viewed as a damaging pest in Norway. However, there is still time to adopt adaptation measures and prevent the eradication of vegetation from the grazing pasture. The corner solution $V^* = 0$ is reached when $\varphi = 1$ in the post-EC scenario as opposed to $\varphi = 0.98$ in the ecological model. This suggests that reindeer slaughter delays the threshold at which the palatable vegetation is eradicated. In contrast, the corner solution is never reached in the Adaptive post-EC scenario, highlighting the role of feeding and crowberry control in preventing the eradication of vegetation (Table 2).

We calculate the numerical comparative statics for the relationships between the controls and other variables at the equilibrium point (Fig. 2). Varying each control by 10%, we calculated how the equilibrium will change, and we plot the changes of each pair of variables. Relationships between the state and control variables are mostly in line with the results of the implicit function theorem in Appendix E and F. The only difference is the relationship between feeding effort and the crowberry population. This is consistent with our theoretical finding that the presence of a native pest that deteriorates pasture quality creates an additional economic benefit of supplementary reindeer feeding. This is an indirect relationship between F and K that manifests through the reindeer dynamics. Figure 2 captures this indirect relationship because it shows how changing one variable changes all other variables. In contrast, the implicit function theorem illustrates the change of only one variable in response to changes in another variable, holding all other variables constant.

Finally, it is important to verify that the cost savings provided from adaptation will persist as the slow dynamics of the change in the allelopathic coefficient continue to evolve. Table 2 shows the unique adaptive post-EC equilibrium when φ changes drastically. The systems equilibrium remains classified as a saddle point as the slow dynamics gradually evolve. Higher φ leads to greater allelopathic effects which drives down the vegetation population. This reduction in competition leaves space for crowberry to expand, which necessitates greater crowberry control by the resource manager. The combination of declining stocks of both forage species reduces the reindeer stock which makes the resource manager reduce feeding and slaughter volume.

Table 2 The effect of changing allelopathic coefficient on the adaptive post-EC saddle point equilibrium for the bioeconomic model

φ	0.8	0.9	1	1.1	1.2
V^*	5.4×10^{10}	5.27×10^{10}	5.16×10^{10}	5.07×10^{10}	4.99×10^{10}
R^*	251 301	244 052	238 147	233 416	229 675
S^*	148 000	143 575	139 951	137 032	134 708
K^*	2.3×10^{10}	2.23×10^{10}	2.15×10^{10}	2.06×10^{10}	1.97×10^{10}
F^*	63	61	59.6	58.5	57.6
T^*	73.1	84.5	95.3	105.2	114.4
Profit (mil NOK)	220.4	210.4	201	192.5	184.8

7 Discussion

Climate change can impact how native species are assessed. Our results highlight the importance of controlling a native species that becomes a pest. Without the aggressive allelopathic encroachment, crowberry's shadow price is positive because the plant is grazed by valuable reindeer. However, allelopathic encroachment leads to a gradual switch in the sign of crowberry's shadow price turning the valuable native species into a damaging pest. Since this transition from valuable native species to pest has not received much attention (Carey et al. 2012; Valéry et al. 2009), we use our results to highlight four characteristics of any government program designed to mitigate these often unacknowledged impacts of climate change.

First, like many consumable invasive species, reindeer pasture management faces competing market failures (Meadows and Sims 2023). Historically, reindeer management has focused on avoiding a "tragedy of the commons" in which herd sizes increase until all rents associated with grazing are dissipated. This market failure tends to increase the total herd size. The Norwegian government is preventing this first market failure by incentivizing reindeer slaughter through subsidies. Yet this management strategy decreases the reindeer herd size while ignoring the changes in habitat quality due to crowberry encroachment. Hence, crowberry encroachment introduces a second market failure. As crowberry is substantially encroaching the grazing pasture (Tuomi et al. 2024), our chosen value for φ reflects specifically the current encroaching situation. As shown in Table 1. and Appendix B, without rapid control, the actual φ can be expected to exceed the critical threshold when vegetation is eradicated from the pasture. Yet, private herders will underprovide crowberry control since it is a public good (Horan et al. 2002; Perrings et al. 2002). Government support of crowberry control will thus likely be necessary to achieve the socially optimal levels of crowberry control suggested from our theoretical model. Effective management of the reindeer husbandry will require accounting for both market failures. For example, current government efforts to decrease the herd size to avoid overgrazing are reducing the benefits of crowberry control by reducing the number of reindeer to be slaughtered in the future.

Second, any government subsidy program designed to correct the free-rider problem associated with crowberry control will need to develop new metrics for pasture quality. The Norwegian government's current pasture management strategy focuses on quantity but not quality (Ministry of Agriculture and Food 2008; Reinert et al. 2009; Tuomi et al. 2024). Such metrics based on total native vegetation could be misleading since it would not account for the shift from palatable vegetation to less palatable crowberry. Percentage of grazing land encroached by crowberry will also be incomplete since biomass measures do not account for changes in the shadow value of crowberry. While crowberry is less valuable as a forage species, incentivizing crowberry control when its shadow value is positive is effectively government-funded reduction in pasture biodiversity. Subsidies for controlling crowberry should not be considered until the crowberry shadow value turns negative. Ensuring that management decisions are well-informed and effective in promoting long-term sustainability for the reindeer husbandry will require closer attention to climate-induced changes in the shadow value of native species.

Third, supplementary feeding of reindeer can be sustainable in the face of encroachment of a native pest. Our adaptive post-EC bioeconomic equilibrium includes positive levels of supplementary feeding indicating an economic justification for long-term feeding efforts. The Norwegian government spends about 20 million NOK annually to subsidize supple-

mentary feeding (County Governor 2020). The motivation for this government support is to provide supplemental forage to counter the deteriorating pasture quality created by climate change. As our theoretical and empirical results show, feeding also provides an indirect benefit by controlling crowberry encroachment via increased grazing and trampling effects by reindeer (Tybirk et al. 2000). This suggests that the economic argument for supplementary feeding will increase as crowberry encroachment expands.

Fourth, and related to the previous points, government programs to support feeding and crowberry control should be integrated annually to fully account for the tradeoffs and complementarities associated with these adaptation measures. The total calculated adaptation cost in the numerical application of our model is approximately 35 million NOK and is expected to grow as climate change increases the impacts of allelopathy (higher φ). In contrast, recent government support for husbandry under the climate crisis is 20 million NOK, yet this amount is only to subsidize supplementary feeding. These efforts to subsidize supplementary feeding may increase the incentive to control crowberry since feeding and control are shown to be complements in our model. Likewise, any effort to subsidize crowberry control will increase the optimal amount of feeding.

8 Conclusion

Our study introduces a bioeconomic model that assesses the ecological and economic impacts of native pest species encroachment on grazing lands and highlights the consequences of climate and environmental change affecting grazing habitats. Through our analysis, particularly focusing on Norwegian reindeer herding, we reveal the devastating effects of crowberry encroachment on the herding profit and analyze two climate adaptation strategies for mitigating these impacts. Through our bioeconomic model, we provide several methodological and empirical contributions.

Recently the Norwegian government support for husbandry under the climate crisis has focused only on subsidizing supplementary feeding. This study underscores the need for additional focus on managing native pests, which often cause gradual ecological shifts that may elude immediate attention from resource managers. The study also advocates for the Norwegian government to integrate quantitative indicators of grazing pasture quality into their management strategies, enhancing the effectiveness and sustainability of reindeer husbandry. For example, a subsidy program that is only in effect when the shadow value of crowberry is negative will mitigate the effects of climate change in reindeer husbandry without inadvertently reducing pasture biodiversity through the reduction of a native pest.

Although our analysis captures several critical ecological and economic factors, there are a number of important areas for future work. Other goals need to be included in future studies. For instance, incorporating a spatial perspective in our model could provide further guidance for controlling crowberry where the plant is encroaching the most. Furthermore, many benefits of crowberry are excluded in this paper because we wanted to emphasize the allelopathy characteristic of this native pest. Crowberry has many intrinsic biodiversity benefits, medicinal benefits, gastronomic adoption, and even carbon sequestration benefits (Lorion and Small 2021; Parmentier et al. 2018; Ylanne et al. 2015). Setting a carbon price for both plant species could shift the optimization solution of the problem, which could be included in future studies (for example see Vondolia et al. 2020). Finally, future stud-

ies could benefit from explicitly incorporating dynamic programming and feedback control rules to model stochasticity and uncertainty associated with climate change. Stochastic shocks will become especially important in variations of the model that have multiple stable equilibria where relatively small environmental fluctuations could lead to a shift to an alternative stable steady states. Determining optimal hedging strategies in response to stochastic fluctuations will help identify when managers should double down on conservation or when they should give up and rather allocate limited conservation funding elsewhere.

Appendices

Appendix A: Ecological Model Analysis

The equilibrium solutions to the ecological model include seven equilibrium points $P_i \equiv (K_i, R_i, V_i)$, $i \in [1;7]$, including five corner solutions in which there exist two axial equilibria $P_1 \equiv (0,0,M)$ and $P_3 \equiv (Mx,0,0)$. Three other corner solutions are $P_2 \equiv \left(0, \frac{\alpha_V d_V M}{\alpha_V + d_V g_V M}, \frac{\alpha_V M}{\alpha_V + d_V g_V M}\right)$, $P_4 \equiv \left(\frac{M}{\varphi - 1}, 0, \frac{M(\varphi - 1)}{\varphi - 1}\right)$, and $P_5 \equiv \left(\frac{\alpha_K M}{\alpha_K + d_K g_K M}, \frac{\alpha_K d_K M}{\alpha_K + d_K g_K M}, 0\right)$. Conditions for existence of point 4 is $\varphi \neq 1$, and for point 5 is $\alpha_K > d_K g_K M$. There exist two interior equilibria P_6 and P_7 for which the analytical forms are too complex for intuitive interpretation (Table 3).

Analytical eigenvalues of the axial and corner equilibria are reported in Table 4. Two axial equilibria are saddle points as the eigenvalues are real numbers of opposite sign. The five other equilibria have complex eigenvalues such that it is difficult to analyze the stability of these equilibria. We therefore use our empirical application to analyze the system. Our goal is to gain more understanding of how changes in the crowberry allelopathy coefficient, φ, determines the system's stability (Tables 5 and 6).

Table 3 Analytical solutions of two interior equilibria

Point 6	V	$\frac{\alpha_V\alpha_K+\alpha_K d_V g_V M-\alpha_K d_K g_V M+\alpha_V d_K g_K M+\alpha_V d_K g_K M-\alpha_V\alpha_K\varphi\varphi-\alpha_V d_V g_K M\varphi}{2\alpha_V g_K(d_K-d_V\varphi)}$ $+\frac{\sqrt{4\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V-d_V x\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_V\varphi))^2}}{2\alpha_V g_K(d_K-d_V\varphi)}$
	R	$\frac{\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_K-d_K x+d_V\varphi)}{2g_K(d_K g_V M+\alpha_V\varphi)}$ $-\frac{\sqrt{4\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V-d_V\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_V\varphi))^2}}{2g_K(d_K g_V M+\alpha_V\varphi)}$
	K	$\frac{\alpha_K(\alpha_V+d_V g_V M)((-d_V+d_K)g_V M+\alpha_V(-1+\varphi))+\alpha_V g_K M(\alpha_V(d_V(-2+x)\varphi)+d_V g_V M(d_V\varphi))}{2\alpha_V g_K(d_K g_V M+\alpha_V\varphi)(d_K-d_V\varphi)}$ $-\frac{\alpha_V\sqrt{4\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V+d_K(-1+x)-d_V\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_V-d_K+d_V\varphi))^2}}{2\alpha_V g_K(d_K g_V M+\alpha_V\varphi)(d_K-d_V\varphi)}$ $-\frac{d_V g_V M\sqrt{\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V+d_K(-1+x)-d_V\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_K-d_K+d_V\varphi))^2}}{2\alpha_V g_K(d_K g_V M+\alpha_V\varphi)(d_K-d_V\varphi)}$
Point 7	V	$\frac{\alpha_V\alpha_K+\alpha_K d_V g_V M-\alpha_K d_K g_V M+\alpha_V d_K g_K M+\alpha_V d_K g_K M-\alpha_V\alpha_K\varphi\varphi-\alpha_V d_V g_K M\varphi}{2\alpha_V g_K(d_K-d_V\varphi)}$ $-\frac{\sqrt{4\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V-d_V x\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_V\varphi))^2}}{2\alpha_V g_K(d_K-d_V\varphi)}$
	R	$\frac{\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_K-d_K x+d_V\varphi)}{2g_K(d_K g_V M+\alpha_V\varphi)}$ $+\frac{\sqrt{4\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V-d_V\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_V x\varphi))^2}}{2g_K(d_K g_V M+\alpha_V\varphi)}$
	K	$\frac{\alpha_K(\alpha_V+d_V g_V M)((-d_V+d_K)g_V M+\alpha_V(-1+\varphi))+\alpha_V g_K M(\alpha_V(d_V(-2+x)\varphi)+d_V g_V M(d_V\phi))}{2\alpha_V g_K(d_K g_V M+\alpha_V\varphi)(d_K-d_V\varphi)}$ $+\frac{\alpha_V\sqrt{4\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V+d_K(-1+x)-d_V\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_V-d_K+d_V\varphi))^2}}{2\alpha_V g_K(d_K g_V M+\alpha_V\varphi)(d_K-d_V\varphi)}$ $+\frac{d_V g_V M\sqrt{\alpha_V\alpha_K g_K M(d_K g_V M+\alpha_V\varphi)(d_V+d_K(-1+x)-d_V\varphi)+(\alpha_K(-d_V+d_K)g_V M+\alpha_V\alpha_K(-1+\varphi)+\alpha_V g_K M(d_K-d_K+d_V\varphi))^2}}{2\alpha_V g_K(d_K g_V M+\alpha_V\varphi)(d_K-d_V\varphi)}$

Table 4 Analytical solutions for traces, determinants, and eigenvalues of P_1, P_2, P_3, P_4, and P_5

	Eigenvalues
Point 1	$-\alpha_V$ α_K α_R
Point 2	$\frac{\alpha_V\alpha_K+\alpha_K d_V g_V M-\alpha_V d_V g_K M}{\alpha_V+d_V g_V M}$ $-\frac{\alpha_V{}^2+\alpha_V\alpha_R+\alpha_R d_V g_V M+\sqrt{-4\alpha_V\alpha_R(\alpha_V+d_V g_V M)^2+(\alpha_V(\alpha_V+\alpha_R)+\alpha_R d_V g_V M)^2}}{2(\alpha_V+d_V g_V M)}$ $-\frac{\alpha_V{}^2+\alpha_V\alpha_R+\alpha_R d_V g_V M-\sqrt{-4\alpha_V\alpha_R(\alpha_V+d_V g_V M)^2+(\alpha_V(\alpha_V+\mathrm{a3})+\alpha_R d_V g_V M)^2}}{2(\alpha_V+d_V g_V M)}$
Point 3	$-\alpha_K$ α_R $\alpha_V-\alpha_V\varphi$
Point 4	α_R $\frac{\alpha_V+\alpha_K-\alpha_K\varphi-\alpha_V\varphi-\sqrt{(\alpha_K(-1+\varphi)+\alpha_V(-1+\varphi))^2}}{2(-1+\varphi)}$ $\frac{\alpha_V+\alpha_K-\alpha_K\varphi-\alpha_V\varphi+\sqrt{(\alpha_K(-1+\varphi)+\alpha_V(-1+\varphi))^2}}{2(-1+\varphi)}$
Point 5	$-\frac{\alpha_K(\alpha_K+\alpha_R)+\alpha_R d_K g_K M+\sqrt{\alpha_K{}^2(\alpha_K-\alpha_R)^2+2\alpha_K\alpha_R(-3\alpha_K+\alpha_R)d_K g_K M+\alpha_R(-4\alpha_K+\alpha_R)d_K{}^2 g_K{}^2 M^2}}{2(\alpha_K+d_K g_K M)}$ $\frac{-\alpha_K(\alpha_K+\alpha_R)-\alpha_R d_K g_K M+\sqrt{\alpha_K{}^2(\alpha_K-\alpha_R)^2+2\alpha_K\alpha_R(-3\alpha_K+\alpha_R)d_K g_K M+\alpha_R(-4\alpha_K+\alpha_R)d_K{}^2 g_K{}^2 M^2}}{2(\alpha_K+d_K g_K M)}$ $\frac{-\alpha_K d_K g_V M x+\alpha_V(\alpha_K+d_K g_K M x-\alpha_K\varphi)}{\alpha_K+d_K g_K M}$

Table 5 Ecological parameters for both ecological and bioeconomic models

Parameters	Indicators	Unit	Value		Sources
			Pre-encroachment	Post-encroachment	
α_V	Vegetation intrinsic growth coefficient		0.5	0.6	Moxnes 1993; Skonhoft et al. 2010; Myers-Smith et al. 2020
α_K	Crowberry intrinsic growth coefficient		0.1	0.2	Murguzur et al., 2019, Tuomi et al., 2022
α_R	Reindeer intrinsic growth coefficient		0.7		Moxnes 1993; Skonhoft et al. 2010
M	Pasture carrying capacity	Kg biomass/ total area	8.7×10^{10}		Calculated from Bråthen and Lortie 2015
g_V	Grazing coefficient on vegetation		4×10^{-7}		Bakka et al. 2021; White and Trudell 1980
g_K	Grazing coefficient on crowberry		8×10^{-9}		Danell et al. 1994
d_V	Impact coefficient of vegetation on reindeer carrying capacity from vegetation		2×10^{-5}		Calibrated
d_K	Impact coefficient of crowberry on reindeer carrying capacity		4×10^{-7}		Calibrated
φ	Allelopathic coefficient		0	0.8	Calculated from Bråthen and Lortie 2015; Pilsbacher et al. 2020; Tuomi et al., 2022
$V(t_0)$	Initial condition of vegetation	Kg biomass	6.09×10^{10}		Bråthen et al. 2018
$K(t_0)$	Initial condition of crowberry	Kg biomass	2.61×10^{10}		Bråthen et al. 2018
$R(t_0)$	Initial condition of reindeer	Reindeer	250 000		Landbruksdirektoratet, 2009

Table 6 Economic parameters for the bioeconomic model

Parameters	Meaning	Unit	Value	Source
ε	Burning coefficient	/effort unit	8×10^{-4}	Calibrated
β	Feeding coefficient	/effort unit	8×10^{-4}	Calibrated
c_1	Maintenance cost	NOK/ reindeer	1200	Calculated from Regjeringen, 2021
c_2	Supplementary feeding cost	NOK/ tonne dry feed	6000	Discussion with herders, Pekkarinen et al. 2015
c_3	Crowberry burning cost	NOK/ treatment effort	2000	Calibrated
p	Price per reindeer	NOK/ reindeer	3760	Calculated from Regjeringen, 2021
γ	Non-use value per reindeer	NOK/ reindeer	2540	Calculated from Regjeringen, 2021
δ	Discount rate		0.05	Regjeringen, 2012

Appendix B: Impact of Crowberry Encroachment in the Ecological System

Besides the fitted value for $\varphi = 0.8$, we choose another more extreme value for $\varphi = 1.1$ to represent the extreme post-EC case in which the high-quality palatable vegetation goes extinct due to crowberry encroachment. Table 7 shows the existence of a unique stable equilibrium in all three cases. While point P_7 is the stable equilibrium in the pre- and post-EC cases, it turns into a saddle point in the third case when $\varphi = 1.1$.

Regarding the extreme post-EC case, only the corner solution P_5 is stable, emphasizing the system shift from an interior to a corner solution without the palatable vegetation. Though the eigenvalues of P_7 are too complex to analyze analytically when $\varphi = 1.1$, we can analyze the eigenvalues of P_5. Among three eigenvalues (Table 4), only one contains φ, which is

$$\frac{-\alpha_K d_K g_V M + \alpha_V(\alpha_K + d_K g_K M - \alpha_K x\varphi)}{\alpha_K + d_K g_K M}$$

Since P_5 is stable in the extreme case while a saddle point in the two former cases, we need

$$-\alpha_K d_K g_V M + \alpha_V(\alpha_K + d_K g_K M - \alpha_K\varphi) < 0 \leftrightarrow \varphi > 1 + d_K M\left(\frac{g_K}{\alpha_K} - \frac{g_V}{\alpha_V}\right)$$

for the eigenvalue of P_5 to be negative. Thus, when φ reaches the encroachment threshold of $1 + d_K M\left(\frac{g_K}{\alpha_K} - \frac{g_V}{\alpha_V}\right)$, the system's unique stable equilibrium will turn from an interior to a corner solution of only crowberry and reindeer. At this threshold, the allelopathic encroachment leads to the extinction of the palatable vegetation. The extinction threshold varies based on the magnitudes of the ecological parameters. Specifically, the higher grazing rate of reindeer and impact rate of crowberry, g_K and d_K, the harder crowberry encroachment can drive the system to a corner solution. Numerical results of the eigenvalues in three cases are reported in Table 8.

The number of reindeer in the pre-EC case in P_7 reaches 734 000, which serves as the carrying capacity of reindeer in the bioeconomic model. Crowberry encroachment inflicts

Table 7 Multiple equilibria for the ecological model in three cases: pre-EC, post-EC, and extreme post-EC cases. Unit for biomass and is kg/ whole area while R is reindeer number

Equilibria	Pre-EC ($\varphi = 0$)				Post-EC ($\varphi = 0.8$)				Extreme Post-EC ($\varphi = 1.1$)			
	K	R	V	Stability	K	R	V	Stability	K	R	V	Stability
P_1	0	0	8.7×10^{10}	Saddle	0	0	8.7×10^{10}	Saddle	0	0	8.7×10^{10}	Saddle
P_2	0	727 425	3.64×10^{10}	Saddle	0	805 556	4.03×10^{10}	Saddle	0	805 556	4.03×10^{10}	Saddle
P_3	7.83×10^{10}	0	0	Saddle	7.83×10^{10}	0	0	Saddle	7.83×10^{10}	0	0	Saddle
P_4	-8.7×10^{9}	0	8.7×10^{10}	Saddle	-4.35×10^{10}	0	1.22×10^{11}	Saddle	8.7×10^{10}	0	-8.7×10^{9}	Saddle
P_5	7.81×10^{10}	31 242	0	Saddle	7.82×10^{10}	31 281	0	Saddle	7.82×10^{10}	31 281	0	**Stable**
P_6	-6.36×10^{15}	-1.06×10^{9}	7.4×10^{13}	Saddle	5.57×10^{12}	-3.94×10^{7}	-2.08×10^{12}	Saddle	2.24×10^{12}	-2.16×10^{7}	-1.12×10^{12}	Saddle
P_7	3.99×10^{10}	734 098	3.59×10^{10}	**Stable**	6×10^{10}	372 433	1.74×10^{10}	**Stable**	7.91×10^{10}	14 374	-6.64×10^{8}	Saddle

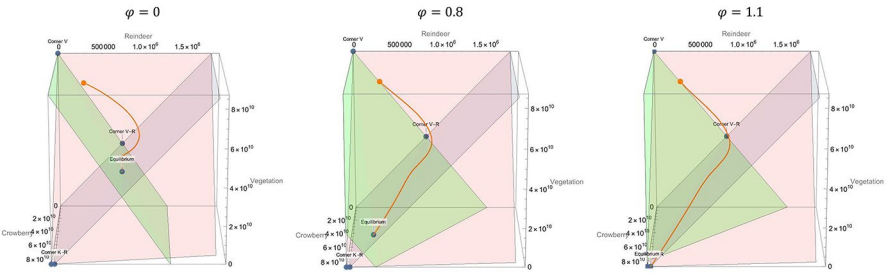


Fig. 3 Visualization of the three-dimensional phase-space of the system, showing the iso-planes of three species and the dynamic trajectory to a stable equilibrium. The green, red, and blue surfaces represent the iso-planes of vegetation, crowberry, and reindeer, respectively. The orange trajectory illustrates system convergence from the starting point (orange dot) to the intersection of all three iso-planes in the three cases: $\varphi = 0$, $\varphi = 0.8$, $\varphi = 1.1$. The other blue dots are the non-negative equilibria of the system. Initial conditions follow those described in the dataset

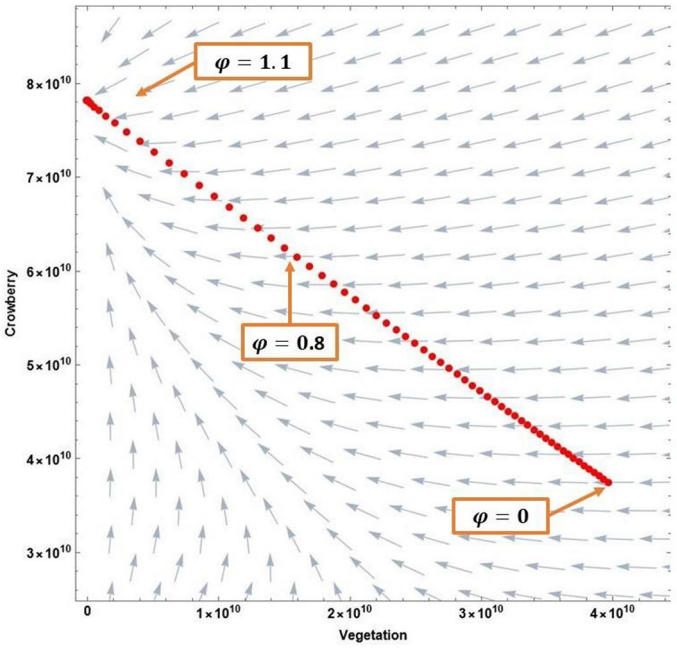


Fig. 4 Phase-plane bifurcation analysis of vegetation and crowberry dynamics with varying allelopathic coefficient φ

a 50% decline in reindeer population in the post-EC case, while vegetation is reduced 50% due to allelopathic impact. Together with reindeer's selective herbivory towards vegetation, the crowberry population increases up to 70% of the total pasture carrying capacity in the post-EC case.

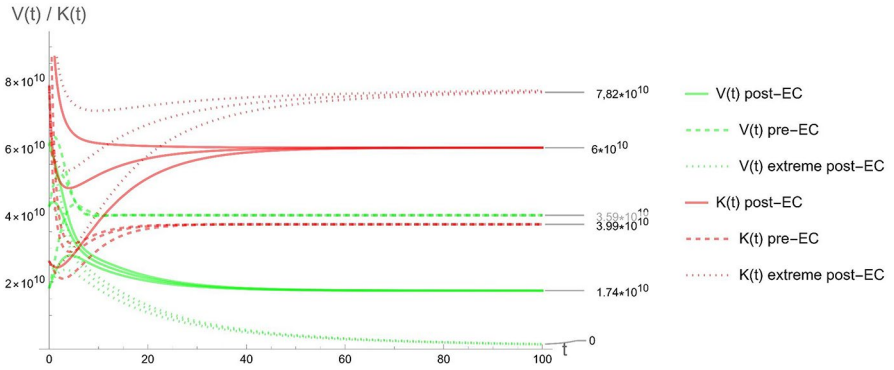


Fig. 5 Dynamic convergence of vegetation and crowberry in three cases with different initial conditions

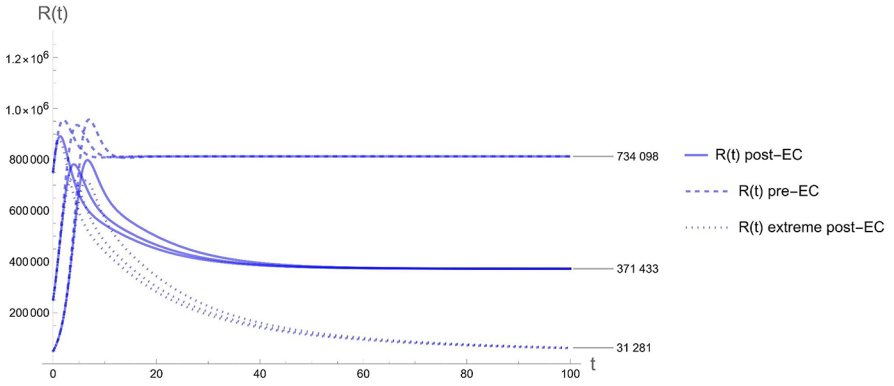


Fig. 6 Dynamic convergence of reindeer in three cases with different initial conditions

The ecological model can be visualized in a three-dimensional phase-space plot, which includes three iso-planes to enhance the depiction of equilibrium points and their stability.

Figure 3 illustrates the impact of the allelopathic coefficient φ on the system's stability. Each isophase plane corresponds to a condition wherein the growth rate of a particular species is constant, indicating a state of equilibrium for that species. The intercept of these planes marks the equilibria of the system. Notably, since φ directly influences the growth dynamics of vegetation (Eq. 1), altering φ results in corresponding shifts solely in the vegetation's iso-plane, while the iso-planes of crowberry and reindeer remain unchanged. The shift in vegetation's iso-plane indicates the decrease in vegetation when φ increases, shifting the stable equilibrium of the system from an interior solution to a corner solution with $V = 0$. A cross-section of this phase-space is the phase-plane of each pair of two species.

Table 8 Numerical solutions for stability of seven ecological equilibria in the pre-, post-, and extreme post-encroachment cases

	Pre-EC		Post-EC		Extreme Post-EC	
	Eigenvalues	Stability	Eigenvalues	Stability	Eigenvalues	Stability
Point 1	– 0.5 0.1 0.7	Saddle point	– 0.6 0.2 0.7	Saddle point	– 0.6 0.2 0.7	Saddle point
Point 2	0.094 – 0.455– 0379 i – 0.455 + 0379 i	Saddle point	0.194 – 0.489–0.425 i – 0.489 + 0.425 i	Saddle point	0.194 – 0.489–0.425 i – 0.489 + 0.425 i	Saddle point
Point 3	0.7 0.5 – 0.1	Saddle point	0.7 – 0.2 0.168	Saddle point	0.7 – 0.2 0.006	Saddle point
Point 4	0.7 – 0.5 – 0.1	Saddle point	– 1.007 0.7 – 0.033	Saddle point	0.7 -0.13 -0.009	Saddle point
Point 5	– 0.7 0.488 – 0.1	Saddle point	– 0.7 – 0.2 0.156	Saddle point	-0.7 -0.2 -0.006	Stable point
Point 6	– 424.328 – 11.547 1.237	Saddle point	14.078 – 1.296 0.373	Saddle point	7.73 –1.26 0.2	Saddle point
Point 7	– 0.453 + 0,375 i – 0.453– 0,375 i – 0.093 + 0. i	Stable point	– 0.464 + 0.162 i – 0.464 + 0.162 i – 0.09 + 0. i	Stable point	-0.71 -0.19 0.006	Saddle point

Figure 4 visualizes the dynamics between vegetation and crowberry populations, in which the directional vector field, illustrated by blue arrows, conveys the trajectory and velocity of population changes at $\varphi = 1.1$. The series of red dots depict the gradual shift of the system's equilibrium in response to the incremental bifurcation of the allelopathic coefficient φ. Notably, the equilibrium shift indicates a consistent and progressive movement rather than an abrupt and drastic change, mirroring the ecological behavior of the crowberry– a native species whose spread is gradually amplified by climate change (Bokhorst et al. 2018; Gonzalez et al. 2021). The absence of rapid encroachment, due to the ecological characteristics of this plant, circumvents any abrupt disruptions to the system's equilibrium. (Bråthen et al. 2018; Maliniemi et al. 2018; Tuomi et al. 2024). We acknowledge that incorporating more complex ecological factors, such as an Allee effect (Drake and Lodge 2006; Salau and Fenichel 2015), into our model might result in an abrupt shift.

Figures 5 and 6 shows the time convergence of three species between 2 cases, pre-EC and post-EC, under different initial conditions. Figure 7 depicts the system convergence to the stable interior equilibrium from different initial conditions.

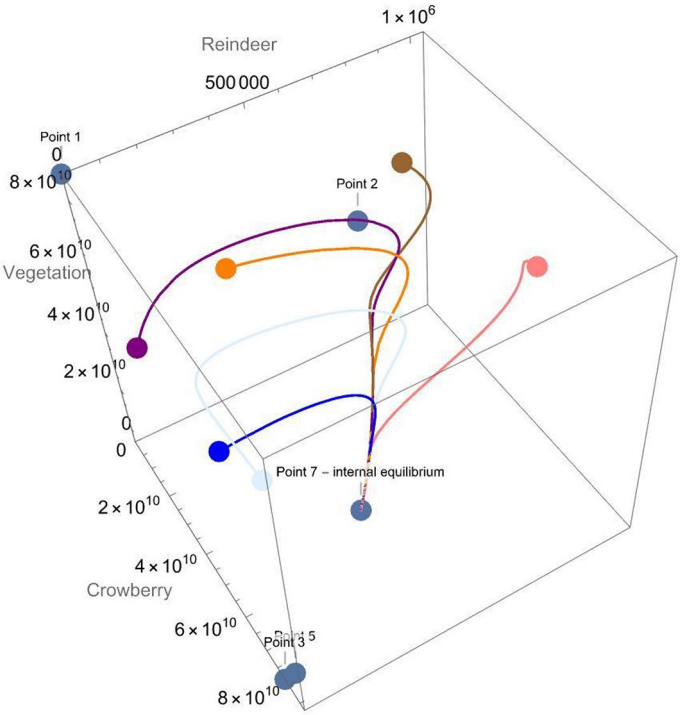


Fig. 7 Convergence to the stable interior equilibrium with different initial conditions

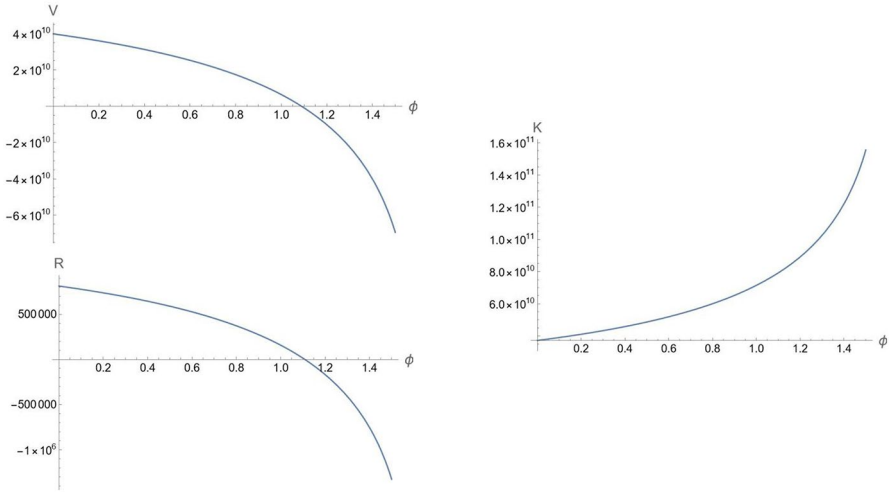


Fig. 8 Relationship between different choice of φ and the equilibrium point

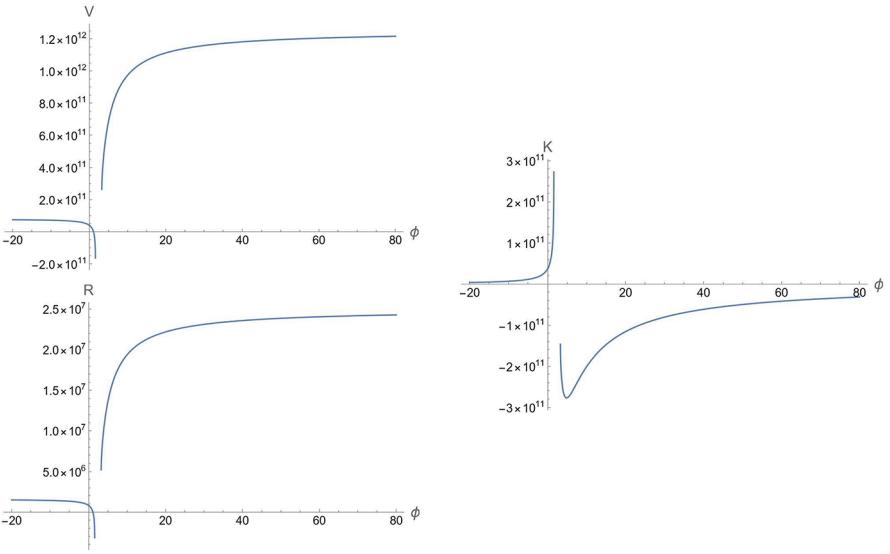


Fig. 9 Relationship between different choice of φ and the equilibrium point

Table 9 Sensitivity analysis in pre-encroachment and post-encroachment cases in relation to 10% increase in each parameter (in percentage)

Indicators	Parameters	Pre-EC			Post-EC			Extreme Post-EC		
		ΔV^*	ΔR^*	ΔK^*	ΔV^*	ΔR^*	ΔK^*	ΔV^*	ΔR^*	ΔK^*
Intrinsic growth rates	α_V	5.78	5.54	-5.22	9.33	8.55	-2.8	-10.15	**11.94**	-0.12
	α_K	-0.007	0.005	0.57	-0.3	-0.26	0.23	0.55	-0.64	0.01
	α_R	0	0	0	0	0	0	0	0	0
Carrying capacity	M	3.75	3.99	**15.007**	0.73	1.49	**12.63**	**19.28**	-0.92	10.1
Grazing coefficients	g_V	-5.69	-5.46	5.18	-8.72	-8	2.62	8.93	-10.5	0.1
	g_K	0.008	-0.006	-0.63	0.33	0.29	-0.25	-0.62	0.71	-0.01
Comparative rate to reindeer carrying capacity	d_V	-5.55	3.9	4.45	-7.88	1.39	2.23	-10.11	1.11	-0.11
Comparative rate to reindeer carrying capacity	d_K	-0.13	0.09	0.11	-0.59	0.1	0.17	**20.66**	-2.26	0.23
Allelopathy coefficient	ϕ	0	0	0	**-22.49**	**-20.61**	6.77	**119.8**	**-140.89**	**13.99**

Sensitivity analysis provides insight regarding how sensitive the variables are in relation to a 10% increase in a single parameter (Table 9). The results provide signs of change as expected, with the exception of the relationship between changes in α_R and the variables. K in both cases and V and R in post- and extreme post-EC cases are sensitive in relation to only the parameters governing carrying capacity M. Due to reindeer's selective herbivory, V and R are robust to a 10% change in M, but not K. Also due to selective herbivory, a 10% increase in g_V and g_K leads to different signs for the change in reindeer stock, for pre- and post-EC. Moreover, a 10% increase in α_K causes sign difference for the change of R in all cases.

The reason why both the pasture and reindeer are sensitive to the allelopathy coefficient φ lies in the analytical solutions, as K increases exponentially with higher φ while V and R develops in the opposite direction. As a function of φ, point 7 takes the form of

$$K_7 = \frac{2.89 \times 10^{12} - 1.35 \times 10^{12}\varphi - 1.13 \times 10^{13}\sqrt{0.07+(-0.06+0.01\varphi)\varphi}}{(0.02+\varphi)(-0.02+\varphi)}$$

$$R_7 = \frac{-2.67 \times 10^{7} + 1.33 \times 10^{7}\varphi + 1.04 \times 10^{8}\sqrt{0.07+(-0.07+0.01\varphi)\varphi}}{0.02+\varphi}$$

$$V_7 = \frac{-1.34 \times 10^{12} + 6.64 \times 10^{11}\varphi + 5.21 \times 10^{12}\sqrt{0.07+(-0.06+0.01\varphi)\varphi}}{-0.02+\varphi}$$

Given all the chosen parameters are fixed, K, R, and V are nonlinear functions of φ where K increases exponentially when φ ranges from 0 to 1, 5, while V and R follows the opposite growth direction (Figs. 8 and 9).

Appendix C: Proof for Singular Control S

Due to the linear S control in the objective functional, S disappears from the FOCs and the adjoint equations. To determine the functional form of S^*, we can take the second time derivative of the first adjoint equation. We use the first adjoint equation in the adaptive scenario since it includes all three control variables. Setting this equation equals to 0, we obtain:

$$
\begin{aligned}
S^* = & \frac{1}{p(-\frac{4\alpha_R}{d_K K^* + d_V V^*} + \frac{p\beta^2}{c_2})} \Bigg[\frac{4\alpha_R{}^2 p R^* (R - d_K K^* - d_V V^*)}{(d_K K^* + d_V V^*)^2} + \frac{F^* p^2 R^* \beta^3}{c_2} - 2 g_V V^* \delta \lambda^*{}_V \\
& - \frac{2\alpha_K K^{*2} (g_V V^* - g_K V^* + g_K M) \lambda^*{}_K}{(V^* - M)^2} - 2 g_K K^* \delta \lambda^*{}_K \\
& - \frac{2\alpha_V V^* \lambda^* V (g_V V^* + g_K K^* \varphi)}{M} + \frac{\alpha_R R^*}{c_2 M (d_K K^* + d_V V^*)^2 (Mx - V^*)} \\
& \times [-4\alpha_K c_2 d_K K^* M p (K^* + V^* - M) - (M - V^*)(-M p^2 (d_K K^* + d_V V^*) \\
& \times (d_K K^* + d_V V^* - R^*) \beta^2 + 2 c_2 d_K K^* M (2p(g_K R^* + F^* \beta + T^* \varepsilon) - g_K p R^*) \\
& + 2 c_2 d_V V^* (2 F^* M p \beta + g_V p M R^* + 2\alpha_V p (V^* + K^* \varphi - M)))] \Bigg]
\end{aligned}
$$

We check whether S^* is a singular control by testing the Legendre-Clebsch condition (Krener 1977), obtaining:

$$
\frac{d}{dS} \frac{d^2}{dt^2} \frac{\partial H}{\partial S} = \frac{p\alpha_R 2(-1)}{(d_K K^* + d_V V^*)} < 0
$$

which satisfies the condition for a maximization problem with singular control, and S^* will be singular through time (see similar example in Clayton et al. 2010).

Appendix D: Stability and Sensitivity Results of Bioeconomic Models

In general, the results are insensitive to the 10% increase of each parameter, except α_R, α_K, and φ in some scenarios (Tables 10 and 11). One seemingly unexpected sign in all scenarios is that 10% increase of α_R leads to an increase in V^* and a decrease in K^*, which does not seem to follow the selective herbivory in our model. However, from Table 11, it becomes clear that the increase in α_R leads to a larger increase in crowberry control than supplementary feeding which triggers the divergence of the V and K values. Another unexpected sign is that V^* increases when α_K increases in the post-EC case whereas R^* and S^* show negative signs. This can be explained as higher α_K boosts crowberry encroachment, leaving the reindeer stock less nutritious food which then alleviates less stress on vegetation stock. Model sensitivity to a 10% increase in φ can be explained by the non-linear effect of crowberry encroachment (see Appendix B).

Combination of both measures reinforces the robustness of the model as the variables are insensitive to 10% change of the parameters. Besides, a 10% increase in maintenance cost c_1 decreases the gained profit, reducing treatment effort, making the sign of change in T^* negative. Another seemingly unexpected sign is that 10% higher reindeer price decreases the optimal vegetation biomass. The reason is that as reindeer meat is valued 10% higher, enhanced feeding effort F^* and controlling effort T^* are needed to increase the herd size, thus decreasing K^*. Table 12 shows the stability of the equilibrium in three scenarios.

Table 10 Sensitivity results between pre-encroachment and post-encroachment cases in relation to 10% increase of each parameter (in percentage). Sensitivity higher than 10% is marked in bold

Parameters	Pre-EC							Post-EC						
	ΔV^*	ΔR^*	ΔS^*	ΔK^*	$\Delta \lambda_R{}^*$	$\Delta \lambda_V{}^*$	$\Delta \lambda_K{}^*$	ΔV^*	ΔR^*	ΔS^*	ΔK^*	$\Delta \lambda_R{}^*$	$\Delta \lambda_V{}^*$	$\Delta \lambda_K{}^*$
α_V	1.94	2.727	2.503	-6.671	0	-8.213	1.69	4.861	5.438	5.246	-4.314	0	-8.044	6.163
α_K	-0.001	0.005	0.003	0.216	0	-0.035	-6.331	0.061	-0.293	-0.207	0.008	0	1.524	-5.947
α_R	8.077	**11.609**	**10.584**	**-27.722**	0	**-32.6**	7.238	**12.7**	**14.439**	**13.866**	**-11.239**	0	**-19.995**	**15.93**
M	7.803	6.808	7.083	**17.303**	0	-0.167	-1.833	5.069	4.054	4.349	**14.311**	0	0.698	-3.817
g_V	-1.998	-2.9	-2.65	6.882	0	-0.169	-1.978	-4.597	-5.546	-5.27	4.085	0	0.792	-3.89
g_K	0.001	-0.002	-0.001	-0.234	0	0.002	0.152	0.122	0.152	0.143	-0.174	0	-0.1	0.074
d_V	-1.981	6.752	7.031	6.584	0	9.902	-1.859	-4.351	3.935	4.237	3.804	0	**10.765**	6.084
d_K	-0.016	0.055	0.051	0.053	0	-0.091	10.03	-0.135	0.122	0.114	0.118	0	-0.013	-0.289
ϕ	0	0	0	0	0	0	0	**-26.482**	**-25.314**	**-25.33**	**23.52**	0	**36.447**	**10.34**
p	-1.758	5.991	3.787	5.843	10	**29.948**	**28.041**	-3.956	3.578	1.637	3.457	10	**30.94**	**25.759**
c_1	2.039	-6.949	-4.787	-6.797	0	**-18.237**	**-16.847**	4.813	-4.343	-2.34	-4.209	0	**-18.978**	**-15.079**
δ	0.287	-0.978	-0.649	-0.956	0	-3.798	-5.699	0.305	-0.275	-0.137	-0.266	0	-6.511	-8.146

Table 11 Sensitivity results in comprehensive scenarios in relation to 10% increase of each parameter (in percentage)

Parameters	Adaptive post-EC								
	ΔV^*	ΔR^*	ΔS^*	ΔK^*	ΔF^*	ΔT^*	$\Delta \lambda_R{}^*$	$\Delta \lambda_V{}^*$	$\Delta \lambda_K{}^*$
α_V	3.806	5.357	5.399	-4.962	5.357	1.076	0	-7.086	6.353
α_K	-1.772	-2.494	-2.494	4.524	-2.494	-2.727	0	3.801	-6.937
α_R	9.72	**14.1**	**14.206**	**-12.517**	**14.1**	2.079	0	**-17.561**	**16.683**
M	7.225	7.037	7.766	**11.303**	7.037	9.695	0	-0.319	-1.445
g_V	-3.559	-5.289	-5.245	4.523	-5.288	-0.277	0	-0.976	-4.592
g_K	0.063	0.017	0.104	-0.171	0.107	-0.083	0	-0.045	0.088
d_V	-2.435	6.969	7.7	1.055	6.969	9.815	0	9.623	8.668
d_K	-0.093	0.07	0.069	0.137	0.07	-0.123	0	0.016	-0.26
ϕ	-5.706	-6.712	-6.964	3.556	-6.712	**17.985**	0	6.86	**13.934**
p	0.008	9.423	8.596	-4.692	**20.366**	**24.576**	10	**26.082**	**30.709**
c_1	1.377	-7.201	-5.68	1.022	-7.201	**-14.471**	0	**-14.745**	**-15.336**
c_2	0.185	-1.057	-1.603	0.182	**-10.052**	-2.149	0	-2.205	-2.327
c_3	-1.113	-1.366	-1.403	2.742	-1.366	-7.13	0	1.572	-0.57
β	-0.419	2.525	3.882	-0.476	**12.777**	5.166	0	5.323	5.669
ε	2.307	2.872	2.95	-5.702	2.872	5.034	0	-3.218	1.259
δ	-0.659	-1.469	-1.371	1.951	-1.469	-5.951	0	-4.145	-7.75

Table 12 Stability results of the dynamic optimized equilibrium under different scenarios

Scenario	Eigenvalues	Stability
pre-EC	0.385; - 0.365; - 0.1	Saddle point
post-EC	- 0.448; 0.411; - 0.017	Saddle point
Adaptive post-EC	- 0.448; 0.413; - 0.031	Saddle point

Appendix E: Analysis of Some Implicit-function Theorems

From the reindeer equilibrium condition (Eq. (13)) we have three implicit functions (17), (18), and (19) with:

$$\frac{\partial V^*}{\partial R^*} = \frac{2\alpha_R(d_K K^* + d_V V^*)\lambda_R^*}{2\alpha_R d_V R^* \lambda_R^* - g_1(d_K K^* + d_V V^*)^2 \lambda_V^*} \tag{17}$$

We expect that the sign of the implicit function of vegetation w.r.t reindeer $\frac{\partial V^*}{\partial R^*} < 0$ if $\frac{\lambda_V^*}{\lambda_R^*} > \frac{2\alpha_R d_V R^*}{g_V(d_K K^* + d_V V^*)^2}$ (derived from the denominator).

$$\frac{\partial K^*}{\partial R^*} = \frac{2\alpha_R(d_K K^* + d_V V^*)\lambda_R^*}{2\alpha_R d_K R^* \lambda_R^* - g_2(d_K K^* + d_V V^*)^2 \lambda_K^*} \tag{18}$$

As $\lambda_K^* < 0$ when $\varphi > \frac{\alpha_R d_K M \lambda_R^* R^{*2}}{(d_K K^* + d_K V^*)^2(\alpha_V V^* \lambda_V^*)}$, $\frac{\partial K^*}{\partial R^*} > 0$. Intuitively, due to reindeer's selective herbivory, reindeer will suppress the high quality pasture, leaving the native pest to grow wild. If $\lambda_K^* > 0$ then $\frac{\partial K^*}{\partial R^*} > 0$ only when $\frac{\lambda_K^*}{\lambda_R^*} < \frac{2\alpha_R d_K R^*}{g_2(d_K K^* + d_V V^*)^2}$ (derived from the denominator).

$$\frac{\partial R^*}{\partial F^*} = \frac{d_K \beta K^* + d_V \beta V^*}{2\alpha_R} \tag{19}$$

From (19) we can have $\frac{\partial F^*}{\partial R^*} > 0$, which indicates that if there is more reindeer, under the climate change situation, more effort is needed.

We get two implicit functions from the equilibrium condition of vegetation Eq. (14):

$$\frac{\partial K^*}{\partial V^*} = \frac{2\left(\alpha_R d_V{}^2 M R^{*2}(M - V^*)^3 \lambda_R^* + (d_K K^* + d_V V^*)^3\left(-\alpha_V(V^* - M)^3 \lambda_V^* + \alpha_K K^{*2} M \lambda_K^*\right)\right)}{(V^* - M)\left(2\alpha_R d_V d_K M R^{*2}(V^* - M)^2 \lambda_R^* + (d_K K^* + d_V V^*)^3\left(2\alpha_K M K^* \lambda_K^* + \alpha_V(V^* - M)^2 \lambda_V^* \varphi\right)\right)} \tag{20}$$

$$\frac{\partial R^*}{\partial V^*} = \frac{2\left(\alpha_R d_V{}^2 M R^{*2}(M - V^*)^3 \lambda_R^* + (d_K K^* + d_V V^*)^3\left(-\alpha_V(V^* - M)^3 \lambda_V^* + \alpha_K K^{*2} M \lambda_K^*\right)\right)}{M(d_K K^* + d_V V^*)(M - V^*)^3\left(2\alpha_R d_V R^* \lambda_R^* - g_V(d_K K^* + d_V V^*)^2 \lambda_V^*\right)} \tag{21}$$

The sign of equation (20) is unsure but we expect it to be negative as crowberry shares the common area with vegetation. However, it can also be positive because crowberry acts as a "predator" on vegetation with its allelopathic effect. The sign of equation (21) depends on the sign of the last parenthesis of the numerator $\left(-\alpha_V(V^* - M)^3 \lambda_V^* + \alpha_K K^{*2} M \lambda_K^*\right)$, given that the denominator is positive as we have the above-mentioned inequality condition $\frac{\lambda_V^*}{\lambda_R^*} > \frac{2\alpha_R d_V R^*}{g_V(d_K K^* + d_V V^*)^2}$. As vegetation is the feed for reindeer, we expect that $\frac{\partial R^*}{\partial V^*} > 0$. Letting $-\alpha_V(V^* - M)^3 \lambda_V^* + \alpha_K K^{*2} M \lambda_K^* > 0$ and re-arranging, we have the con-

dition for positive numerator $-\frac{\lambda^*_V}{\lambda^*_K} < \frac{\alpha_K K^{*2} M}{\alpha_V (M-V^*)^3}$. To attain a positive denominator the condition is in the opposite with equation (17) that $\frac{\lambda^*_V}{\lambda^*_R} < \frac{2\alpha_R d_V R^*}{g_V (d_K K^* + d_V V^*)^2}$. This implies that the sign of $\frac{\partial R^*}{\partial V^*}$ and $\frac{\partial V^*}{\partial R^*}$ must be the same.

Three implicit functions are obtained from the equilibrium condition of crowberry (Eq. (15)):

$$\frac{\partial T^*}{\partial K^*} = \frac{2\left(\frac{\alpha_K}{V^*-M} - \frac{\alpha_R d_K{}^2 R^{*2} \lambda^*_R}{(d_K K^* + d_V V^*)^3 \lambda^*_K}\right)}{\varepsilon} \tag{22}$$

The expected sign of $\frac{\partial T^*}{\partial K^*}$ is positive as the resource manager needs to increase the removal effort when crowberry increases. To satisfy this condition, we need $-\frac{\lambda^*_R}{\lambda^*_K} > \frac{\alpha_K (d_K K^* + d_V V^*)^3}{(M-V^*)\alpha_R d_K{}^2 R^{*2}}$.

$$\frac{\partial R^*}{\partial K^*} = \frac{2\left(\alpha_R d_K{}^2 R^{*2}(M-V^*)\lambda^*_R + \alpha_K (d_K K^* + d_V V^*)^3 \lambda^*_K\right)}{(d_K K^* + d_V V^*)(V^*-M)\left(-2\alpha_R d_K R^* \lambda^*_R + g_2 (d_K K^* + d_V V^*)^2 \lambda^*_K\right)} \tag{23}$$

As crowberry is a native pest to the husbandry, we expect that $\frac{\partial R^*}{\partial K^*} < 0$. Setting the numerator negative, we need the condition $\frac{\lambda^*_R}{\lambda^*_K} < \frac{\alpha_K (d_K K^* + d_V V^*)^3}{(M-V^*)\alpha_R d_K{}^2 R^2}$, which is always given the negative λ^*_K. To attain a positive denominator, we need

$$2\alpha_R d_K R^* \lambda^*_R - g_K (d_K K^* + d_V V^*)^2 \lambda^*_K > 0 \leftrightarrow \frac{\lambda^*_K}{\lambda^*_R} < \frac{2\alpha_R d_K R^*}{g_K (d_K K^* + d_V V^*)^2} \quad \text{which}$$

always holds. Thus $\frac{\partial R^*}{\partial K^*} < 0$, and from the same calculation that $\frac{\partial K^*}{\partial R^*} < 0$, reflecting the negative impact of crowberry on reindeer stock size. We also have

$$\frac{\partial V^*}{\partial K^*} = -\frac{2M(Mx - V^*)\left(\alpha_R d_K{}^2 R^{*2}(M-V^*)\lambda^*_R + \alpha_K (d_K K^* + d_V V^*)^3 \lambda^*_K\right)}{2\alpha_R d_V d_K M R^{*2}(V^*-M)^2 \lambda^*_R + (d_K K^* + d_V V^*)^3 \left(2\alpha_K K^* M \lambda^*_K + \alpha_V (V^*-M)^2 \lambda^*_V \varphi\right)} \tag{24}$$

We expect the sign of equation (24) to be negative, thus we need both numerator and denominator to be positive. Then the condition for the numerator is $-\frac{\lambda^*_V}{\lambda^*_K} < \frac{\alpha_K (d_K K^* + d_V V^*)^3}{(M-V^*)\alpha_R d_K{}^2 R^{*2}}$, which then contrasts with the condition for equation (22). For the denominator the last parenthesis needs to be positive, which then leads to $-\frac{\lambda^*_K}{\lambda^*_V} < \frac{\alpha_V (V^*-M)^2 \varphi}{2\alpha_K K^* M}$.

Appendix F: Indirect Deductions of the Implicit-function Theorem

$$\frac{\partial F^*}{\partial V^*} = \frac{\partial F^*}{\partial K}\frac{\partial K}{\partial V^*} < 0 \qquad \frac{\partial F^*}{\partial T^*} = \frac{\partial F^*}{\partial K^*}\frac{\partial K^*}{\partial T^*} > 0$$

$$\frac{\partial F^*}{\partial K^*} = \frac{\partial F^*}{\partial R^*}\frac{\partial R^*}{\partial K^*} < 0 \qquad \frac{\partial T^*}{\partial V^*} = \frac{\partial T^*}{\partial R^*}\frac{\partial R^*}{\partial V^*} > 0$$

$$\frac{\partial T^*}{\partial R^*} = \frac{\partial T^*}{\partial K^*}\frac{\partial K^*}{\partial R^*} > 0 \qquad \frac{\partial T^*}{\partial F^*} = \frac{\partial T^*}{\partial R^*}\frac{\partial R^*}{\partial F^*} > 0$$

$$\frac{\partial R^*}{\partial T^*} = \frac{\partial R^*}{\partial K^*}\frac{\partial K^*}{\partial T^*} > 0 \qquad \frac{\partial K^*}{\partial F^*} = \frac{\partial K^*}{\partial R^*}\frac{\partial R^*}{\partial F^*} > 0$$

$\frac{\partial V^*}{\partial F^*} = \frac{\partial V^*}{\partial K^*}\frac{\partial K^*}{\partial F^*} < 0$ $\frac{\partial V^*}{\partial T^*} = \frac{\partial V^*}{\partial K}\frac{\partial K^*}{\partial T^*} > 0$

Appendix G: The role of Reindeer's Non-use Value in Managing Crowberry

Incorporation of reindeer non-use value shows more realistic slaughter results in which the slaughter percentage is roughly 35% of the total herd. The non-use value also emphasizes the importance of existence of reindeer in that the herd size in both Post-EC and Adaptive Post-EC scenarios are larger than the results of the same scenarios where non-use value of reindeer is omitted. Especially, in the Adaptive Post-EC scenario, due to reindeer non-use value, more feeding and controlling efforts are invested than for the scenario without reindeer non-use value, leading to a minimal crowberry population (Table 13).

Table 13 Differences in the results at equilibrium with and without reindeer's non-use value

	Without reindeer's non-use value		With reindeer's non-use value	
	Post-EC	Adaptive Post-EC	Post-EC	Adaptive Post-EC
V^*	4.07×10^{10}	5.4×10^{10}	2.16×10^{10}	4.8×10^{10}
R^*	163 722	251 301	234 102	587 627
S^*	92 081	148 000	79 956	229 873
K^*	4.6×10^{10}	2.3×10^{10}	6.5×10^{10}	6.0×10^{9}
F^*	--	63	--	147.3
T^*	--	73.1	--	205.3
$\lambda_R{}^*$	3760	3760	3760	3760
$\lambda_V{}^*$	0.013	0.011	0.13	0.05
$\lambda_K{}^*$	−0.012	−0.016	−0.06	−0.17
$\bar{\varphi}$	0.011	0.014	0.014	0.02
Social welfare (mil NOK)	149.8	220.4	614.32	1 437.3

Acknowledgements We want to thank the Norwegian Research Council for funding this research (FRIPRO project MONEC, code 302749), and Dr. Asle Gauteplass from the Social Research Centre at the Norwegian University of Science and Technology (NTNU) for helping us in the model part to connect carrying capacities of two plants on one common pasture. We also want to thank Prof. Dr. Suzanne Lenhart at the Mathematics Department, University of Tennessee, Knoxville for assisting us in examining the singular case of the linear slaughter control. Finally, we would like to express our sincere gratitude to the six anonymous reviewers for their invaluable comments and feedback, which significantly contributed to the enhancement of this manuscript.

Author Contributions All authors designed the study. PN, CS, and CA conducted modelling while all authors contributed to parameter choice and analysis. The first draft was written by all authors, and all authors contributed to the final version.

Funding Open access funding provided by UiT The Arctic University of Norway (incl University Hospital of North Norway)
Research Council of Norway (project number 302749).

Declarations

Conflict of interest The authors have no conflict of interest to declare.

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