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ORIGINAL ARTICLE

Whole Stand Yield and Density Equations for the Basswood Forest Type in the Lake States

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Abstract

Whole stand yield models serve forest managers by providing projections of useful stand metrics such as height and volume. Climate shifts are currently being observed in the Lake States, including changes in forest composition and species' ranges. Basswood (*Tilia americana* L.) has potential to become a climate winner but lacks whole stand yield model parameters to adequately predict forest conditions. This paper aims to extend the utility of an existing whole stand yield model to basswood dominated stands in the Lake States to enable adaptation to shifting climatic conditions. Basal area, quadratic mean diameter, trees per acre, volume, and height models were fit using nonlinear least squares estimation. Models were validated with cross-validation, independent stand inventory data collected from a local forest management area, and national forest inventory data from more southerly states to evaluate model performance under climate conditions the Lake States could see in the future. Fit statistics for these models were consistent with other modeling efforts in the region, and validation statistics showed applicability of the models to broad basswood ranges and potential stability under changing climates. Forest managers can now utilize these new yield models to improve current and future management of the increasingly important basswood forest type.

Keywords Basswood · *Tilia americana* L. · Whole stand yield models · Lake States · Climate change · Species adaptation

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Introduction

One of the most important tools a forest manager can possess is a growth and yield model for stand development. Model projections inform decisions around timing and intensity of harvest and other silvicultural treatments. Statistical modeling is particularly helpful where managers may not have the resources to conduct a full survey of their target forests, assisting managers in visualizing current conditions and predicting future stand values such as height or volume (Peng 2000). Growth and yield models are not new in forest management, with many covering different tree species and forest types from conifers to maple-birch forests. In Minnesota, Walters and Ek (1993) modeled various stand conditions for fourteen different forest types. Using Forest Inventory and Analysis (FIA) data, they created whole stand yield models for basal area, height, and volume, among others. These models are widely used and applied in the Lake States region (e.g., Gilmore and Palik 2006; Stander et al. 2018).

As the climate shifts, forest habitat has already begun to change (McDowell et al. 2020). Having the ability to adapt old methods to fit current needs will be critically important to effectively manage future forests sustainably. In addition, the availability and/or utility of specific forest types may also change under altered climate conditions, leading to potentially new forest types filling the historical roles of current commercial forest types. For example, one Lake States forest type projected to respond favorably to future climate conditions is American basswood (*Tilia americana* L.), commonly known as basswood (Handler et al. 2014).

Basswood is a hardwood commonly found in the Lake States region to the corners of the east coast, United States. Basswood thrives in areas of cold winters (0˚F) to warm and humid summers (65˚F—85˚F). Thriving in sandy/silty loam soils, basswood inhabits river valleys and rocky bluffs (Burns and Honkala 1990). Basswood can reach heights between 75 and 135 feet and a diameter at breast height (DBH) between 36 and 48 inches at 50 years and older (Burns and Honkala 1990). Growing quickly and persistently, basswood supports local wildlife through the food chain (Von Althen 1971). Basswood has potential to supply harvested wood products due to its success in maintaining dominant status and larger than average DBH sizes in mixed forests (Shea and Helgeson 2018), although historical widespread management for basswood is limited in the Lake States. Basswood has no serious insect infestations but is very susceptible to wildfires (Nesom 2017).

The qualities of basswood make it potentially attractive to future forest management in the Lake States. Basswood's resiliency to changing climatic conditions is critical to a rapidly changing environment. The region is predicted to see temperatures rise in both summer and winter by 5 °C and 6 °C, respectively (Liess et al. 2022). Basswood has the potential to become a new dominant species in forest inventories as a result of warming conditions, with high projected growth in biomass in comparison with northern conifers in the future (Handler et al. 2014).

The commonly used whole stand yield equations of Walters and Ek (1993) did not model basswood directly but through the associated northern hardwood forest type. However, with the potential for basswood to increase in importance, there remains

an information gap when modeling stands dominated by basswood. This paper seeks to 1) extend the utility of an existing whole stand yield model system to basswood that will enable management under shifting climate conditions and 2) validate these models both locally (within the Lake States) and within more southerly regions with current climates that are projected to resemble those in the region in the future.

Methods

Data

The data used to develop the models was compiled from the US Department of Agriculture, FIA program (USDA 2024). This inventory collects detailed tree and stand level information on plots in the study region, with each plot representing roughly 3,000–6,000 acres. Approximately 14–20% of the plots are measured annually, leading to complete inventories collected every 5–7 years. See Bechtold and Patterson (2005) for further details on FIA protocols. To determine sites with a relatively dominant basswood component, units were selected if basswood was present and if the field crew 1) assigned the plots or FIA conditions the hard maple/basswood forest type in the field and 2) selected basswood as the associated site tree for determining site index. For the Lakes States (Minnesota, Wisconsin, and Michigan), this yielded $n = 281$ independent plots for model fitting across the years 2017–2021. All trees $\geq$1-in DBH were measured, with stand level variables compiled for all live trees on the selected plots, including basal area, quadratic mean diameter, number of trees, mean height, volume, age, and site index (from Carmean et al. 1989). Note that similar to other whole stand models (e.g., Walters and Ek 1993), all species on the plot were included in the stand metrics. Across all plots, the basswood component averaged 33.0%, ranging from 1.5% to 98.0%. Common associated species included sugar maple (*Acer saccharum*), eastern hophornbeam (*Ostrya virginiana*), red maple (*Acer rubrum*), black ash (*Fraxinus nigra*), bur oak (*Quercus macrocarpa*), American elm (*Ulmus americana*), and northern red oak (*Quercus rubra*). Figure 1 shows the approximate locations of the selected FIA plots.

Model Fitting

To model the growth and yield of basswood, five equations were utilized from Walters and Ek (1993).

$$B = a_1 S^{a_2} A^{a_3} \tag{1}$$

$$D = b_0 + b_1 A + b_2 S \tag{2}$$

$$N = exp\left(c_0 + c_1 lnD\right) \tag{3}$$

$$V = d_1 B^{d_2} H^{d_3} \tag{4}$$

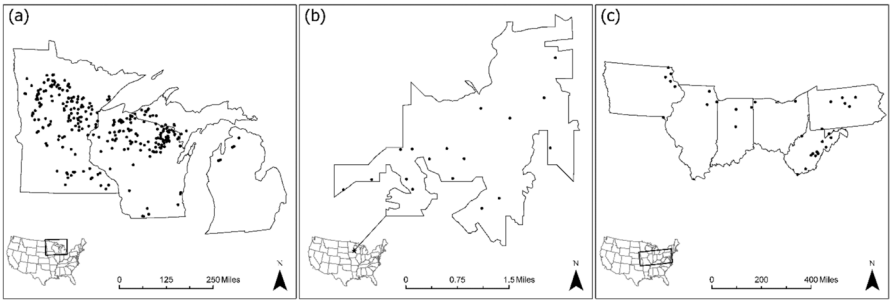


Fig. 1 Maps showing (**a**) basswood Forest Inventory and Analysis (FIA) plot locations (approximate) in the Lake States used in model fitting, (**b**) basswood forest inventory plot locations on the Zumbro Bottoms Management Unit used in model validation, and (**c**) basswood FIA plot locations (approximate) in the central states used in model validation

$$H = g_1 S^{g_2}(1 - exp(g_3 A))^{g_4 S^{g_5}} \quad (5)$$

where $B =$ basal area (ft^2 ac^{-1}), $S =$ site index (ft), $A =$ stand age (years), $D =$ quadratic mean diameter (in), $N =$ number of trees ac^{-1}, $V =$ gross volume (ft^3 ac^{-1}), $H =$ average total height of dominant/codominant trees (ft), and a_i, b_i, c_i, d_i, $g_i =$ specific model coefficients. Note that only two of the three models B, D, and N are necessary, given the relationship $B = 0.005454154 D^2 N$. Similar to Walters and Ek (1993), all three models were included to meet varying modeling needs, with users cautioned to use only two models during any given application.

For each of the variables of interest, summary statistics were calculated to screen the data for outliers and assess general data utility for modeling building. The nonlinear models were then fit using the nls function (Nash and Bhattacharjee 2024) in the R statistical program (R Core Team 2024). Diagnostic plots were examined for regression assumptions, and fit statistics including root mean square error (RSME), mean absolute deviation (MAD), and mean bias (MB) were calculated to evaluate model performance. Each fitted line was also examined for expected model behavior.

Validation

Three model validation procedures were employed to evaluate model performance beyond the data used to fit the models. First, leave-one-out cross-validation or predicted residual error sum of squares (PRESS) generated independent estimates from the original data. Second, the models were applied to data gathered between May – August 2023 by University of Minnesota personnel at the Zumbro Management Unit of the Richard J. Dorer Memorial Hardwood State Forest (Fig. 1). The Minnesota Department of Natural Resources (MN DNR) manages this state forest in southeast Minnesota, and data was collected using MN DNR plot based inventory protocols (MN DNR 2022). Filtering the data yielded $n = 17$ plots with an associated basswood site index tree.

Third, to test the models for future applicability under changing climate conditions, additional FIA data was compiled from the central US, including Iowa,

Illinois, Indiana, Ohio, West Virginia, and Pennsylvania (Fig. 1). The $n = 34$ independent plots were selected from 2015–2021 using the same protocols applied as the original data, with basswood comprising an average of 39.0% of the basal area (ranging from 3.9% to 100%). This data from more southerly latitudes that still fall within the distribution of basswood will help evaluate model performance in warmer climates, mimicking a potential future for Minnesota's own climate. Validation statistics RMSE, MAD, and MB were calculated for all three validation approaches.

Results

Parameter estimates for each model and associated fit and validation statistics are shown in Table 1 and footnotes. When compared against other local whole stand yield models (e.g., Walter and Ek 1993), model fit statistics appeared reasonable. Examination of residual plots suggested no substantial issues, particularly since these models will be used predictively (Gifford et al. 2023; Robinson and Wykoff 2004). The validation statistics for the three different validation approaches were all relatively close to the fit statistic values from the original models, except for mean bias for the two independent datasets (Table 1). Most of these biases were $< 10\%$, but in particular the quadratic mean diameter model showed relatively high biases (35–37%). This suggests the model tended to over-predict diameters in Zumbro and the central states, and users should consider adjusting or recalibrating the models before use in these locations. Still, overall the validation results demonstrate the applicability of the models for a variety of current and future basswood datasets.

Discussion

The whole stand yield model parameters appear to adequately estimate the attributes of interest for basswood forests. The fit and validation statistics compare to other whole stand models used in the region (e.g., Walters and Ek 1993; Zobel et al. 2015; Wilson and Ek 2019; Gifford et al. 2023). The exception was the quadratic mean diameter model that performed relatively poorly on the independent datasets, suggesting caution or recalibration before use on non-FIA data or use outside the Lake States.

The small sample sizes in the southeastern Minnesota dataset ($n = 17$) and central states ($n = 34$) reflect the present lack of dominance of the species in those regions and limits the strength of the validation. Even still, the validation results closely resembled the fit statistics. While this is not surprising for the cross-validation results (Kozak and Kozak 2003), the consistent results for the independent datasets in southeastern Minnesota and the central states suggest broad model transferability. In particular, the validation of the models on data from the more southerly central states is novel, considering current climate changes occurring in the region. A University of Minnesota Climate Adaptation Partnership used statistical downscaling to model climate shifts, including a degree difference of $+6^{o}$ F in parts of the state for maximum winter temperature (Handler et al. 2014). Since basswood currently thrives in the southern and more temperate regions of Minnesota, it is likely that

Table 1 Summary statistics for the Lake States (Minnesota, Wisconsin, and Michigan) Forest Inventory and Analysis (FIA) data (2017–2021). Also included are the fit and validation statistics[1] for each model and validation approach. Parameter estimates are included as a footnote[2]

	Statistic	Basal area ($ft^2\ ac^{-1}$)	Quadratic Mean Diameter (in)	Trees $acre^{-1}$ (TPA)	Volume ($ft^3\ ac^{-1}$)	Height (ft)	Age (years)	Site Index (ft at base age 50 years)
Summary statistics	Mean	127	6.7	654	3081	69	80	64
	St. Dev	43	2.4	412	1395	11	18	11
	Min	27	2.7	29	182	35	11	34
	Max	339	18.5	2538	11557	103	145	94
Fit statistics	RSME	42	2.3	233	388	8		
	MAD	30	1.7	158	290	7		
	MB	0	0.0	−3	−6	0		
Validation (Cross-validation)	RSME	42	2.3	236	404	9		
	MAD	30	1.7	159	296	7		
	MB	0	0.0	−3	−6	0		
Validation (Zumbro)	RSME	43	5.2	180	326	11		
	MAD	32	3.9	143	240	9		
	MB	3	2.4	−36	−30	−7		
Validation (Central states[3])	RSME	48	4.2	140	408	12		
	MAD	41	3.0	99	275	10		
	MB	16	2.5	−15	−180	5		

[1]RMSE = root mean square error; MAD = mean absolute deviation; MB = mean bias

[2]Model parameter estimates:

$a_1 = 23.02497$, $a_2 = 0.05171$, $a_3 = 0.34294$

$b_0 = 2.38393$, $b_1 = 0.02976$, $b_2 = 0.03108$

$c_0 = 9.163$, $c_1 = -1.507$

$d_1 = 0.09628$, $d_2 = 0.99072$, $d_3 = 1.30885$

$g_1 = 8.6947$, $g_2 = 0.53374$, $g_3 = -0.01925$, $g_4 = 0.14814$, $g_5 = 0.29675$

[3]States include Iowa, Illinois, Indiana, Ohio, West Virginia, and Pennsylvania

the range of basswood will spread north as temperatures warm. Considering basswood has potential to become a more common species found in the Lake States, these broadly applicable yield equations will serve both current and future modeling efforts by forest managers. This study fills a modeling gap that will benefit current and future management of basswood in the region.

Others have explored the adaptation of forest management strategies to shifting species ranges and changing forest compositions (e.g., Nagel et al. 2017; Crotteau et al. 2019; Muller et al. 2019). While much previous climate adaptation research has focused on silvicultural treatments, this work explored a potential strategy to adapt growth and yield models to a shifting climate. By extending the utility of well-established models (i.e., Walters and Ek 1993) and using southerly data to represent potential future conditions, this work provides an example for exploring climate adaptation in growth and yield modeling.

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Data Availability Forest Inventory and Analysis data is publicly available from the USDA Forest Service at https://research.fs.usda.gov/products/dataandtools/tools/fia-datamart. Additional data and material used in this work can be obtained upon reasonable request to the corresponding author.

Declarations

Competing interests The authors have no competing interests to declare.

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