INSTITUT DE FRANCE
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Comptes Rendus
Mécanique

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Volume 348, issue 6-7 (2020), p. 659-678

Published online: 16 November 2020
Issue date: 16 November 2020

https://doi.org/10.5802/crmeca.47

Part of Special Issue: Tribute to an exemplary man: Yves Couder
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Les Comptes Rendus. Mécanique *sont membres du Centre Mersenne pour l'édition scientifique ouverte*
www.centre-mersenne.org
e-ISSN : 1873-7234

Comptes Rendus
Mécanique
2020, 348, nº 6-7, p. 659-678
https://doi.org/10.5802/crmeca.47

Tribute to an exemplary man: Yves Couder

Morphogenesis, elasticity

A work on reticulated patterns

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Abstract. A work on analyzing and modeling reticulated pattern is told through a personal point of view, showing its development trough many people inputs, including that of Yves Couder. The question of how to describe a reticulated pattern as *Ficus religiosa* leaf veins and make a physical model reproducing it, led to study cracks in colloids or clay, city streets plans, gorgon skeletons and variation in fern vein patterns.

Résumé. Un travail d'analyse et de modélisation de motifs réticulés est raconté à travers un point de vue personnel, montrant son développement à travers l'apport de nombreuses personnes, dont Yves Couder. La question de savoir comment décrire un motif reticulé, comme les veines d'une feuille de *Ficus religiosa*, et réaliser un modèle physique le reproduisant, à conduit à l'étude des fissures dans les colloïdes ou l'argile, du plan des rues des villes, du squelette des gorgones et de la variation des motifs des veines de fougères.

Keywords. Reticulated patterns, Leaves veination, Cities, Ferns, Cracks, Gorgonia.

Mots-clés. Réseaux réticulés, Veination de feuilles, Villes, Fougères, Craquelures, Gorgones.

> *"[...] the isolated man does not develop any intellectual power. It is necessary for him to be immersed in an environment of other men, whose techniques he absorbs during the first twenty years of his life. He may then perhaps do a little research on his own and make a few discoveries which are passed on to other men. From this point of view the search for new techniques must be regarded as carried out by the human community as a whole, rather than by individuals."* Alan Turing, *Intelligent machinery*, NPL report, 1948.

* Corresponding author.

1. Introduction

The story of scientific discoveries and the meandering that can lead to them is rarely told, and fade compared to the reconstructed logical discourse, if not mythical (mostly fabricated) communication stories, like Newton's apple or Flemming's petri dishes.[1] Let it then be a little example, seen from a single personal point of view (Stéphane Douady), but involving many people.

After the work with Yves Couder on phyllotaxis, we were invited to a conference in Calcutta (India) in 1993. I used this invitation to travel afterward, in particular to BodhGaya, in Bihar. There the MahaBodhi temple was build around the tree under which Buddha obtained his enlightenment. The tree is a *Ficus* named afterward *Ficus religiosa*, as it ended up being planted in all Buddhist temples in Asia. It is anyway common around the tropic, and is used for instance as alignment trees in la Havana, Cuba. The temple, as an antique Hindu temple, presents pools of water for ritual ablutions. The leaves of the tree fall in the pool water. These leaves are very thick, and in the water the upper and lower epidermis detach, and only the skeleton of lignified veins remains. Since antiquity it is traditional to take such clarified leaves, sometimes painted, as souvenir. So did I.

Once back in Paris, I started to look more closely at them (Figure 1), and wondered how such pattern could be quantitatively described. There were good descriptions of tree structures, but not of reticulated ones that I knew. And how one could build a model that would produce such patterns, since the models I knew, like Saffman–Taylor from Yves' work, would produce only trees, and fingers would precisely never reconnect [1]. For me, these two questions are still running.

2. Quantification—1: vein angles

The structural analysis of trees is essentially based on the fact that they do not make loops, or at least that they are clearly oriented. The traditional way, for instance used to gives the names in river systems, would start from the exit of the tree, its root, in the sea. Then climbing up, and, at each bifurcation, deciding with a local criteria, like the largest mean flow, which element remains the same, and which is a new element. Some geographers made it more complicated as noticing the flows irregularity, they would rely on another criteria, the overall length, assuming that the rain being equal, the longer element would collect globally the most water. This makes it more complicated since it needs a global measurement on the whole tree to decide at each bifurcation which are the new longest elements. This is one of the origin of the controversies of the still running question of the spring of the Nile.[2]

To look at this question differently, a hydrologist, Hotton, proposed a different way, starting this time from all the springs of a river basin [2]. Strahler later refined it, saying that the status of the element down the confluence is increased only if the two converging elements have the same status [3]. This gives a good quantification of the tree, revealing many power laws, and is widely

[1] The infamous paragraph of Newton's letter remembering looking young at an apple falling under a tree is probably a forgery, during his battle of recognition with Hooke, who came to the idea from the planets motion. Flemming's petri dish story is a patriotic story build at the end of the second world war. Flemming can be credited of selecting the right mould allowing the industrial production of penicillin to be provided to the mass of wounded during WWII, but the discovery itself was made several time, for example by Ernest Duchêne or René Dubos, from indigenous north African knowledge and ecological reasoning respectively.

[2] See for instance "The unsettling sources of the Nile", Christopher Ondaatje, Geographical, Published in Explorers 13 Jul 2016, https://geographical.co.uk/people/explorers/item/1799-the-unsettling-sources-of-the-nile

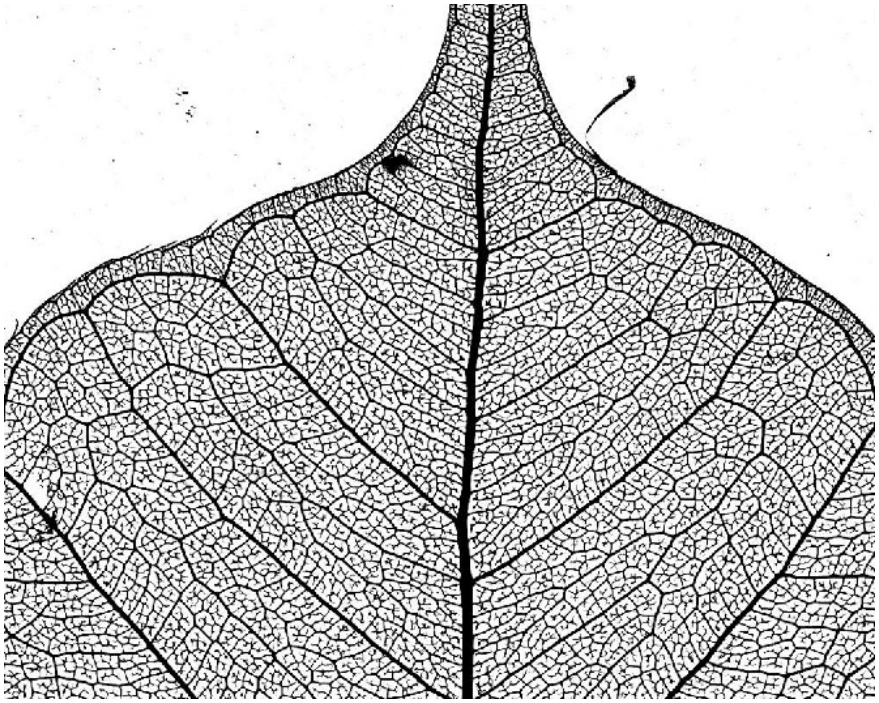

Figure 1. A typical part of a clarified leaf of *Ficus religiosa* (from BodhGaya). One can see the reconnections of the lateral secondary veins, the round top with the beginning of the pointed tip, and the small reticulation running along the border (S. Douady).

used in many domains where trees appears, like mathematics and informatics. But this second method needs dead ends, and does not work when loops are present.[3]

The question was how to treat the loops directly, and describe the shape. Looking at the details of the veins, it seemed that the angles at which they connect were varying depending on the respective size of the veins. I was precisely looking at this when Yves, passing by the corridor, asked me casually if there would be a thesis subject for a young trainee. I answered that quantifying these patterns was a deep and wide subject, and looking at the angles was a good starting point. This is how the thesis of Steffen Bohn started.

Looking at the angles needed a vectorisation of the network, and this was done in a kind of primitive Voronoï-middle line techniques, as would be later developed properly by Etienne Couturier for leaves shapes [4]. With this Steffen showed that indeed the angles were varying continuously, being perpendicular for a small vein connecting to a big one, up to $2\pi/3$ for three veins of same size merging (Figure 2) [5]. Surprisingly enough he noticed that it corresponds simply to the case of connecting together three veins of various size cut perpendicularly: it forms a unique triangle, or a unique set of angles between the veins.

The origin of this fact could be interpreted as the presence of tension in the veins, and that the tension is proportional to the vein diameter. This tension could arise from the fact that the whole leaf is expanding, and that the veins are already more rigid than the other tissues, and more difficult to stretch. In this way the veins resist the expansion and the surrounding growing tissues

[3] However this method could be applied with loops, as soon as a root is decided, giving an orientation to each element, and for instance choosing at a 3 element crossing that the element with the longest distance to the root is conserved at the junction while the one shorter is merging into it. By removing these merging a tree can be recovered.

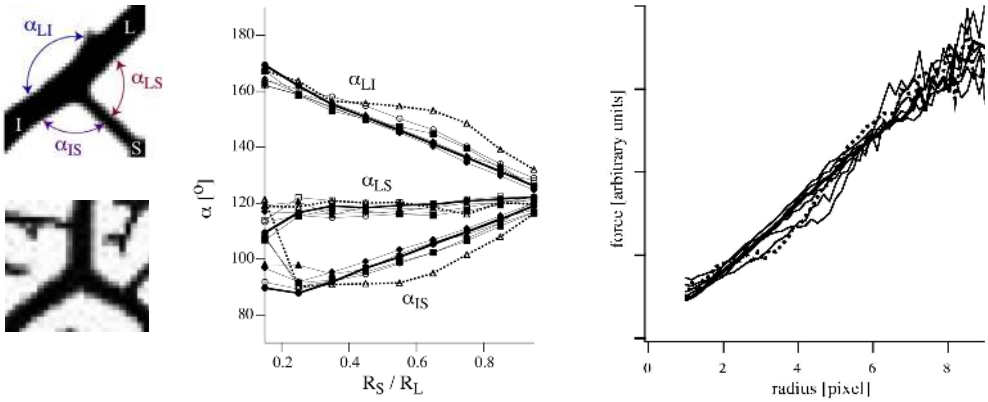

Figure 2. At each vein connections, one can define the large, intermediate and small radius, and measure the corresponding angles. Depending on the ratio of large to small radius, one finds a linear variation of these angles, going from nearly right angles for a very small connecting radius (as in the upper left detail), similarly to porcelain cracks, to nearly equal 120° for similar radius (lower left detail), similarly to soap bubbles. The continuous variation is similar for different species (center graph, various lines). This effect can be interpreted as a pulling force proportional to the radius (right graph) (Steffen Bohn).

are creating a tension in them. As the veins expand with age it creates a larger lignin cortex and offer more resistance.

But this nice result would not give a global description of the pattern, and a particular characterization of each leaf, especially because it was found to be universal, applying similarly to leaves with patterns looking very different at the eye. Further attempt of defining veins by following their size, or direction, were not judged fruitful at the time.

3. Mechanisms—1: cracks

If the growth of the leaf could explain the varying angles, it would not explain why the reconnections would occur in the first place. The best model for complex pattern formation I knew, based on Yves's work, was then the Safmann–Taylor type, or Laplacian growth, and it would form precisely only a tree. It is formed when a local front speed depends on the gradient of a field, which itself obeys a null Laplacian equation. It corresponds to the original Saffman–Taylor problem, with air pushing oil in a Hele–Show cell, where the Laplacian field is the field of pressure in the oil, or crystal [6] or bacteria growth [7], where the Laplacian field is a field of concentration, of the crystal component or bacteria food. The pattern forming abilities come from the fact that a forward disturbance of the front would grow quicker, as it captures more gradient from the Laplacian field due to the needle effect well know in electrostatic.

As the pattern forming field obeys a Laplacian equation, when two part of the front become close, the gradient between the two vanishes, so does their speeds. Thus two branches can never connect to each other, and such instability creates only trees (this was recently shown not to be always true [8]). Looking for a simple model that would produce loops was thus motivating. Looking for other examples, I found the case of river deltas and mangroves (Figure 3). I realized that if the flow goes only in one direction (to the sea), it creates only a river basin tree, where the possible dynamical connection of two rivers would be instable: only one branch, the easiest going to the sea, would capture the flow, and the other branch would dry up. But if the flow can be reversed with the rise of the tide, then at least the connection could be preserved. Making a

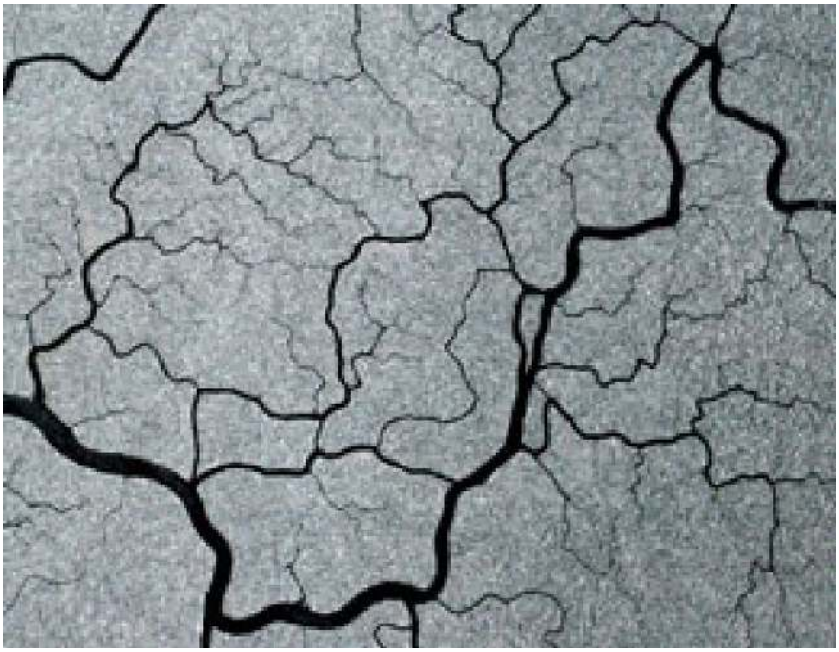

Figure 3. Satellite image (reflectometry) of a river delta. The tree shape gathering in the trunk of the river spread again in the roots of the delta, the river depositing sediments at the sea level. The roots connect together under the influence of the flow of the rising tides.

reverse flow means that the pattern forming field is no more a scalar but at least a vector (with the time oscillation).

I remember I was explaining this to the group during a sandwich lunch in (unusually) the "Jardin des Plantes", sitting in the grass, with Yves sitting further away on a bench, not taking part in the discussion. However, some time after, he came back with the example of the cracks formation he was working on with Ludovic Pauchard at Orsay. He explained the fact that the tension on a surface being a 2D tensor (matrix), if a crack releases the tension in one direction, the tension in the other direction remains, and this asymmetry even attracts other nearby crack tips to rotate and connect to the first one perpendicularly. This is very visible on the cracked ceramic glaze. He then explained that to form a reticulated pattern one needs a tensor. This led to the further work of Steffen Bohn. Yves, a skilled experimentalist, also played with Ludovic's drying colloid system to produce different types of reticulated patterns, very reminiscent of leaves veins patterns (even if the connection angle remained perpendicular), depending on the geometry variable thickness, and different history of drying (Figure 4) [9].

Working then on this crack patterns Steffen Bohn showed that the T right angle connection allows to deduce from a pattern roughly its sequential history of construction [10]. The reconstruction of the history was later redeveloped by Eleni Katifori to construct, from the point of view of the tiles, a history asymmetry coefficient that is promising to analyze loops patterns [11]. Steffen also showed that the typical tile shape was simply a quadrangle [12], while their contact neighbours is typically hexagonal as demonstrated by Euler.

4. Quantification—2: city streets

In this article [12] Yves remarked that the crack pattern is also reminiscent of another reticulated pattern: that of city streets. It also appears successively, with shorter road streets connecting to

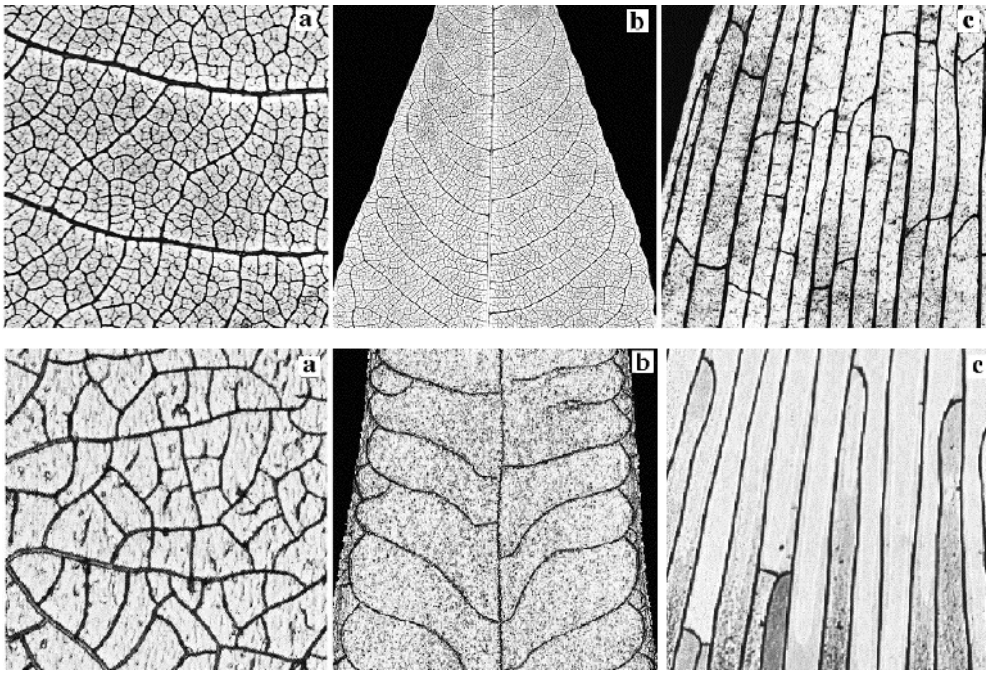

Figure 4. Above, images of leaf networks (a and b dicotyledons, c from a monocotyledon, lily of the valley). Below, silica beads cracks made by Yves Couder with Ludovic Pauchard, similar to the above (in particular the monocot one), by manipulating the drying process (variable thickness, borders, drying fronts) [8].

long older ones. This suggested that the dynamics is the same and that the pattern should be similar too. This analogy remained undeveloped at the time.

Years after, a friend Bernard Hennion, a researcher of France-Telecom working then at the CRI, proposed me to co-direct a student thesis.[4] The original problem of a colleague of him was to predict the length of fiber needed to connect the buildings of a city neighborhood, for good planning. As the fiber and other cables mainly pass under and along the streets, this requires a good understanding of the street network. I then proposed to make a model for the growth of the city road network, as the best way to understand a structure is to understand its growth. This is the idea of morphogenesis that can be traced back to Turing: any complex object (be the brain thoughts or a flower organization) can be build by letting a simple mechanism unfold in time. This was the beginning of the Thesis of Thomas Courtat.

But before making a model one has to characterize first the pattern, otherwise any result of a model would just "look like" the aim. To be really demonstrative one has to use a sensitive quantitative measurement to make the comparison. We thus went back to the problem of rebuilding the continuity of the veins, here the streets. Intuitively a street continues at a crossing when there is another element rather aligned in front. Thomas built an algorithm to do so. Then, with the larger elements constructed, one can measure how they are connected to each other. It is very different from looking at the reticulated pattern itself directly, meaning to look only street segment between two crossings. This last pattern is rather homogeneous, with in average square

[4]At that time the original group had split, because of the move from ENS physics lab—LPS, to the new physics laboratory of Paris 7—MSC, and of internal fight on the (singing) sand dunes.

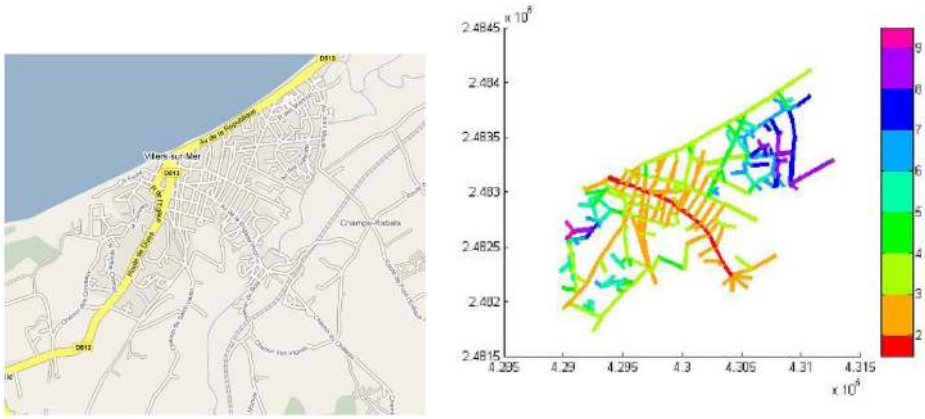

Figure 5. Analysis by Th. Courtat of Villers-sur-Mer. From the street network, arcs connected at crossings, one can recompose streets using the alignments at the crossings, and then organize the streets with respect to each other by the number of turns to go from one to the others. The central street is the one minimizing these average numbers of turn. Here it is colored in red (the historical road going from the cemetery to the harbor), and then the other ones by number of turns away (Thomas Courtat).

cells and mostly T connections (degree 3), as found in cracks, distributed with a short-range exponential decay (with a typical number of 0.65). But with the reconstructed streets, a much broader distribution is found, from very long streets crossing the whole map sample, to small streets with only one segment. The long streets have also a very large number of connections, giving a power law distribution similar to a small world case.

The most fascinating part is that doing so, looking at how it is inhomogeneous, gives a result that can be read as the history of the growth of the network (Figure 5). Andrea Perna, then a post-doc from a group of Ethologists working on termites' and ants' nests, showed that the distribution of distance, in number of connections, or turns, is very different from random to hierarchical patterns as city streets or leaf veins. Applying this result to Venice, it allows to show that without the canals, the street network is close to random, while introducing back the canals as streets, one recover a normal hierarchical distribution of distances (Figure 6). This shows that Venice was first built with canals and gondolas, and that the streets were secondary space left between the houses on the little islets, later connected with small bridges [13].

Thomas then characterized the score of each street as the mean connecting distance from this street to all the others, which is called a "centrality" [14]. And its value allows roughly to guess the growth history of the city (Figure 5).

After doing it, as is very common, we discovered that we have redone previous works. The intuition that the alignment of street segment is essential to their perception is he basis of a large work from Hillier that founded with it, in 1967, the "space syntax". The original method was very intuitive, but has progressed a lot over the years, to make it finally objective and robust [15, 16]. From this initial work, Jiang and Claramunt [17] tried to use the names given to streets to define them more objectively. After, Porta *et al.* started to use also the relative angles to define automatically the continuity at the crossing [18]. But as always, rediscovering by yourself the possibilities offers a deeply rooted personal interpretation and understanding, and a little originality, that have to be put in value and cultivated.

Then I was pushed by my father, an architect and town planner, to discuss these subjects in a work group, called MorphoCity, with town planners, architects and anthropologist, to think about

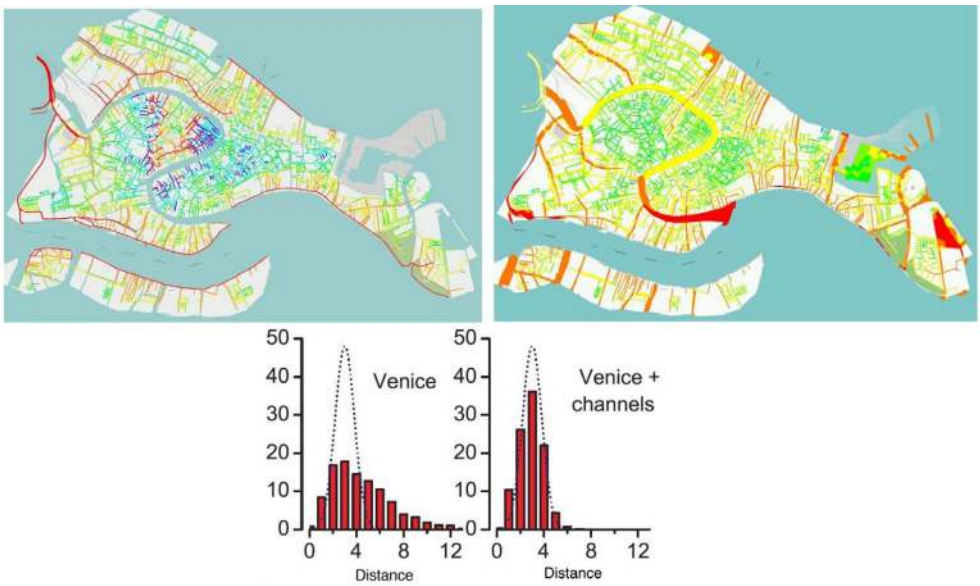

Figure 6. The topological distance map (number of turns) from the most central street, for Venice taking only account of the walking streets, left and right, when the canals are added to the network (Andrea Perna). Below the distance distributions to the continent bridge. When only the streets are taken into account, the distance span on an unusually large number of turns, giving its characteristic feeling of getting lost. But when the original mean of transport is put back, the gondola and its canals, one recovers a normally structured city, with the distance distribution falling perfectly in the predicted Gaussian curve (dotted curve).

the growth of cities.[5] The most surprising result is that the topological distance centrality, called accessibility or closeness, when computed on the streets, gives a good estimate of the history of the city (Figure 7). It was very chocking for the human scientists that a simple computer program looking only at the skeleton of the city streets could reveal the history of this city, that seems to depends so much on the decision of humans, who have the essential qualities of free will, consciousness, knowledge and intelligence (particularly ourselves). This had to be balanced with self-organization, and the realization that a good town-planner is in fact guided by the current state of the city. In this group and the help of an ANR-MN-2012, Claire Lagesse, a geomatician, did her thesis. In her thesis, that received a price from town-planners,[6] she clarified the algorithm to reconstruct the roads, stating the best parameters (which limit angle accepted for continuity) [19]. She also showed that with the reconstructed streets, called "ways" to include also countryside roads, the numerous centralities that can be computed are stable, and not sensitive on the chosen borders of the map sample, contrary to measurements made on the street segments. She further went on to show that many centralities are correlated, giving essentially the same information. This is surprising especially in the case of a local indicator, computed on one

[5]The core of the group was composed by Clément-Noël Douady, Philippe Bonnin, Pierre Vincent, Jean-Pierre Frey, Stéphane Douady, and extended for some time with Thomas Courtat, Magali Watteaux, Marc Barthelemy, Patricia Bordin, Claire Lagesse, Wang Xi, Estelle Degouys, Robin Brigand, Cedric Lavigne, Mohammad Atashinbar, Romain Pousse.

[6]Prix spécial pour "Lire les Lignes de la Ville. Méthodologie de caractérisation des graphes spatiaux", Claire Lagesse, Prix de thèse sur la ville 2018, APERAU / Institut CDC pour la Recherche, Caisse des Dépôts / PUCA.

Figure 7. (top) Reconstruction of the streets of Paris, and then classification of them by the number of turns to go from them to all the other streets, (Claire Lagesse on a map from Mohammad Atashinbar). (bottom) A drawing of the ages of the streets by Michel Huard. One recognizes in the above classification of streets the old historical streets, on which another tilted grid (of Haussmann type) had been superimposed.

way (such as the degree of a way), being correlated to a global one, needing the computation over the whole network (such as the betweenness). In this way one can propose a very limited number of significant centralities.

5. Modeling—2: city growth model and human understanding

In the group, Thomas made the first model for city growth. His assumption was that a street segment attracts houses to be built around, but also attracts other people from far to settle close, and that this settlement induces the creation of new street segment [14]. In practice a segment would generate a locally repulsive potential (the house build), and further away a attracting potential, similar to a Lennard-Jones'. This allows to compute a potential field of attraction for a new settlement. On this field an algorithm can decide a new position, with a parameter allowing to pass continuously from random position to absolute minimum place. Once decided, this place is connected with street segments to the previous network, also with a parameter describing how many times. With this various type of local patterns can be generated, which resembles some actual local patterns.

This type of modeling was also deeply discussed in the group, as it makes a self-generating pattern, already existing streets inducing new ones. It seems obvious for a physicist, but more difficult to grasp for a town-planner that the intermediate steps, the construction of buildings and the will of people, could be shortcut. There were also many discussions between the tenants of the streets (the "line people"), and the tenants of the surface (the "tile people"), where more work had been done, saying that the evolution of the building surfaces was the important aspect. This is parallel to the discussion on the cracks connections or on the tiles shapes.

The fact that looking at the accessibility of ways could reveal roughly the historical development of a city was also debated in length. The validity of such interpretations was first checked in the field, by doing blind studies on some cities and then going to discuss with local town planners. It worked very well on the cities studied (Avignon, Toulouse, Bordeaux, Le Creuzot), where we could determine the original part and its different developments. One result we found, in the city of Avignon (and Oulan Bator), is that some not well connected places, looking more like trees, were found either in slums or in rich villas. In both cases the civil administration has not the power to enforce an efficient reticulation. The first case comes from the construction of the minimal functional structure, and the second one from the desire of being locally quiet, without any thru traffic.

The reason for the fact that analyzing blindly the street skeleton could give a hint on history was also thoroughly discussed. The conclusion is that the streets are built historically in succession, and that they have a double objective: to distribute the people in their buildings, but also to link the city to the rest of the world. Thus the oldest roads are going far, always crossing the whole map, and cities often settled at crossing of roads. The next roads extend around, but from the first ones, or create reconnection between roads [20, 21]. Thus we end up into dissecting with smaller and smaller roads. This can be understood also not only from a purely urban growth, but it happens already at some distance of a city, the large grazing fields or crops ones, turning to smaller fresh vegetables ones near the city and factory space, and finally smaller urban blocs.

This creates a typical hierarchical pattern in the street network, very close indeed to the process of cracks, subdividing already existing blocs, although inhomogeneously in space. The fact now to understand is that these hierarchical structures that appear through a normal successive development is preserved in time, that there is no strong reorganization. The conclusion of the group work is that actually this hierarchical structure fits the human brain, by giving at the large scale a simple structure that can be known and remembered, and that fine details are necessary only locally. This allows to move efficiently in the city without needing too much knowledge and memory of it [22].

This conclusion, on the general non-planed growth of cities, could chock town-planners and historians, traditionally putting in front the will of some decision-makers. However, even if such events are important in the historical representation one can have of a city, they are efficient if

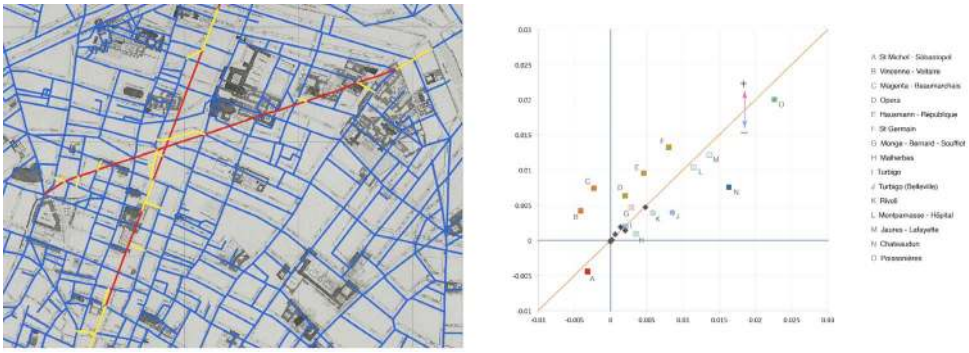

Figure 8. Left, a detail (around Turbigo and Sebastopol piercings), showing in red the new street and in yellow the street segments suppressed (some superposed). It shows that many streets were broken close to the new piercing. Right, the graph of the relative increase of circulation ease (global closeness) in Paris for the "Haussmann piercing" (in the wide sense). The horizontal axis is the change obtained when the piercing is added alone (without any other one done), and the vertical one is the changed obtained when the piercing is added to all the other ones. This allows to judge the efficiency of the piercing by itself on the unchanged city, or its effect in the collective. Some piercing behaves better collectively (above the diagonal), but some other not (below), and one piercing—St Michel/Sébastopol—has always a negative impact, because of the rupture of ancient roads it induced (Mohammad Atashinbar).

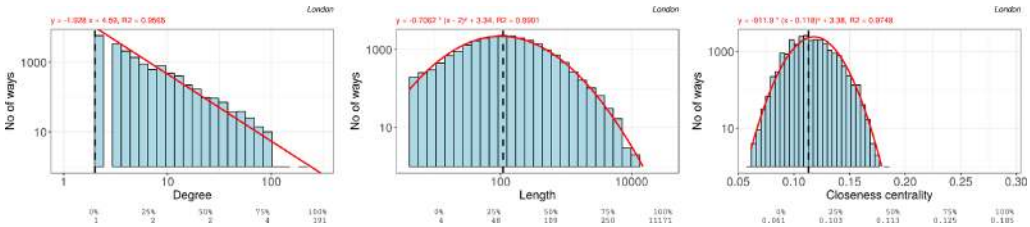

Figure 9. Distribution of characteristics of ways of London. We see, left, that the degree distribution is a power law, coherent with a small world, while, middle, the distribution of length is a log-normal, and, right, the distribution of closeness is a Gaussian. While the first two distributions are very similar to the ones obtained for other cities, the closeness is much smaller, indicating a globally less efficient structure (Claire Lagesse).

they follow the same spontaneous type of organization. It also appears that their effect is always limited both in space and time. On the long run, they get diluted and do not show over lasting impact.

This is even true for the piercings done by Haussmann (and precursors and followers) in Paris. We studied the impact of these piercings with Mohammad (Babak) Atashinbar, a post-doc from Nazar Center of Teheran. It turns out that although they are important for modern transportation, essentially for their large width, these roads are limited in length, not going through the whole city, nor far away. Even worse, by destroying the surrounding houses and rebuilding new ones, old structural roads end up blocked by the new buildings, and thus important old transportation structures have been erased at some places (Figure 8).

One of the main results of Claire Lagesse is that the length of ways is in good approximation close to a log-normal distribution (Figure 9). It could look natural but in fact a stationary

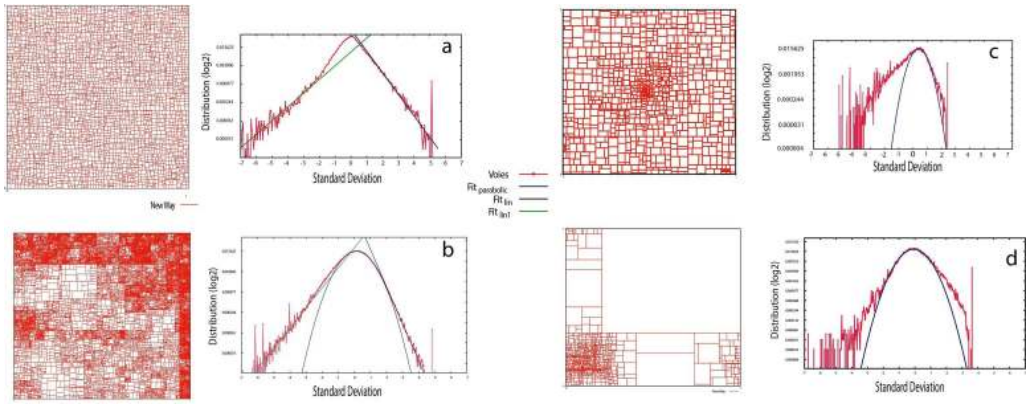

Figure 10. Simple space division models. (a) Cut (with some noise) the largest cell, leading to an homogeneous pattern, but a length distribution close to power laws. (b) Cut all the cells successively regardless of their size, giving a very inhomogeneous and random pattern, and a more rounded distribution. (c) Cut the cell with highest potential, depending on its size and proximity (Euclidian distance) with other cells. This leads to the appearance of centers. (d) Cut the cell with highest potential depending on their size, but also their distance with other cells using the topological distance (number of turns). It gets very close to the actual quasi log-normal distribution of city street length, in particular the higher number of large cells, well visible in Paris or Los-Angeles (Romain Pousse).

distribution of subdividing surfaces would lead naturally a power law (typically l^{-2} for bloc iteratively cut in 2). Constructing a model that can give rise to the right distribution did not turn to be easy. This is the thesis of Romain Pousse, on a Cifre contract with an Architect of the group, Pierre Vincent. The present model is based on dividing space. Following Thomas Courtat, the idea is to decide where to place a new road, here just cutting a bloc in two, by computing a potential on the existing network. The potential for cutting a given bloc is computed of the distance of the crossings of the whole network to this bloc. The main result is that in order to have a statistic of length of streets that is reasonable, one has to introduce in the potential computation the topological distance (number of turns) in the evaluation of the distance to the other crossings (Figure 10). This reinforces the idea that this distance is psychologically very important to evaluate distance by the user, and not only the metric (Euclidian) distance on the network as put forward with the travel time.

The success of the discussions within the Morphocity group, in particular its cross-culturing, is highlighted by the fact that an anthropologist-architect of the group, Philippe Bonnin, is now spending time making crack patterns in clay, trying to reproduce and compare it with real cities (Figure 11). Mastering this material and process, he now find ways length distributions which are also close to real city ones.

6. Observations—2: gorgons

I was invited in Cuba for a conference, and the organizer, Ernesto Altshuler, was kind enough to put me in his father's apartment. There I looked at a small collection of sea life collected on the shore. In them was some gorgon skeletons, and I was caught by the particular structure of their reticulated pattern. Long lines are clearly visible, as the main veins of a leaf, but they often appear interrupted while secondary veins take over and become dominating. One can also observes reinforcement of the veins, but again, contrarily to the leaves, these reinforcements are not linked

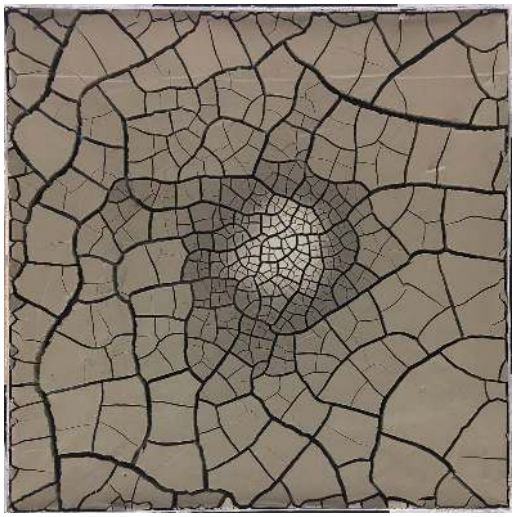

Figure 11. A snapshot of a cracks formation in drying clay. The center is thinner, with smaller tiles, and already dried (whitish), while the border is darker, and the surroundings still humid, even if already cracked (Philippe Bonnin).

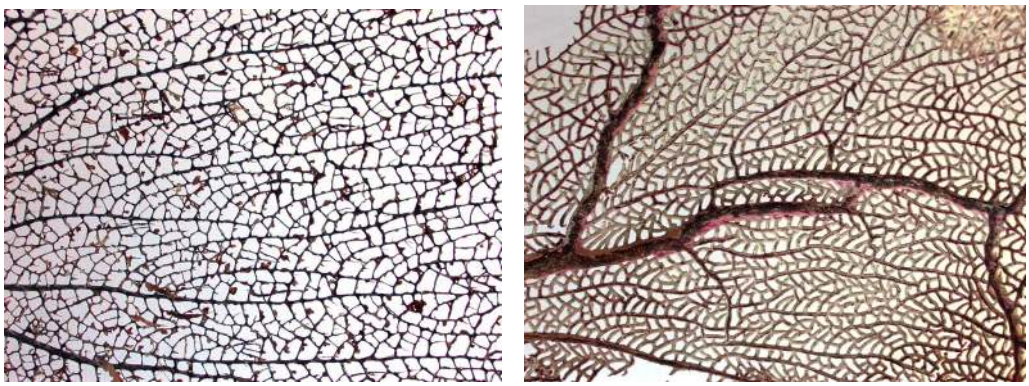

Figure 12. Left, gorgonin skeleton of *Gorgonia flabellum*. On can see the dominant branches, surrounded by secondary branches and third level reconnections, but with changing dominance. Right, reinforcements on *Gorgonia ventalina*, the thin skeleton becoming locally a large ellipsoid to resist the deformation from drag of the swell. It does not follow the logic of the first pattern, and can even split in two (from Altshuler's collection).

essentially to the age or order of formation, but run freely along the network, independently of the historical structure (Figure 12).

Years after, while discussing with Pascal Jean Lopez on diatom growth, the possibility of studying directly these animals became possible at the sea biology laboratory in Point à Pitre, Guadeloupe, in collaboration with Claude and Yolande Bouchon. This work will lead to the thesis of Paul Valcke.

Gorgons are little animals in the form of polyps, living in clonal colonies sharing mechanical support and digestion. They are the type of coral with inside skeleton (like animals) but not made with calcium carbonate (like our skeleton), but with a protein similar to chitin or keratin (like insect external skeleton). Their morphologies vary, but seem to depend essentially on

the polyp size. For large polyps, the size of the skeleton is large and strong enough to resist the motion of the swell. It then grows with a typical chandelier shape. The oscillating water motion, that can be also only with a period of half a day with the tide at larger depth, tends however to force the alignment of the branches perpendicularly to the motion. This can be related to a hydrodynamical instability: if a plane is inclined in the flow, the lift will rotate it to make it more perpendicular. For smaller polyps and thinner skeleton, there is not enough mechanical resistance and the danger is then that the gorgon is pushed down the sea floor. One finalistic interpretation is that it leads to reticulation: the tree branches attach together in the perpendicular plane, to form a globally resisting surface, in the shape of a leaf (sea-fan).

The particular shape of the gorgon can be ascribed to its particular way of growing: it grows only on its outside perimeter, with no expansion of the already grown parts, like a crystal. On the perimeter little growing axes expand like little trees with lateral axes turning up. These trees reconnect and form the reticulated network. Once the gorgon expands it endures more drag and then it can be speculated that it is sensitive to the local mechanical bending, and reinforce the parts that are too bent, anywhere they are. Any reinforcement of a segment will focus the deformation at its ends, and thus favors the reinforcement of next segment, creating lines. Along these lines the surrounding fabric profits from its rigidity and does not need to be reinforced. This can be convenient for the global colony, as only few parts have to pay the price of being reinforced, especially since the reinforced segments do not bear any more polyps.

As well as for the cities, such history of growth can be recovered by looking at the accessibility of the reconstructed ways (Figure 13). Reconstructing ways for gorgons is a particular challenge since they are very rounded at the connection, making the local identification of the orientation and their continuity very difficult. Paul Valcke succeeded to overpass this difficulty on looking at slightly larger alignments, in a way very reminiscent of early intuitions of Hillier.

7. Observations—3: ferns

While on a trip in the CNRS station of Nourragues, in the middle of the tropical forest of French Guyana, to record the growth motions of stems and leaves inspired by observations from Etienne Couturier, I started to look at the veins of ferns. Their veins, much more visible than that of angiosperms, can take the shape of trees or more or less reticulated networks, most of the time very stereotyped. It then happens that looking at a common local species of *Adiantum*, I found leaves that are connecting the vein tree in few places. It then seems that these particular species, depending on the local environment, was doing the transition from tree to reticulated network (Figure 14).

This is very exciting since it shows the possibility to have the same growth mechanism creating both type of patterns, and even intermediary ones. This is surprising since a priori the mechanisms to form a tree and a reticulated pattern are different, for instance the difference between the boundary growth based on the Laplacian of a scalar and the tensor of cracks, as precisely highlighted before.

Looking then at fern venation one can see that there are different levels of connections. The assumption is that the ancestral pattern is that of parallel veins, then a tree type, and finally a more recent reticulated type. The evolutionary interest of doing so can be guessed: the local growth of a plant tissue is locally constant, leading to locally exponential increase. But in practice it happens only for some time and thus remains localized at a certain distance from the growing border. This gives an expansion which is globally linear in time. This is true for stems and for such primitive blades. Then the total growth of the surface remains also linear in time, proportional to the perimeter length [23]. This is dangerous for the blade, since it exposes a large fragile surface directly to the outside conditions. In order to reduce this time of exposure a solution is to increase

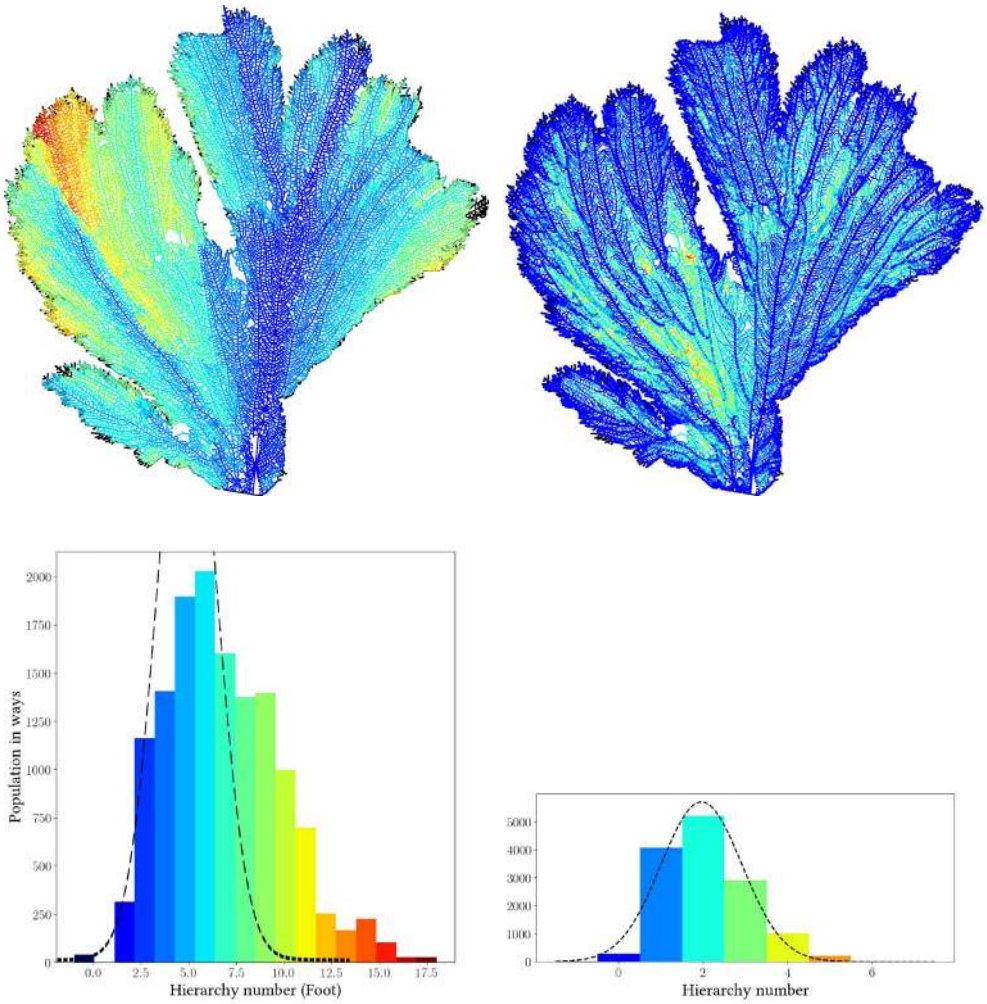

Figure 13. A well developed *Gorgonia ventalina* showing separated fingers. On the top left the ways have been analyzed and colored by topological distance to the foot. One can see central lines, but the lateral branches become very distant by successive branching out, especially on the left palm, leading to a pattern that does not look hierarchical. This is confirmed by the distance histogram in the lower left, showing a deviation away from the expected hierarchical Gaussian of A. Perna, as in the streets of Venice (Figure 6). On the contrary, on the right, the distance to the border gives a typical hierarchical pattern, with the distance distribution falling back on the predicted Gaussian. This shows that the main growth element is not the foot but each tip of locally dominant vein at the border. (Paul Valcke).

the perimeter, with a perimeter instability, creating side fingers. Since this instability is not due to an external (Laplacian) field, one can see that it is related to the elongation of the main vein. The axis length increases quicker and the slowly growing blade can only fill up a part of the surface and finally leaves separated fingers (Figure 15). This transversal growth of the blade becomes associated with the growth of transversal veins, forming trees, where the same process can be repeated. This eventually leads, by further repetition to a fractal-like border with fractal vein tree.

Figure 14. Left, a leaflet of *Adiantum sp.* from the CNRS Nourrague station in French Guyana. The dotted lines in the tissue from the first vein bifurcations (in yellow) leading the opening on the perimeter have been drawn, as well as the unusual reconnection places (along these lines) binding locally the otherwise independent vein trees. Right, a detail showing the bump in the lamina just before the binding.

Figure 15. Three different types of fern leaf veins. Right shows an ancestral one, with parallel veins going to the border. Middle shows the division between secondary veins that start to divide also (in a fractal way) forming local trees. Right shows a more recent reconnected blade with reticulated veins.

Now this border has a new problem. Creating lateral fingers growing independently, without intervention of any outside media that could harmonize their relative growth, can lead to fingers covering each other (top of Figure 15 middle). One can marvel that this does not happen often, and that the respective fingers can come close, collectively covering nearly the whole surface, without covering each other. One can then imagine complex regulation mechanism to achieve such a feast. But it seems the ferns have come up with a more radical solution: reconnecting the fingers between them to form a new single surface.

It appears that this happens through another invention: the reconnection of the vein network, the reticulation, forming loops. What is particular to ferns is that their veins pattern is very

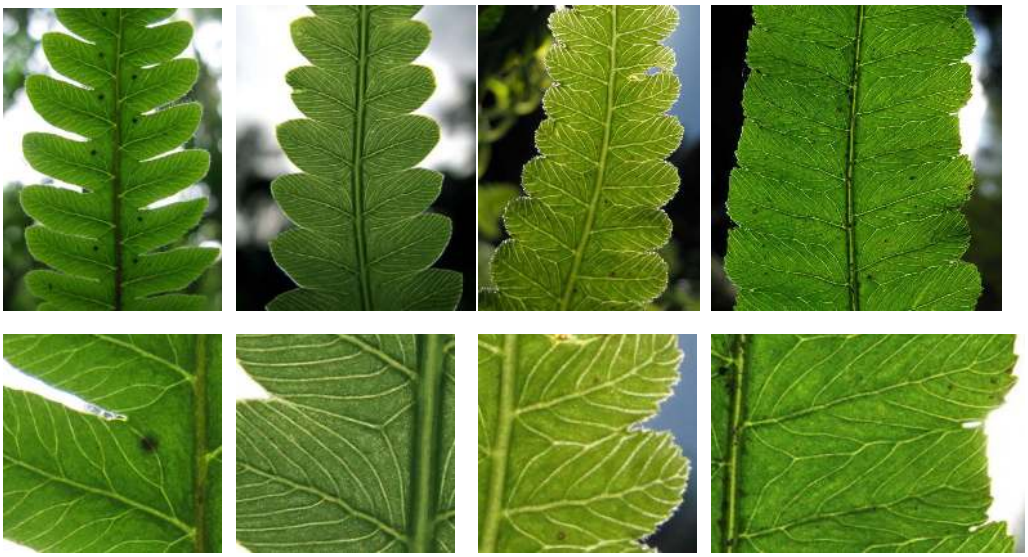

Figure 16. Different blades (from different species and genus). Left shows a pure tree with independent fingers that starts to have a new lamina between the fingers. The next one, middle left, shows a first vein reconnection in this large lamina space. Middle right have three reconnection and finally, right, nearly all the blade is reconnected.

Figure 17. A blade where the little trees growing out from the lateral veins can be seen as reconnecting with the neighbor tree at the orange points. Then the full original tree with two bifurcations level can be recovered (dotted lines).

stereotyped. After the first obvious reconnections (Figure 16), one can also analyze more complex but stereotyped reticulation (Figure 15 right) as reconnected trees. It works very well, allowing to recognize the fusion of trees of different level of reiterations (Figure 17).

If we follow an evolutionary path, we then reach recent fern leaves that are reticulated exactly as angiosperm leaves (Figure 15 right). This new type of leaves is able to maintain an exponential growth on its whole surface, with the internal creation of new veins. With the preserved global exponential growth the leaves can fully develop during a short spring period. The final difference between fern and angiosperms is then not on the structure of the pattern but on the fact that the angiosperm one remains very fine, a sign that could be interpreted as a Neoteny [24].

Figure 18. Growing blade front. One can observe particular growing places around the attachment of veins ate the borer, reminiscent of apical meristems in stems. The growth is strong enough to make higher ridges. But between these growing place, one also observe an "anti-meristem", as a white dot, where the growth is smaller, leaving to deep sinuses.

To finish this evolutionary image, one can point to the most recent evolution of grasses (Poaceae). One solution to protect even more the growing leaf from the outside environment is to keep the growing part inside the protection of the previous ones. Poaceae leaves thus form tubes at their base, which enroll the next ones. The growing part of the leaf is now reduced to a place close to its base inside the previous leaf tubes. Geometrically, being blocked between an already old part, fixed, and the stem base, means that the only real expansion possibility is along the length. This is why theses leaves grow considerably in length, and present long thin blades. Their vein pattern is then mostly of parallel veins, as in the original primitive type (Figure 4 top right), as obtained by Yves in directed drying (Figure 4 bottom right). But there remains little transverse connection, trace of the reticulated pattern that is just considerably stretched in one direction.

8. Modeling—3: toward a new vein model?

Growing ferns from spores leads to interesting observations: as proposed by Haeckel, the Ontogeny resumes (roughly) the Phylogeny. In this case some ferns with adult reticulated leaves start to grow in the shape of separated trees [25]. As the plant itself grows older, and its leaves develop to larger forms, reticulation starts to appear, before the final reticulated leaves becomes stable. This transient state on a single specimen is very interesting. In particular it shows that the reconnection appear progressively, first by an attraction of side veins.

Looking at the growing leaf periphery also shows not only special places at the tips of the parallel veins, similar to 2D apical meristems, but also their negative counterparts, as the places with less growth between two vein tips (Figure 18). Clearly the growth around the vein tips is larger, but the dynamics of the growth around the counterparts could be more essential to the final shape of the structure. If it stops growing, the two growing sides will not join anymore after. In other words it creates a dent in the leaf. The relationships of the size of these dents with the order of the previous bifurcations in the tree was already noticed by Bower [25].

One interpretation for the junction of the side veins, as observed in the original leaves of Guyana, is that this anti-meristem does not stop to grow in length but induce a more limited growth laterally. This closes the blade left behind, creating a visible fold in the lamina (Figure 14 right). If this process is extended to a disappearance of the growth, but only when the two side veins touch each other, then this create the binding of the two lateral veins. The reconnection between front veins can be interpreted in the same way, at the condition to add that reducing the lateral growth induces also the appearance of a new vein, that comes to close the surface.

In this way, the connection and space between the veins is not due to a particular dynamics of the veins, the lamina following as it can in between, but rather it is the lamina between the veins, and its particular growth, that direct and position the veins, up to the point of reconnecting them.

9. Conclusion

Even after all this work it should be clear that the analyze of reticulated patterns, the understanding of their dynamics, and also the distinction and possible transition between tree like structure and reticulated one, within the same frame, is still an open question. The question of analyze has made big progress, but the tools remain to be used, and the various cases compared. The success of this step is necessary before going in the next phase of making the link between the growth dynamics and the final pattern. This question, "can we recover a possible growth dynamics from the analyze of a pattern?" is the initial question of the thesis of Paul Jeammet. After the analyze of the fern dynamics and patterns would open a new way to build a common frame producing both tree like structure and reticulated ones, depending on few parameters. This is the starting aim of the thesis of Camille Le Scao. This would bridge the gap that was initially thought as impossible to cross, of two patterns coming from two different physical worlds. In a way this would show that starting from a strong statement ("it is impossible to create reticulated patterns with scalar growth models, so the formation of reticulated patterns has to be physically very different"), has led to a big journey taking many interesting turns and detours, but might finally come back to a solution dissolving part of the initial statement.

It is also clear from looking at this personal history that the development of a scientific question is essentially a collective effort, as underlined by Turing. The starting point, the initial question, is built from the confrontation of an already made statement by some person with someone else observation. Once the ball started to roll, many people interfered in its movements, pushing it in different directions. The ideas develop using every person different point of view, experience, abilities, and will, in many little confrontations and synergies. In this process each person has its own style, and it takes all the people to make the question progress the best way possible. The intuitive person that gets some idea, based on his experiences, analyzed through his understanding of other's work and remarks, will be developed more professionally by more production oriented persons, before it can be applied by even more practical persons. Its does not need the same abilities, and few persons have several of them. In this time of promotion of a unique type of production oriented researcher, we can see that this selection is dangerous for the whole research life. Especially since the meandering of thoughts giving rise to new questions and findings needs both time and freedom, that can be obtained only in a benevolent environment.

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