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Statistical Inference for Diameter-Height Curves using Varying Coefficient Model

Tetsuji Tonda[1*], Ken-ichi Kamo[2], Masayoshi Takahashi[3]

Abstract: Diameter-height curve is a function that defines the relationship between diameter at breast height and tree height. It is the mean trend on a scatter plot that shows the relationship between tree diameter at breast height and tree height. This curve shifts to the upper right corner as the tree ages in a uniformly aged plantation forest. Hence, it is commonly used to estimate the parameters of a model that describes the diameter-height curve by tree age. The transition in diameter-height curve with age can be interpreted as a change in the parameters of the model over time. However, the behavior of the parameters estimated independently from age-specific data may not accurately capture the features in the transition of the diameter-height curves, due to the variabilities of observations within each age of a forest stand. Against this background, in this paper, we introduce varying coefficients into the regression model for diameter-height curve to describe the transition of diameter-height curves as a function of time. The proposed method is then applied to a forest growth data of sugi (*Cryptomeria japonica*) stand in a village in Hoshino, Japan.

Keywords: confidence interval, diameter-height curve, varying coefficient

1. Introduction

The diameter-height curve, also known as a stand height curve, is a mean trend which describes the relationship between tree height and tree diameter at breast height (abbreviated as "DBH"); at a given point in time, in a forest stand. The DBH can be measured directly using a measuring instrument, such as a diameter tape. On the other hand, the height of a tree can either be measured directly using an instrument such as a Suunto Clinometer, or indirectly by using diameter-height curves, because of the difficulty of direct height measurements. The diameter-height curve is the mean trend on the scatter plot showing the relationship between tree height and DBH. In general, as tree height increases, the DBH also increases, and thus over time, shifts the diameter-height curve in the upper right direction on the scatter plot. The diameter-height curve is useful in the estimation of tree volume, using the two-way volume equation. See Osumi (1971) and Nagumo and Minowa (1990) for detailed explanation of diameter-height curve.

The Näslund model (Näslund 1936) is one of the most commonly used diameter-height curves and has often been used in the analysis of even-aged plantation forests in Japan. Takata (1958) pointed out that the Näslund model is highly accurate in showing the relationship between DBH and tree height in sugi (*Cryptomeria japonica*) stand. The Näslund model has also been used to develop techniques for measuring individual sugi trees using high-resolution satellites (Hirata et al. 2009), LiDAR (Nakajima et al. 2008), and UAVs (Ota et al. 2017). Mathematically, the Näslund model describes the relationship between DBH and tree height as follows:

$$[1] \qquad H = \alpha + \frac{D^2}{(\beta + \gamma D)^2}$$

Note that the variables D and H are the observations for DBH [cm] and height of tree [m], respectively. The value of the intercept α is fixed at 1.2 [m] above ground, which corresponds to height at breast height. Thus, the unknown parameters in the Näslund model are β and γ. As mentioned above, in a uniformly aged plantation forest, the diameter-height curve shifts in the upper right direction as the tree ages. It is often used to estimate the parameters of the diameter-height curve

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[1]Prefectural University of Hiroshima, Japan
[2]Sapporo Medical University, Japan
[3]Forestry and Forest Products Research Institute, Japan
*Corresponding Author: ttetsuji@pu-hiroshima.ac.jp
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for each tree age, as explained in section 3. Even though past measurement data were accumulated, only the data on tree age at the time of the survey were used for parameter estimation. The fitting of the diameter-height curve to the data varied with the age of the trees, due to environmental factors such as the climatic condition at the time of growth.

Let D_{it} and H_{it} be the observed values of DBH and tree height at age $t(= t_1, t_2, \ldots, t_m)$ for sample i in a forest stand of n trees; and let β_t and γ_t be the values of each parameter in Eq.[1] at age t. The upper right transition of the diameter-height curve with increasing tree age is represented by the temporal change of parameters β_t and γ_t. However, the estimated values of parameters β_t and γ_t, based only on the observed data at tree age t, do not adequately capture the changes of β_t and γ_t over time, due to the influence of the variability of the observed data at each point in time. In this study, we introduce varying coefficients (Hastie and Tibshirani 1993). A model with varying coefficients is a class of regression models in which the coefficients are allowed to vary as smooth functions of other variables, such as time. For example, the "time" varying coefficient can be used for visualizations or interpretations of the time-dependent effect of the explanatory variables. Konoshima et al. (2021) evaluated the effect of surrounding land uses on the extents of freshwater body over time in Madagascar using the "time" varying coefficients. Incorporating the time varying coefficients into the Näslund model allows us to describe the temporal change of the diameter-height curve with increasing tree age, in terms of time varying coefficients $\beta(t)$ and $\gamma(t)$.

The rest of the paper is organized as follows. In Section 2, we introduce the varying coefficients into the Näslund model for the diameter-height curve as expressed in Eq.[1]. We then construct an estimating method of varying coefficient, with confidence intervals. In Section 3, we apply both the conventional (see the discussion section for details), and the proposed methods to the data on sugi (*Cryptomeria japonica*) stands in Hoshino village, Japan (Yoshimoto et al. 2012). Finally, we summarize the usefulness of the proposed method and future issues by comparing the results of the conventional and proposed methods in Section 4.

2. Method

In this section, we explain the model building process using varying coefficients (Hasti and Tibshirani 1993), and explain the estimating method for varying coefficient for the Näslund model, as shown in Eq.[1]. To estimate the unknown parameters in Eq.[1], the following linear representation is used.

$$[2] \qquad \frac{D}{\sqrt{H-\alpha}} = \beta + \gamma D$$

By substituting Y for $D/\sqrt{H-\alpha}$, the parameter β and γ can be estimated using the least-squares method. We now introduce the varying coefficients (Hastie and Tibshirani 1993) into the Näslund model for the transformed scale $Y = D/\sqrt{H-\alpha}$ in Eq.[2]. Let D_{it} and $Y_{it} = D_{it}/\sqrt{H_{it}-\alpha}$ be the observed values of DBH and transformed height of tree at age $t(= t_1, t_2, \ldots, t_m)$ in a forest stand of n trees. Then, the Näslund model for transformed scale can be defined as:

$$[3] \qquad E[Y_{it}] = \beta(t) + \gamma(t) D_{it},$$

where $\beta(t)$ and $\gamma(t)$ are varying coefficients which depend on tree age t.

Several estimating methods for varying coefficients have been proposed. Satoh and Yanagihara (2010) proposed an estimating method in the framework of growth curve model (Potthoff and Roy 1964) by assuming a linear structure in the shape of the varying coefficient. A similar linear structure which is assumed for $\beta(t)$ and $\gamma(t)$ is defined by:

$$[4] \qquad \beta(t) = \boldsymbol{b}'\boldsymbol{x}(t), \quad \gamma(t) = \boldsymbol{c}'\boldsymbol{x}(t),$$

where $\boldsymbol{b}$ and $\boldsymbol{c}$ are unknown parameter vectors with length q, and $\boldsymbol{x}(t)$ is a vector of basis functions with length q that defines the shape of the varying coefficient. For example, if we fit a polynomial curve of degree $q-1$ in terms of time, those are taken to be polynomial basis functions, i.e., $\boldsymbol{x}(t) = (1, t, \ldots, t^{q-1})'$, then, Eq.[3] becomes:

$$[5] \qquad E[Y_{it}] = \boldsymbol{b}'\boldsymbol{x}(t) + \boldsymbol{c}'\boldsymbol{x}(t) D_{it} = \begin{bmatrix} \boldsymbol{b}' & \boldsymbol{c}' \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t) D_{it} \end{bmatrix} = \boldsymbol{\theta}' \boldsymbol{x}_{it},$$

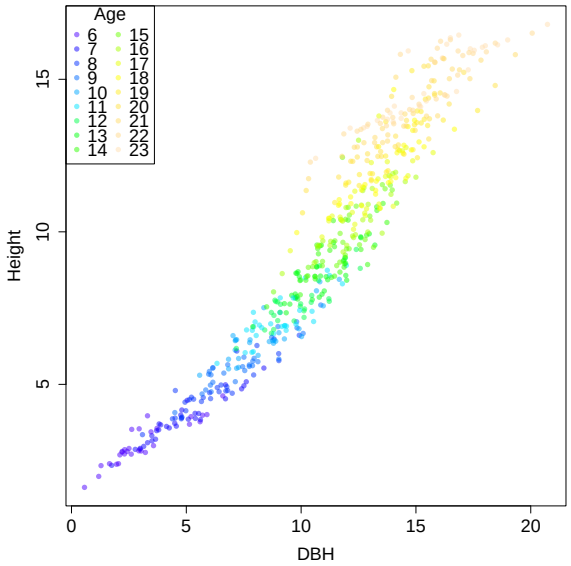


Figure 1. Scatter plot of height of tree against DBH by ages.

where $\boldsymbol{\theta} = (\boldsymbol{b}', \boldsymbol{c})'$, $\boldsymbol{x}_{it} = (1, D_{it})' \otimes \boldsymbol{x}(t)$ and the symbol $\otimes$ is a matrix operator called the Kronecker product (Scott 2016). The model in Eq.[3] is represented by the linear regression model in Eq.[5] and the unknown parameter $\boldsymbol{\theta}$ can be estimated by the least-squares method. Once the estimation of $\boldsymbol{\theta}$ as $\hat{\boldsymbol{\theta}}$ is known, the varying coefficient $\beta(t)$ and $\gamma(t)$ can be estimated as follows:

$$[6] \qquad \hat{\beta}(t) = \hat{\boldsymbol{b}}'\boldsymbol{x}(t), \quad \hat{\gamma}(t) = \hat{\boldsymbol{c}}'\boldsymbol{x}(t).$$

From the asymptotic normality of $\hat{\boldsymbol{\theta}}$ based on the theories of linear models (see e.g., Rao 2002; Kollo and von Rosen 2005), the sample distributions of $\hat{\beta}(t)$ and $\hat{\gamma}(t)$ are as follows:

$$[7] \qquad \hat{\beta}(t) \sim N\left(\beta(t), \lambda(t)^2\right), \quad \hat{\gamma}(t) \sim N\left(\gamma(t), \omega(t)^2\right),$$

where $\lambda(t)^2 = \boldsymbol{x}(t)'\boldsymbol{\Sigma}\boldsymbol{x}(t)$, $\omega(t)^2 = \boldsymbol{x}(t)'\boldsymbol{\Gamma}\boldsymbol{x}(t)$, and $\boldsymbol{\Sigma}$, $\boldsymbol{\Gamma}$ are the asymptotic covariance metrices for $\hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{c}}$.

Next, we construct $100\alpha\%$ confidence intervals for varying coefficients $\beta(t)$ and $\gamma(t)$ which is expressed as follows:

$$[8] \qquad \left[\hat{\beta}(t) - u_\alpha\hat{\lambda}(t),\ \hat{\beta}(t) + u_\alpha\hat{\lambda}(t)\right], \quad \left[\hat{\gamma}(t) - v_\alpha\hat{\omega}(t),\ \hat{\gamma}(t) + v_\alpha\hat{\omega}(t)\right],$$

where u_α and v_α are thresholds determined by the confidence level α. Several methods have been proposed for deriving the thresholds for confidence intervals for varying coefficients. Here, we adopt the method of constructing confidence intervals proposed by Satoh and Yanagihara (2010), Tonda and Satoh (2013) because of the simplicity of the derivation. The threshold of the simultaneous confidence interval for varying coefficient is obtained by the squared root of the upper $100(1-\alpha)\%$ quantile for chi-square distribution with q degrees of freedoms.

3. An example for analysis using real data

In this section, we apply the conventional and proposed methods to data on sugi (*Cryptomeria japonica*) stands in a village in Hoshino, Japan (Yoshimoto et al. 2012). This data was collected from measurements of tree height, DBH, and tree volume for 30 trees in Hoshino Village, Fukuoka Prefecture, Japan. The minimum distance between individual sugi trees is 1.2 meters. The densities of sugi trees inside a circle of radius 2 meters around the target sugi tree were distributed in a range of less than 0.11 [trees/m^2] to 0.48 [trees/m^2]. Figure 1 shows a scatterplot of tree height against DBH, colored by tree age. Data for trees older than 6 years old with non-zero DBH observations

for all trees were included in the analysis. Figure 1 shows the upper right transition of the points in the scatterplot of tree height against DBH as tree age increases.

In the existing methods (Osumi 1971; Nagumo and Minowa 1990), the unknown parameters are estimated by the least-squares method for the transformed scale of tree height as expressed in Eq.[2], using data by tree age. Figure 2 shows the fitted diameter-height curves using the Näslund model for data by tree age, with its coefficient of determination for regression line shown in red. The black dots represent data for a particular tree age, as against all data shown by gray dots. The fitted results in Figure 2 show that the coefficient of determination was 0.95 at tree age 6 and 0.97 at tree age 7. Thereafter, the coefficient of determination gradually decreased with age, and reached 0.70 at tree age 15. However, the trend of coefficient of determination changed and recovered to 0.96 at tree age 23.

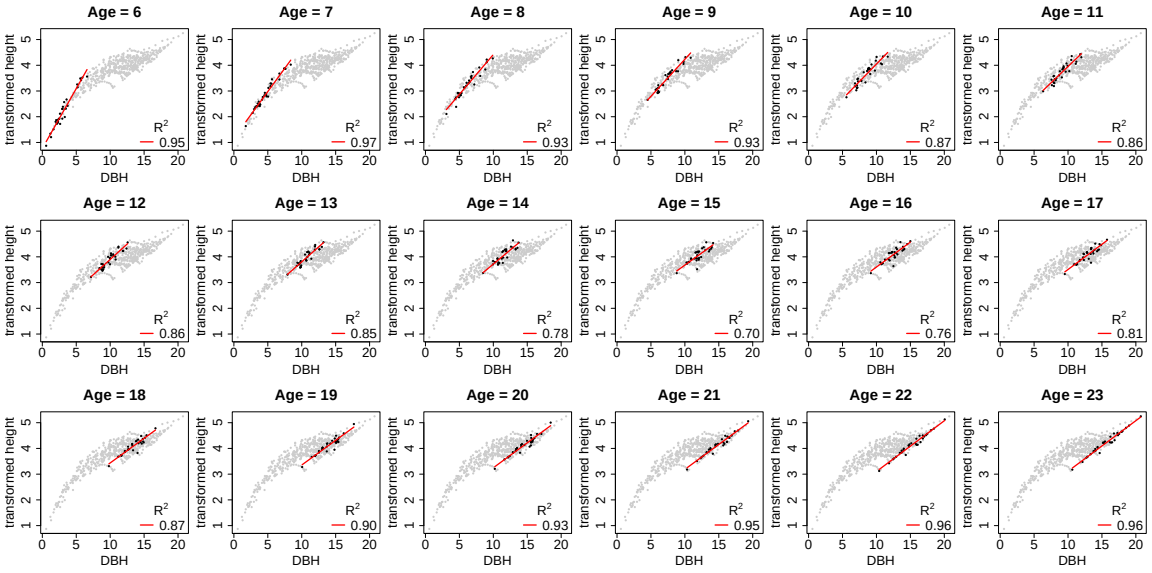


Figure 2. Diameter-height curve fitted to the transformed height by Näslund model for data by tree age.

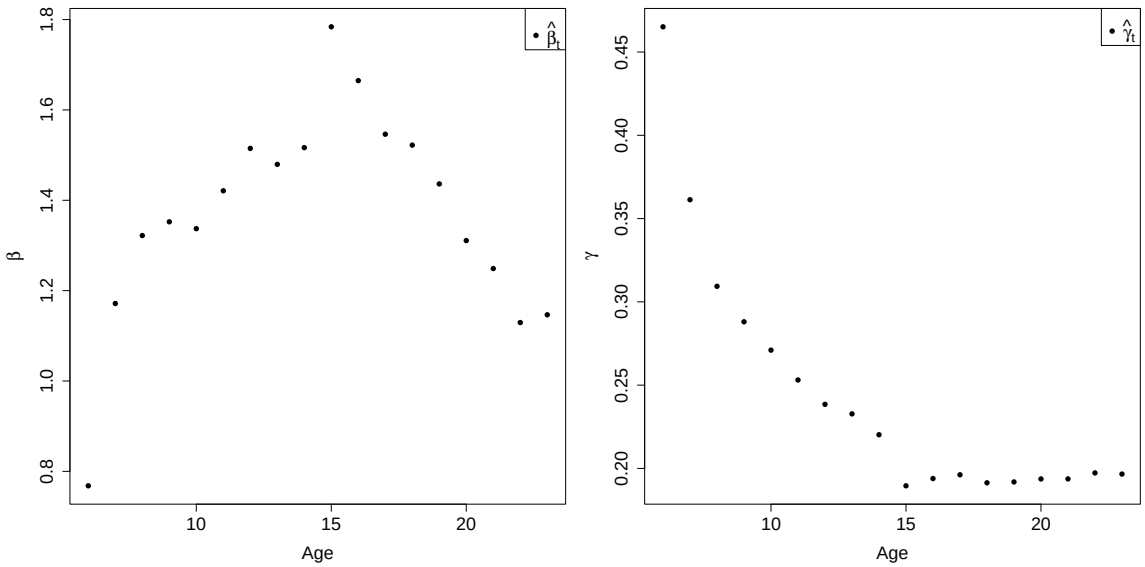


Figure 3. Estimates of parameters in Näslund model for data by tree age.

Figure 3 shows the scatterplot of estimated values of β_t, γ_t against the tree age t. The estimates of β_t increased with tree age and peaked at tree age 15, and then decreased. The estimates of γ_t decreased until tree age 15, and then stayed nearly constant as tree age increases. As mentioned in the introduction, the parameter β_t and γ_t in Näslund model at tree age t is estimated only by observed data at a given tree age t. Therefore, the result in Figure 3 may not adequately capture the changes of β_t and γ_t over time due to the influence of variability of the observed data at each point in time.

The proposed method introduces the varying coefficient $\beta(t)$ and $\gamma(t)$ in Eq.[4] to describe temporal changes in the parameters of Näslund model. A polynomial curve was employed for the structure of varying coefficients $\beta(t)$ and $\gamma(t)$ as shown in Eq.[5], and model selection was performed using AIC (Akaike, 1973) for the optimal degrees. Figure 4 shows the values of AIC against several degrees of polynomial on the shape of varying coefficients $\beta(t)$ and $\gamma(t)$. Figure 4 suggests that the optimal degree is $q-1=6$. The estimated curves of the varying coefficients $\beta(t)$ and $\gamma(t)$ based on the optimal model and their 95% confidence intervals are shown in Figure 5.

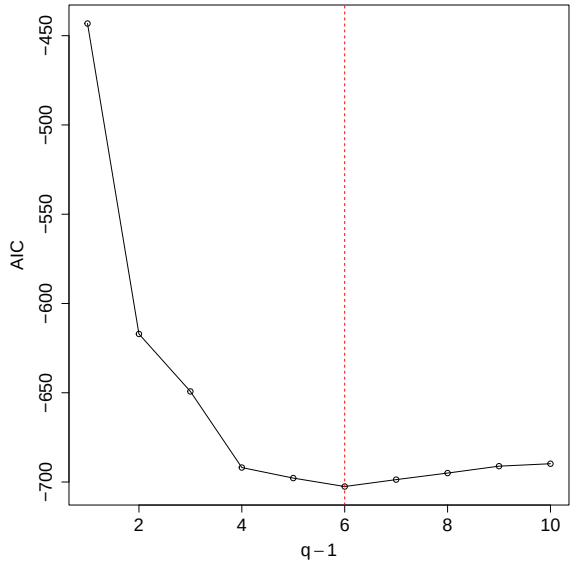


Figure 4. Model selection for varying coefficient in Näslund model.

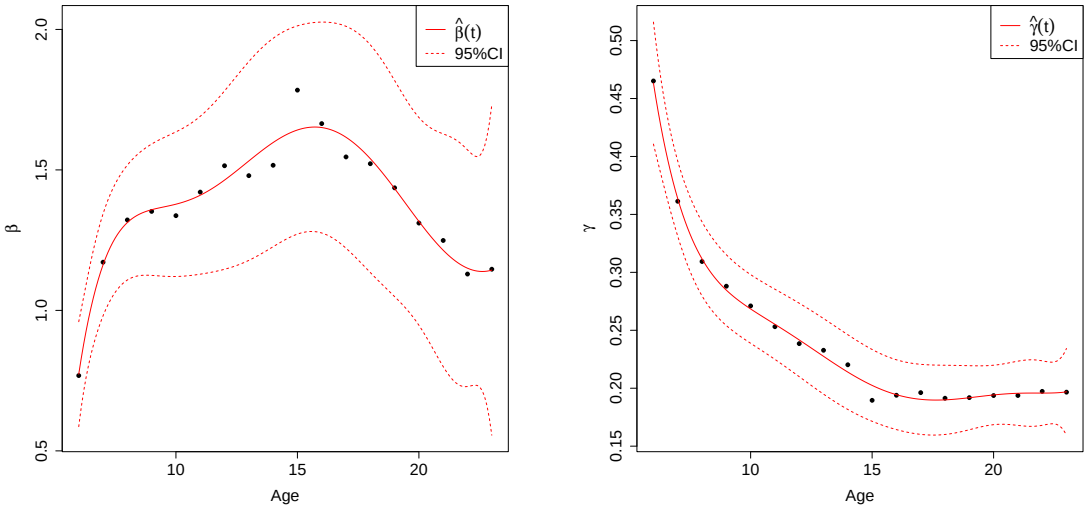


Figure 5. Estimated varying coefficient with its confidence interval.

Figure 6 shows the estimated diameter-height curves based on Näslund model using varying coefficients.

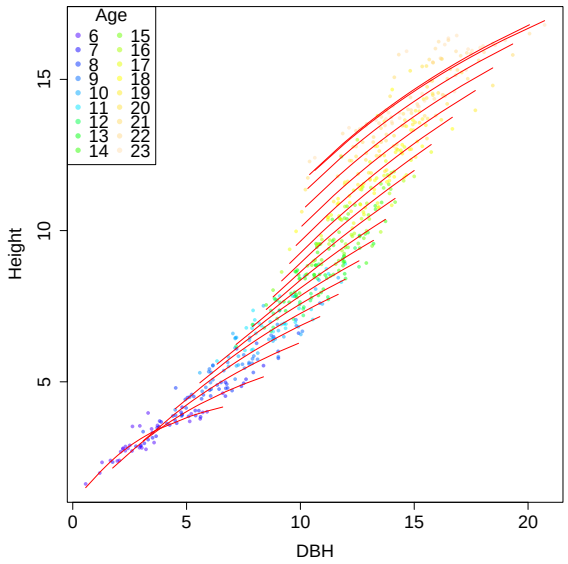


Figure 6. Estimated diameter-height curves in Näslund model with varying coefficients.

4. Discussion

In this paper we introduced a varying coefficient into the Näslund model for diameter-height curve and then constructed the estimating method of varying coefficient with confidence interval. The diameter-height curve moves in the upper right direction as the trees ages in a uniformly aged plantation forest (Figure 1). Several models have been proposed to describe this relationship between DBH and tree height. Nagumo and Minowa (1990) introduced the commonly used five diameter-height curves in Table 1. Note that the Henrickson and Stoffels models use the natural logarithm of DBH as the explanatory variable, and Torey model restricts the intercept to zero in the linear representation. The proposed method is also applicable to the other models in Table 1. Selecting the appropriate model from several candidate models would require the use of the information criterion (e.g., Kamo and Yoshimoto, 2013).

Table 1. The models for diameter-height curves with transformed scales for linear representations.

Model	Equation	Linear representation
Inverse model	$H = \alpha + \dfrac{D}{\beta + \gamma D}$	$\dfrac{D}{H-\alpha} = \beta + \gamma D$
Henrickson model	$H = \beta + \gamma \log D$	$H = \beta + \gamma \log D$
Näslund model	$H = \alpha + \dfrac{D^2}{(\beta + \gamma D)^2}$	$\dfrac{D}{\sqrt{H-\alpha}} = \beta + \gamma D$
Stoffels model	$H = \alpha + \beta D^{\gamma}$	$\log(H-\alpha) = \log\beta + \gamma \log D$
Torey model	$H = \alpha + \beta D - \gamma D^2$	$H - \alpha = \beta D - \gamma D^2$

It is common practice to estimate the parameters of the diameter-height curve using data specific to a given tree age (e.g., Osumi 1971; Nagumo and Minowa 1990). The upper right transition of the diameter-height curve for a given age, is described as a temporal change of parameters in

the model. However, the behavior of the parameters, estimated independently from age-specific data, may not accurately capture the features of the upper right transition of the diameter-height curves due to the variabilities of observations for each age in the forest stand. In addition, the accuracy of the estimation becomes low because the data are divided by tree age. This is because, the smaller the sample size due to splitting of the data, the more unstable the estimation becomes. This is a drawback. To overcome these problems, we proposed modeling the upper right transition of the diameter-height curve using the varying coefficients. The proposed method can estimate the transition of the diameter-height curve to the upper right, via varying coefficients which are estimated for the whole data.

Results from Figure 5 shows that the estimated curves of varying coefficients $\beta(t)$ and $\gamma(t)$ were similar to the behaviors of the estimates shown in Figure 3. The advantage of the proposed method compared to the existing method is the ability to construct a confidence interval for the curve of temporal variation. The behavior of $\hat{\gamma}(t)$ tends to decline until age 15 and then remains fairly constant thereafter. Thus, the diameter-height curve shows an upward convex monotonically increasing trend at younger ages. This convex trend tends to decrease with age until 15. The larger the value of $\gamma(t)$ in the young plantation stands, indicates that the curvature of the diameter-height curve will be large. This means that the positive gradient between tree height and DBH tends to decrease as tree height increases. This phenomenon in young plantation stands is called height convergence (Nagashima and Hikosaka 2011; Kunisaki 2001). It can be regarded as a kind of ecological interruption among young plantations stands. The decrease in $\gamma(t)$ with increasing tree age means that the ecological interruption tends to disappear. In Figure 5, we can see that there are divergences between estimates based on age-specific data (black dots) and the estimated curve of varying coefficient (red curve) around tree age 15. This means that the relatively large variation in the observations around tree age 15 makes the estimates of existing method less accurate. In fact, the values of the coefficient of determination around age 15 in Figure 2 were relatively low.

This change in trend before and after tree age 15 suggests the occurrence of a change in factors related to tree growth of sugi trees. The growth data used were collected from a plantation forest of sugi trees in Hoshino Village. Unfortunately, the detailed information on how this forest was planted and managed was not provided. The standard method of planting sugi tree during this period is as follows. First, sugi trees are planted using a planting density of 3000[trees/ha]. Weeding is required for tree sapling (young trees) for a period of 4 to 5 years to reduce tree competition with weeds. As the planted trees grow, there will be less competition between the planted trees and weeds. Further growth in tree height will only see competition among trees. To mitigate tree competition and enhance tree growth thinning treatment is employed as a stand management technique. In other words, in the initial growth cycle of the plantation, the competition is between the planted trees and the weeds, but as the trees ages, the competition shifts to, between trees. In Hoshino plantation forest, tree age of 15 might be the tree age where there is a shift (transition) from inter-species competition to intra-species competition.

The conventional method of estimating diameter-height curves by age is equivalent to fitting the following regression model with different coefficients for each tree age to the whole data.

$$[9] \qquad E[Y_{it}] = \beta_t + \gamma_t D_{it}, \quad t = t_1, t_2, \ldots, t_m, \quad i = 1, 2, \ldots, n.$$

Here, we refer to Eq.[9] as the conventional model. The conventional model contains 36 parameters in total, as there are 18 observation time points. In contrast, the proposed model, which corresponds to Eq.[3] with $q - 1 = 6$, contains 14 parameters. The residual variances for the conventional and proposed models were 0.016 and 0.015, respectively. While the number of parameters in the proposed model is fewer than that of the conventional model, the residual variance was slightly smaller in the proposed model. In addition, the AIC values of the conventional and proposed models were -663.1 and -702.5, respectively. The proposed model is superior to the conventional model in terms of model selection for prediction purposes.

Plantation forest of sugi trees is generally planted at regular intervals for a given tree density. Just after planting, saplings must compete with fast-growing weeds and grasses including fast growing tree species. As individual saplings gradually grow up, another competition occurs between planted sugi trees. In the case of plantation forest of sugi trees in Hoshino Village which was used in this study, competition among planted sugi trees began to occur around tree age 15. We speculate that the growth trend may have changed as a consequence. In Fukuoka prefecture, the first thinning is

recommended at tree age 12, at a planting density of $3,000$ [trees/ha]. At tree age 16, the recommendation is to target a planting density of $2,500$ [trees/ha] (Fukuoka Prefecture 2014). The aim of thinning is to reduce competition between planted trees. It is inevitable that the recommended timing of the first thinning based on empirical findings coincides with the point of change in the trend in the present sample data. In addition, the timing of thinning recommended by Fukuoka Prefecture is likely to be the best time to reduce competition among planted trees in the forests in Hoshino Village, from which the sample data was obtained.

Finally, we summarize the issues and future development of the proposed method in this paper. The first issue is the choice of the basis function that describes the shape of the varying coefficient. In this paper, polynomial bases were employed for simplicity of mathematical processing, but there are difficulties in describing temporal changes using polynomial bases when observations are collected at many points in time. Various alternative basis functions have been proposed in the framework of regression models (e.g., Ruppert et al. 2003). The problem of overfitting arises when more flexible basis functions are used, then the penalty terms to control flexibility of varying coefficients are needed.

In this paper, we constructed the method estimating varying coefficients for diameter-height curves within the framework of a linear model using the transformation in Eq.[2]. This is convenient in the sense that the varying coefficients can be estimated using the same method as the conventional method. However, as Kubo (2003) pointed out, the transformation with explanatory variables may lead to pseudo correlations. Therefore, the second issue is to develop an estimating method under the framework of nonlinear model as in Eq.[1].

The third issue is modeling the effects of environmental factors that affect the temporal changes in the diameter-height curve. The model building process of growth curve models (e.g., Kollo and von Rosen 2005) could be used to construct a model that incorporates regression structures to the parameters of the varying coefficients in Eq.[4]. It is expected that the estimating method of diameter-height curves using the varying coefficients will be extended to the prediction of diameter-height curves at future ages based on past accumulated data.

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Disclosure Statement

The authors declare there are no competing interests.

Data Availability

This manuscript does not report data.

Author Contribution

Conceptualization: TT, KK, TM
Funding acquisition: TT, KK, TM
Investigation: TT, TM
Methodology: TT, KK
Project administration: TT, KK, TM
Supervision: N/A
Writing - original draft: TT
Writing - review & editing: TT, KK, TM

ORCID iDs

Tetsuji Tonda https://orcid.org/0009-0005-9093-4142
Ken-ichi Kamo https://orcid.org/0000-0002-6788-7684

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