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Allometric relationships between total height and diameter for volume estimation of bottomland hardwood oaks in Arkansas

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Bottomland hardwood (BLH) forests are a unique ecosystem that plays an important role in the provision of ecosystem services, both ecologically and economically. This study modeled allometric relationships between total tree height and diameter at breast height (DBH) for seven oak species (*Quercus* spp.), including overcup (*Quercus lyrata*), cherry bark (*Quercus pagoda*), Nuttall (*Quercus texana*), Shumard (*Quercus shumardii*), water (*Quercus nigra*), willow (*Quercus phellos*), and post oaks (*Quercus stellata*) that prevail in BLH forests in the Lower Mississippi Alluvial Valley region of eastern Arkansas. Twelve candidate models (Schumacher, Bufford, Curtis, Meyer, Micment, Power, Logistic, Weibull, Chapman, Gompertz, Ratkowsky, and Prodan) were fitted using data from 2,762 individual trees across 34 Forest Inventory and Analysis (FIA) plots collected between 2015 and 2023. The results showed that the three-parameter Weibull model was the best-fitting model based on the relative rank sum scores and was therefore selected as the base model for further analysis. To account for interspecific variability, a nonlinear mixed-effects model was developed using the Weibull function with species as a random effect. The mixed-effects model performed better than the fixed-effects global models, with $R^2 = 0.79$ and RMSE = 3.34 m. Furthermore, volume validation analysis revealed that mixed-effects model estimates were more consistent with the observed data. In contrast, global model estimates tended to underestimate volume in larger diameter classes (DBH > 50 cm). Additionally, applying response calibration to a subsample of trees localized these predictions and consistently reduced prediction errors relative to the uncalibrated population-average model. These findings suggest that the mixed-effects Weibull model is a robust and precise tool for predicting tree height and estimating volume for oak species in BLH forests of this region, providing a baseline for predicting biomass stock of these oak species and contributing to the development of biometric models to guide forest inventory, silvicultural practices, and forest management.

KEYWORDS

height-diameter modeling, mixed-effects model, model calibration, *Quercus*, Weibull distribution

1 Introduction

Bottomland hardwood (BLH) forests, predominantly distributed throughout the Lower Mississippi Alluvial Valley (LMAV) in the southeastern United States, are deciduous forested wetlands found in floodplains of rivers and streams (Lockhart et al., 2010). These ecosystems provide a wide range of critical ecosystem services, including deep aquifer recharge, nutrient cycling, soil stabilization, and wintering habitat for migratory waterfowl (Muir, 2020), representing one of North America's most productive and ecologically significant ecosystems (Banzhaf, 2009). Unfortunately, this unique ecosystem has experienced substantial degradation over the past centuries due to widespread deforestation, land-use conversion, altered hydrology, poor management practices, agricultural expansion, and urbanization (Hossain et al., 2021). Specifically, BLHs once covered 30 million acres; however, land-use change and expansion of agriculture resulted in a 70% loss in area and severe degradation of their quality over the past century. The remnant BLH tracts are under severe threat because of water management practices and the continued expansion of agriculture (Schneider et al., 2011; Zarfl et al., 2015; Wohl et al., 2017). To mitigate BLH loss and restore hydrological processes and ecosystem services, restoration and reforestation efforts are currently among the highest conservation priorities within the region.

Among the most important tree species in BLHs are oak species such as overcup (*Quercus lyrata*), cherry bark (*Quercus pagoda*), Nuttall (*Quercus texana*), Shumard (*Quercus shumardii*), water (*Quercus nigra*), willow (*Quercus phellos*), and post oaks (*Quercus stellata*) (Kellison and Young, 1997; Lockhart et al., 2010), which provide food resources for wildlife and possess economic value, for example, as a source of timber for producing furniture, flooring, wine barrels, and other products. Acorns of red oaks provide important energetic resources for mallards and wood ducks, as they contain high levels of saturated fatty acids and other micronutrients that are not readily available in other food sources (Heitmeyer et al., 1989). Mast production of bottomland oaks is desired by managers for enhancing wildlife habitat and high-quality wood. However, managers and landowners often face significant challenges in estimating the yield and growth of the BLH forests (Jeffreys, 2014), largely due to the absence of practical, species-specific growth and allometric models. Such models are essential for generating reliable estimates of periodic growth, informing decisions about growing stock, and identifying a suitable management plan (Gschwantner et al., 2024; Kim and Lee, 2017).

Forest inventories routinely measure total tree height (H) (ground to apical meristem) and diameter at breast height (DBH) to monitor tree- and stand-level growth dynamics. These fundamental variables are essential for characterizing stand structure, tree architecture, and the competitive position of individual trees within the vertical canopy profile (Ogana, 2019; West, 2015). Moreover, height and DBH measurements are frequently used to calibrate and validate outputs from remote sensing technologies widely applied in forest monitoring (Colgan et al., 2013; Kunneke et al., 2014). Significant relationships between tree height and stem diameter have been reported across numerous studies (Gompertz, 1833; Meyer, 1940; Curtis, 1967; Burkhart and Tomé, 2012a, 2012b). Height-diameter (H-DBH) models capitalize on this relationship by using a subset of precisely measured tree heights as the dependent variable and DBH as the predictor (Kalbi et al., 2018). These models are widely applied to estimate missing height values and to interpolate height distributions within forest inventories. In addition, predicted heights derived from H-DBH

models serve as key explanatory variables in more complex allometric equations used to estimate stem taper, merchantable volume, and aboveground biomass (Chenge, 2021).

Several approaches have been used to model H-DBH relationships. One of the earliest and most commonly employed techniques involves fitting simple linear models using ordinary least squares after applying logarithmic transformations to linearize the variables (Mensah et al., 2018; Lindner, 2014). However, these linear models implicitly assume a proportional increase in height with increasing diameter, an assumption that often contradicts biological reality. In natural stands, tree height typically exhibits an asymptotic pattern, approaching a species-specific maximum as diameter increases (Watt and Kirschbaum, 2011). Given the inherently nonlinear relationship between diameter and height growth, nonlinear models generally provide a more biologically realistic representation of H-DBH dynamics (Paine et al., 2012). Numerous nonlinear models using only DBH as a predictor have been developed, including exponential and Weibull forms with two or three parameters, as well as widely applied functional forms such as the power law, Gompertz, logistic, and Chapman-Richards equations (Weibull, 1951; Curtis, 1967; Bailey, 1973; Zeide, 1993; Sánchez et al., 2003).

In addition, mixed-effects models have become increasingly important for modeling H-DBH relationships, as they allow for partitioning variability attributable to differences among sites, forest types, or species (Chenge, 2021). Mixed-effects frameworks estimate fixed effects that describe the average trend across the population, while random effects capture deviations associated with individual subjects or groups (Pinheiro, 2011). A growing body of research has applied mixed-effects H-DBH models using DBH as the primary predictor variable (Sharma and Parton, 2007; Budhathoki et al., 2008; Zhang et al., 2015; Kalbi et al., 2018; Ogana, 2019). Several studies further demonstrate that mixed-effects models can effectively account for variability in H-DBH relationships even without incorporating stand-level explanatory covariates, highlighting their capacity to represent complex hierarchical structure in forest data (Trincado et al., 2007; Temesgen et al., 2014; Zang et al., 2016).

Several studies (Sullivan et al., 1983; Clatterbuck, 1987; Clatterbuck and Hodges, 1988) have documented stand-level tree height attributes for oak-dominated forests in the LMAV; however, studies examining the H-DBH relationship for oak species in BLH forests are limited (Lynch et al., 2005). For example, Sullivan et al. (1983) developed variable-density yield tables for red oak-sweetgum mixtures using data from minor bottomlands in central Mississippi. Although these tables allow for the prediction of cubic foot and board foot volume per acre based on age, basal area, and average oak height, they did not address H-DBH relationships for oaks (Lynch et al., 2005). Existing tools, such as the southern variant of the Forest Vegetation Simulator (FVS), include height-diameter models for key hardwood species in the southeast United States (FVS Staff, 2008). However, these models provide only a general representation of height-diameter relationships and do not adequately account for variation arising from species-specific characteristics or stand structural attributes (Lhotka, 2012).

Therefore, to model the allometric H-DBH relationships for oak species of overcup (OCUP, *Quercus lyrata*), cherry bark (CBO, *Quercus pagoda*), Nuttall (NUT, *Quercus texana*), Shumard (SHU, *Quercus shumardii*), water (WATER, *Quercus nigra*), willow (WILLOW, *Quercus phellos*), and post oaks (POST, *Quercus*

stellata) in the BLH ecosystems of the LMAV region of Arkansas, twelve nonlinear models, including Schumacher, Bufford, Curtis, Meyer, Michment, Power, Logistic, Weibull, Chapman-Richards, Gompertz, Ratkowsky, and Prodan were developed and fitted in this study. Tree-level DBH and total height data were collected from the U.S. Forest Service Forest Inventory and Analysis (FIA) database. Based on the best-performing nonlinear function, a mixed-effects model incorporating species as a random effect was then developed to quantify species-specific variability in H-DBH relationships. In addition, the best-fitted H-DBH model was further validated using tree volume data by comparing observed volumes with volumes predicted using the model-generated height estimates. Furthermore, a response calibration using a subsample was conducted to estimate random effects through best linear unbiased prediction (BLUP), improving the operational utility of the models.

2 Methods

2.1 Study area and data collection

The study was conducted in the BLH Forests in the LMAV of eastern Arkansas. Sample plots, distributed across multiple counties covering a geographic range of 33.0°N to 36.5°N latitude and 94°W to 90°W longitude, were collected (Figure 1). This region has a humid subtropical climate with mild winters and hot, humid summers; specifically, the mean annual temperature ranges from 14 °C to 21 °C, and the average annual precipitation is 1,395 mm, with values ranging from 1,140 mm to 1,760 mm (Woods et al., 2004). Regarding the hydrological condition, this region is characterized by low-gradient streams and rivers, numerous swamps, marshes, and isolated oxbow lakes; meanwhile, the extensive construction of greentree reservoirs has further altered natural hydrology conditions. With regards to soil, this region is primarily fine-textured and

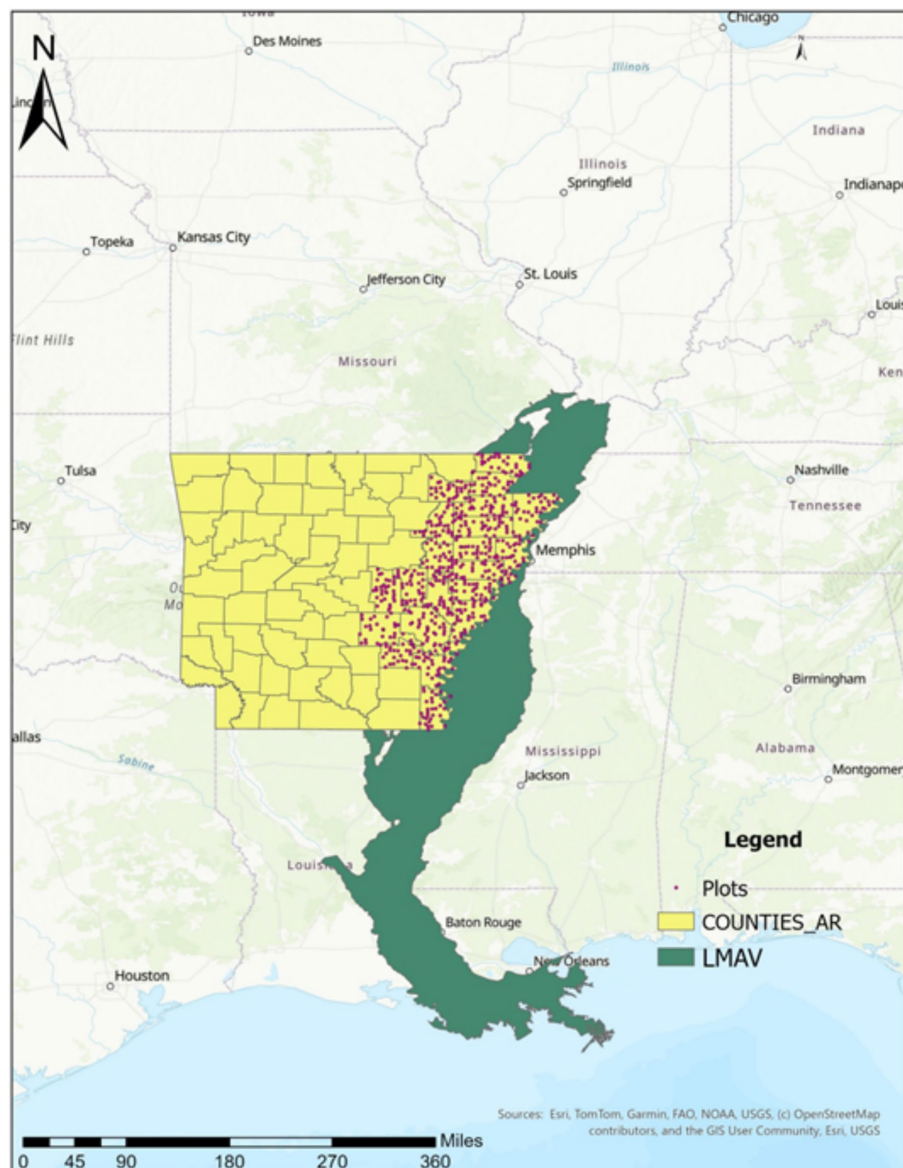


FIGURE 1

Study area and plot sites (red dots) drawn from the Forest Inventory and Analysis database to assess individual tree allometric models for oak species in the bottomland hardwood forest of the Lower Mississippi Alluvial Valley of eastern Arkansas.

TABLE 1 Nonlinear height-diameter models fitted using FIA data for the oak species.

Model	Allometric equation	References
Schumacher	$H = 1.3 + b_0 \times \exp(-b_1 / DBH)$	Schumacher (1939)
Bufford	$H = 1.3 + b_0 \times \exp(b_1 / DBH)$	Hafley and Buford (1985)
Curtis	$H = 1.3 + b_0 \times \frac{DBH}{(1 + DBH)^{b_1}}$	Curtis (1967)
Meyer	$H = 1.3 + b_0 \times (1 - \exp(-b_1 \times DBH))$	Meyer (1940)
Micment	$H = 1.3 + \frac{b_0 \times DBH}{b_1 + DBH}$	Michaelis and Menten (1913)
Power	$H = 1.3 + (b_0 \times DBH^{b_1})$	Stoffels and van Soest (1953)
Logistic	$H = 1.3 + \frac{b_0}{1 + (b_1 \times \exp(-b_2 \times DBH))}$	Pearl and Reed (1920)
Weibull	$H = 1.3 + b_0 \times [1 - \exp(-b_1 \times DBH^{b_2})]$	Weibull (1951)
Chapman	$H = 1.3 + b_0 \times [1 - \exp(-b_1 \times DBH)]^{b_2}$	Richards (1959)
Gompertz	$H = 1.3 + b_0 \times \exp(-b_1 \times \exp(-b_2 \times DBH))$	Gompertz (1833)
Ratkowsky	$H = 1.3 + b_0 \times \exp\left(-\frac{b_1}{DBH + b_2}\right)$	Ratkowsky (1990)
Prodan	$H = 1.3 + \frac{DBH^2}{b_0 + b_1 \times DBH + b_2 \times DBH^2}$	Strand (1958)

H represents total tree height (m); DBH represents diameter at breast height (cm); b_0 , b_1 , and b_2 , are the model parameters to be estimated; and 1.3 is added to avoid prediction of zero height when DBH approaches zero.

poorly drained soils, and the major soil types include Alfisols, Vertisols, Inceptisols, and Entisols, with a thermic temperature regime. According to Woods et al. (2004), soil moisture in this region was primarily aquic and udic with moist soil conditions.

Tree-level measurements for LMAV oak species were obtained from the Forest Inventory and Analysis (FIA) database.¹ The FIA program utilizes a continuous, fixed-radius plot design to monitor the United States forest resources (Bechtold et al., 1998; Woudenberg et al., 2010). The 1998 Farm Bill changed the FIA from periodic inventories to annual surveys, measuring approximately 10 to 15% of the plots annually, depending on the region (Smith, 2002). This approach takes seven to ten years to complete the inventory for a state. Thus, data for the recent eight years (2015–2023) were used in this study.

Data from the Southern FIA unit for the LMAV region of Arkansas was downloaded and combined. The dataset was filtered to include: (1) bottomland or floodplain stands dominated by oaks; (2) natural stand origin; (3) living trees; (4) operable sites (excluding steep ridges or permanently flooded areas); and (5) stands with no recent silvicultural interventions, though natural disturbances were allowed.

The resulting dataset contained 2,762 tree records from 34 FIA plots across seven oak species: Nuttall (21.2%), post (17.5%), water (15.5%), overcup (14.8%), willow (14.7%), and cherrybark (14.3%), whereas Shumard oak showed less representation with only 2.0% of

the observations. Missing data and outliers were removed before the analysis. Selected models were fitted to DBH and H. Initial measurements in the acquired data were in imperial units (inches and feet), which were converted to metric units (centimeters and meters) for all subsequent modeling.

2.2 Nonlinear models and evaluation

The dataset was randomly divided into training (75%) and validation/test (25%) subsets for model development and evaluation. Twelve candidate nonlinear H-DBH models (Table 1) were fitted using ordinary nonlinear least squares (NLS). The pooled training data of all seven oak species were used to establish a global base model. These candidate models were selected based on their parsimony and mathematical robustness in modeling the H-DBH relationship (Newton and Amponsah, 2007; Sharma, 2009; Wang and Hann, 1988). In this dataset, all species did not have equal representation; for instance, Shumard oak had very few observations. Developing an independent model for each species is challenging for the minority species that lack sufficient data for stable parameter estimation. Fitting candidate models to pooled data leverages the statistical power of the entire dataset for estimating stable parameters. It captures the average population-level allometric relationship, even for the species having inadequate representation (Lam et al., 2016). To account for interspecific variability arising from the pooled dataset, a mixed-effects model with species as a random effect was developed using the pooled data to

¹ <https://apps.fs.usda.gov/fia/datamart/datamart.html>

separate out the variation in the H-DBH relationship among species. In addition, using a mixed-effects model reduces model complexity and ensures reliable, species-specific predictions, which was useful in our dataset with unequal representation of species (Vázquez-Veloso et al., 2025). Moreover, evaluating these global base models allows for a baseline to quantify the prediction improvements resulting from the subsequent mixed-effects model. For this study, data preparation, statistical analysis, and model fitting were done using R version 4.4.1 (R Core Team, 2008). The fixed-effects models were developed using the *nls2* package (Grothendieck, 2013). The parameters for the evaluated twelve models were estimated using the NLS method. This method uses an iterative optimization algorithm to minimize the sum of squared differences between the observed and predicted tree heights (Dalggaard, 2008).

To evaluate model performance, multiple goodness-of-fit metrics were used in this study to ensure reliability and applicability. These evaluation metrics included Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Coefficient of Determination (R^2), and Akaike Information Criterion (AIC) (Akaike, 2003). To evaluate empirical support among competing models, ΔAIC , change in AIC was calculated. Because absolute AIC values are less informative than their differences, ΔAIC was computed as the difference between each model's AIC and the minimum observed AIC. The specific mathematical expressions were displayed in Equations 1–5, respectively.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-p}} \quad (1)$$

$$MAE = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{n} \quad (2)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (3)$$

$$AIC = n \times \ln \left(\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n} \right) + 2p \quad (4)$$

$$\Delta AIC_i = AIC_i - AIC_{\min} \quad (5)$$

Where y_i represents the observed total tree height for the i^{th} tree; $\hat{y}_i$ represents the predicted total tree height for the i^{th} tree; $\bar{y}$ is the mean of the observed total tree heights; n is the total number of tree observations; and p represents the number of estimated parameters within the specific model. AIC_i is the AIC value of the i^{th} model, $AIC_{\min}$ is the minimum AIC value among all evaluated candidate models.

A higher R^2 value suggests that the model explains a larger proportion of variance, indicating a better fit of the model. The smaller values for other metrics (MAE and RMSE) suggest the models have higher accuracy in their predictions. Following Burnham and Anderson (2002), the models with $\Delta AIC \leq 2$ indicates very strong

empirical support, while $\Delta AIC \geq 5$ suggests less support for those models. A relative rank sum was calculated following Poudel and Cao (2013) for evaluating the overall performance of these models across the goodness-of-fit metrics assessed. The ranks for the models across the four metrics (MAE, RMSE, R^2 , and ΔAIC) were summed to produce a final relative rank sum. A lower score indicates a better overall fit. The relative rank (R_i) for each model was computed as Equation 6:

$$R_i = \frac{(k-1)(S_i - S_{\min})}{S_{\max} - S_{\min}} \quad (6)$$

Where R_i = the relative rank of method i ($i = 1, 2, \dots, k$) with k = the number of methods evaluated, S_i = goodness of fit produced by method i , $S_{\min}$ = the minimum value of S_i , $S_{\max}$ = the maximum value of S_i .

The optimal model was then selected based on the relative rank sum and the biological realism of the model. In addition, these non-linear models were further evaluated by checking their assumptions. Normality was evaluated using the histogram of residuals and Q-Q plots (Supplementary Figures S1, S2). Homogeneity of variance in model residuals was assessed with the residual plots of the standardized residuals across diameter classes (Supplementary Figure S3). These plots showed that these assumptions of the models were met.

2.3 Nonlinear mixed-effects modeling and functional validation

To account for the species-specific variability in H-DBH relationships, a nonlinear mixed-effects model was developed for the selected base model. Mixed-effects models capture the average trend across the entire population (fixed effects). It also accounts for the unique variations among the studied subjects (random effects) from that population average. This approach partitions variance into components explained by fixed effects and residual variance attributed to random effects, which may be caused by unobserved covariates such as species-level differences and plot- or site-level heterogeneity (Mensah et al., 2018; Lindner, 2014).

Following the initial evaluation and model selection, which is detailed later in the Result subsection 3.2, the three-parameter Weibull model was selected as the base function. A species-level random effect was incorporated into the slope parameter (b_1), which governs the rate of height growth, and added to the base model for addressing interspecific variability. The mixed-effects model was developed using the *nlme* package (Pinheiro, 2011). The statistical significance of the random effect was evaluated using the Likelihood Ratio Test (LRT). Following the parameterization principles outlined by Pinheiro and Bates (2000), the nonlinear mixed-effects model is expressed as:

$$H_{ij} = 1.3 + b_o \times \left(1 - \exp \left(-(b_1 + u_i) \times DBH_{ij}^{b_2} \right) \right) + \varepsilon_{ij} \quad (7)$$

Where H_{ij} is the total tree height (m) for the j^{th} tree of the i^{th} species; DBH_{ij} is the diameter at breast height (cm) for the j^{th} tree of the i^{th} species; b_o , b_1 , and b_2 are the population-level fixed effect parameters; u_i is the random effect parameter specific to the i^{th} species, assuming $u_i \sim N(0, \sigma_u^2)$, where σ_u^2 is the variance of the random species effect; and ε_{ij} is the within-group random error term. To

evaluate and compare height predictions from the mixed-effects model against the fixed-effects model, three fit measures were used: RMSE, MAE, and the R^2 . Additionally, species-specific performance of the mixed-effects model was reported using two metrics of RMSE and bias.

In addition, the mixed-effects model was assessed for its practical utility in volume estimations. Since predicted heights derived from H-DBH models are often used as key explanatory variables for estimating merchantable volume and biomass (Chenge, 2021; Ledo et al., 2016), the assessment of the model-predicted heights was done for their applications in volume estimations. The comparison was done for the differences in tree volume estimated using the observed heights against volumes estimated using the model-predicted heights. Tree volume was calculated using the Schumacher-Hall model (Equation 8; Schumacher, 1933), where the volume estimations from observed heights of the FIA dataset were used as a baseline for comparison. The Schumacher-Hall model was used for its parsimonious structure and consistency in estimations across diverse data sources (Westfall et al., 2024). It takes the following form:

$$V_i = a \times D_i^b \times H_i^c + \varepsilon_i \quad (8)$$

Where a , b , and c are coefficients provided for the model (Supplementary Table S1), ε_i is the random residual error, V_i denotes the volume of the i^{th} tree, H_i is the total tree height for the i^{th} tree, and D_i is the DBH for the i^{th} tree. Finally, MAPE was used to evaluate volume estimates, and the volume validation was done across all DBH classes.

2.4 Response calibration of the mixed-effects model

Height predictions from the mixed-effects models were executed with and without random effect parameters. Excluding the random effects results in a population-average prediction that relies entirely on fixed effects (Mehtätalo, 2004). Response calibration was applied to estimate and incorporate random effects, thereby improving the predictive accuracy of the models. The random effect parameters were estimated using the empirical best linear unbiased prediction (EBLUP) method (Vonesh and Chinchilli, 1996). This iterative procedure initializes the species-level random effect vector u_i to zero before converging on an optimal solution (Arias-Rodil et al., 2015):

$$\hat{u}_i = \hat{D}Z_i' \left(\hat{R}_i + Z_i \hat{D}Z_i' \right)^{-1} \left[y_i - f \left(x_i, \hat{\beta}, \hat{u}_i \right) + Z_i' \hat{u}_i \right] \quad (9)$$

Where $\hat{u}_i$ is the vector of predicted random effects for the i^{th} species (corresponding directly to the species-specific parameter); u_i in the height diameter model; Z_i is the design matrix of the random components for species i , derived from the partial derivative of the mixed effects function with respect to the fixed effect parameters containing random effects; $\hat{D}$ is the estimated variance–covariance matrix of the species-level random effects; $\hat{R}_i$ is the estimated variance–covariance matrix of the residual errors for species i ; y_i is the vector of observed tree heights for species i (corresponding to the response variable H_{ij}); $f(\cdot)$ represents the specified mixed effects function; $\hat{\beta}$ represents the vector of the estimated fixed-effects parameters (b_1 , b_2 , and b_3).

To calibrate the mixed-effect model, this study adapted portions of the codes provided by Arias-Rodil et al. (2015). To evaluate the number and size of trees required for the calibration process, different subsample combinations based on the diameter distribution were tested (Bronisz and Mehtätalo, 2020). Four calibration selection methods were assessed by sampling specific diameter sizes for each species: the smallest tree, the medium-sized tree, the largest tree, and a combination of all three tree sizes. For each oak species, the single plot containing the highest number of individuals for that species was identified. These seven targeted plots were then used to extract the calibration subsamples for each size category, resulting in three trees per species and a total of 21 calibration trees.

Using the complete calibration subsample (all selected trees across the size categories for each species), the random effects were re-estimated. The resulting EBLUP-calibrated model was then used to predict the tree heights. The predictive performance of this localized model was evaluated against the uncalibrated population-average model. The evaluation of these models was conducted across two distinct datasets: the first dataset included the remaining trees within the seven targeted plots (excluding the selected calibration trees), and the second dataset included the remaining trees (excluding the seven targeted plots from the overall data). The calibration was evaluated by estimating the reduction in RMSE and MAE.

3 Results

3.1 Descriptive data summary

The descriptive statistics of DBH and total height (H) for the seven oak species, overcup oak (OCUP), cherry bark oak (CBO), Nuttall oak (NUT), shumard oak (SHU), water oak (WATER), willow oak (WILLOW), and post oak (POST) are summarized in Table 2. The DBH for all observed trees ranged from 2.54 to 69.59 cm, while H ranged from 2.13 to 38.10 m. Overcup oak has the highest average height (19.98 m) and DBH (33.46 cm). Water oak has the lowest average height (16.02 m), and Shumard oak has the lowest average DBH (21.33 cm).

3.2 Fitted nonlinear fixed-effects models and base model selection

All twelve nonlinear models evaluated in this study showed statistically significant parameter estimates ($p < 0.05$) (Table 3). Model performance was assessed using multiple goodness-of-fit metrics, including MAE, RMSE, R^2 , and ΔAIC . The results indicated that the Ratkowsky model produced the lowest MAE (2.5922 m), while the Weibull model showed the highest R^2 (0.7672) and the lowest RMSE (3.3583 m). In contrast, the Schumacher and Curtis models overestimated H; for a tree with a 25 cm DBH (observed H = 16.75 m), these models predicted 18.44 m and 20.02 m, respectively. The Weibull (17.13 m) and Meyer (17.34 m) models estimated heights that aligned with observed heights. For ΔAIC , the Chapman, Curtis, and Weibull models were given strong empirical support, with $\Delta AIC < 2$ (Table 4).

To evaluate these metrics collectively, a relative rank sum analysis (Poudel and Cao, 2013) was computed, which showed similar overall statistical performance among the top three models: Gompertz (5.03), Meyer (5.15), and Weibull (5.07) (Table 4). Given these similar rank sum scores, the models were further evaluated for biological realism.

TABLE 2 Summary statistics of DBH and total tree height for the data used in the study.

Oak species	Observations		DBH (cm)	H (m)
OCUP	409	Min	2.54	3.96
		Max	69.34	38.10
		Mean	33.46	19.98
		CV	49.69	34.80
CBO	396	Min	2.54	3.05
		Max	68.83	36.58
		Mean	24.52	16.19
		CV	64.93	44.44
NUT	585	Min	2.79	2.74
		Max	69.34	36.57
		Mean	24.38	16.54
		CV	58.41	43.56
SHU	56	Min	2.54	2.13
		Max	69.59	38.10
		Mean	21.33	16.59
		CV	66.67	44.70
WATER	428	Min	2.54	3.96
		Max	69.59	36.57
		Mean	23.77	16.02
		CV	55.19	42.44
WILLOW	405	Min	2.79	3.35
		Max	68.83	32.61
		Mean	27.30	17.93
		CV	55.18	41.27
POST	483	Min	2.54	2.43
		Max	69.08	37.49
		Mean	26.48	16.82
		CV	41.93	30.61

To be noted, overcup (OCUP, *Quercus lyrata*), cherry bark (CBO, *Quercus pagoda*), Nuttall (NUT, *Quercus texana*), Shumard (SHU, *Quercus shumardii*), water (WATER, *Quercus nigra*), willow (WILLOW, *Quercus phellos*), and post oaks (POST, *Quercus stellata*).

The Weibull function reflects a sigmoid growth shape with an early-stage inflection point and a positive height increment (Supplementary Figure S4). Furthermore, the Weibull model offers the structural flexibility necessary to model the diverse diameter distributions (Bailey, 1973), which is suitable for multilayered oak-dominated bottomland hardwood forests. The Weibull model was therefore recommended as the base model for further study due to its combined relative rank sum performance and biological realism.

3.3 Nonlinear mixed-effects model

Species was incorporated as a random effect in the nonlinear mixed-effects model, and the parameter estimates are presented in Table 5. Results showed a statistically significant population-level fixed-effects model, with a base slope parameter ($b_1 = 0.026$, $SE = 0.0012$, $t = 21.825$, $p < 0.0001$). Meanwhile, results revealed a slight variation in the slope parameter (b_1) across species, with variance = 7×10^{-7} and $SD = 8 \times 10^{-4}$. The residual variance was estimated at 11.1842 ($SD = 3.344$), reflecting moderate unexplained

variability in the model. The Likelihood Ratio Test ($LRT = 92.43172$, $p < 0.0001$) indicated that using species as a random effect could improve the H-DBH model. The effective slope, calculated as the sum of the population-level fixed effect and the species-specific random effect ($b + u_1$), represents the rate of height growth with increasing diameter, where higher values indicate more rapid vertical growth. For example, as presented in Table 6, the effective slope of Nuttall oak was 0.02705 ($b_1 = 0.02652$, $u_1 = 0.00053$) while it was 0.02517 ($b_1 = 0.02558$, $u_1 = -0.00041$) for overcup oak.

Table 7 summarizes model performance on the 25% validation dataset. The mixed-effects model yielded lower error ($RMSE = 3.3401$ vs. 3.3784; $MAE = 2.5861$ vs. 2.6474) and a higher R^2 (0.7913 vs. 0.7488) compared to the fixed-effects model. Species-specific error metrics for the mixed-effects model are reported in Table 8. Nuttall oak showed the lowest RMSE (2.54 m), and RMSE was highest for post oak (3.76 m). Bias metrics showed a slight overestimation in height for six species, where willow oak showed the largest negative bias (−1.151 m). However, Shumard oak showed a positive bias (1.550 m), indicating a slight underestimation of height.

TABLE 3 Parameter estimates for nonlinear H-DBH allometric models.

Model	Allometric equation	Coefficient		
		b_0	b_1	b_2
Schumacher	$H = 1.3 + b_0 \times \exp(-b_1 / DBH)$	35.02***	17.87***	
Bufford	$H = 1.3 + b_0 \times \exp(b_1 / DBH)$	35.02***	-17.87***	
Curtis	$H = 1.3 + b_0 \times \frac{DBH}{(1 + DBH)^{b_1}}$	1.99***	0.35***	
Meyer	$H = 1.3 + b_0 \times (1 - \exp(-b_1 \times DBH))$	35.54***	0.02***	
Micment	$H = 1.3 + \frac{b_0 \times DBH}{b_1 + DBH}$	55.01***	59.53***	
Power	$H = 1.3 + (b_0 \times DBH^{b_1})$	1.80***	0.66***	
Logistic	$H = 1.3 + b_0 / (1 + (b_1 \times \exp(-b_2 \times DBH)))$	28.70***	5.03***	0.07***
Weibull	$H = 1.3 + b_0 \times [1 - \exp(-b_1 \times DBH^{b_2})]$	30.00***	0.03***	1.00***
Chapman	$H = 1.3 + b_0 \times [1 - \exp(-b_1 \times DBH)]^{b_2}$	31.00***	0.03***	1.04***
Gompertz	$H = 1.3 + b_0 \times \exp(-b_1 \times \exp(-b_2 \times DBH))$	30.42***	2.16***	0.04***
Ratkowsky	$H = 1.3 + b_0 \times \exp\left(-\frac{b_1}{DBH + b_2}\right)$	49.30***	42.69***	13.45***
Prodan	$H = 1.3 + (DBH^2 \times (b_0 + b_1 \times DBH + b_2 \times DBH^2))$	-0.08*	1.15***	0.01***

* indicates p -value < 0.05, and *** indicates p -value < 0.001.

TABLE 4 Goodness of fit measures based on test data for fitted nonlinear models across validation data.

Model	MAE (m)	RMSE (m)	R^2	Δ AIC	Rank sum
Schumacher	2.8435	3.5315	0.1660	1074.5008	39.29
Bufford	2.9435	3.7315	0.4860	704.6365	38.36
Curtis	2.6135	3.3760	0.7491	1.3326	6.10
Meyer	2.6052	3.3421	0.7551	37.4746	5.15
Micment	2.6011	3.3439	0.7544	559.7775	10.44
Power	2.6183	3.3803	0.7485	2.0795	6.39
Logistic	2.636	3.3607	0.7514	13.5180	6.46
Weibull	2.6074	3.3583	0.7672	0.3016	5.07
Chapman	2.6474	3.3783	0.7512	0.0000	7.17
Gompertz	2.6039	3.3370	0.7550	42.9825	5.03
Ratkowsky	2.5922	3.3760	0.7552	87.8970	6.21
Prodan	2.5938	3.3818	0.7543	6.1344	5.60

3.4 Functional validation through volume estimation

To functionally validate the practical application of the best-fitted H-DBH model, Schumacher-Hall was utilized for making volume estimations based on the predicted heights of these best-fitted H-DBH models. The volume estimations were consistent

when estimated using the observed height (black line) and the height from the mixed-effects model (blue line) (Figure 2). The volume estimates from the two heights were nearly identical across different diameter classes. For trees in the 0–25 cm DBH class, the mean volume from the mixed-effects model was 10 m³ compared to 9.86 m³ from the observed data. For trees in the 25–50 cm DBH class, the height from the mixed-effect model

TABLE 5 Mixed effect Weibull model summary.

Component	Parameter	Estimate	SE	<i>p</i>	Variance	SD
Fixed effects						
Intercept	b_0	40.89	3.48	<0.0001		
Slope	b_1	0.02	0.00	<0.0001		
Slope	b_2	0.92	0.03	<0.0001		
Random effects						
σ_{u_i} (species on b_1)					7×10^{-7}	8×10^{-4}
σ_{ε} residual					11.18	3.34
Likelihood ratio test						
Comparison	LRT statistic	<i>p</i>				
Mixed v Fixed slope	92.43	<0.0001				

Where $\sigma_{u_i}^2$ is the variance component of the species level random effect applied to the slope of parameter(b_1), while σ_{u_i} (reported in the SD column) is the standard deviation; σ_{ε} is the variance of the within group residual error; b_0 , b_1 , b_2 are the population level fixed effect parameters.

TABLE 6 Parameter estimates for the mixed-effects Weibull model across seven oak species.

Species	b_{1_random}	b_0	b_1	b_2
CBO	−0.00016	40.89383	0.02584	0.92374
NUT	0.00053	40.89383	0.02652	0.92374
OCUP	−0.00041	40.89383	0.02558	0.92374
POST	−0.00023	40.89383	0.02576	0.92374
SHU	0.00144	40.89383	0.02743	0.92374
WATER	−0.00018	40.89383	0.02581	0.92374
WILLOW	−0.00100	40.89383	0.02499	0.92374

TABLE 7 Goodness of fit measures from the mixed-effects and fixed-effects Weibull models.

Model	RMSE(m)	MAE(m)	R^2
Mixed-effects (random slopes)	3.3401	2.5861	0.7913
Fixed-effects	3.3784	2.6474	0.7488

TABLE 8 Species-specific performance metrics (RMSE and bias) for the mixed-effects height-diameter model based on the 25% validation dataset.

Species	RMSE(m)	Bias(m)
NUT	2.54	−0.232
WATER	2.81	−0.389
CBO	3.07	−0.073
WILLOW	3.45	−1.151
SHU	3.69	1.550
OCUP	3.75	−0.958
POST	3.76	−0.236

yielded a mean volume of 60 m³, matching closely with that (60.6 m³) from the observed heights (Table 9).

However, the volume calculated using the height from the fixed-effects model (red line) was consistently underestimated compared to that from the other two approaches (Figure 2). This underestimation in volume was more evident in larger trees (DBH > 50 cm), where the mean volume from predicted heights of the fixed-effects model was 189 m³ compared to 215 m³ and 211 m³ from the observed height and the

mixed-effects model, respectively. Furthermore, the fixed-effects model showed bigger deviations in the upper range of volume, with a maximum of 364 m³ in the largest DBH class compared to the observed maximum of 427 m³ (Table 9). These results indicated that the fixed-effects model yielded less precise volume estimates than the mixed-effects model.

To further evaluate the accuracy of volume estimations derived from the best-fitted mixed-effects model, Mean Absolute Percentage Error (MAPE) was computed for the volume

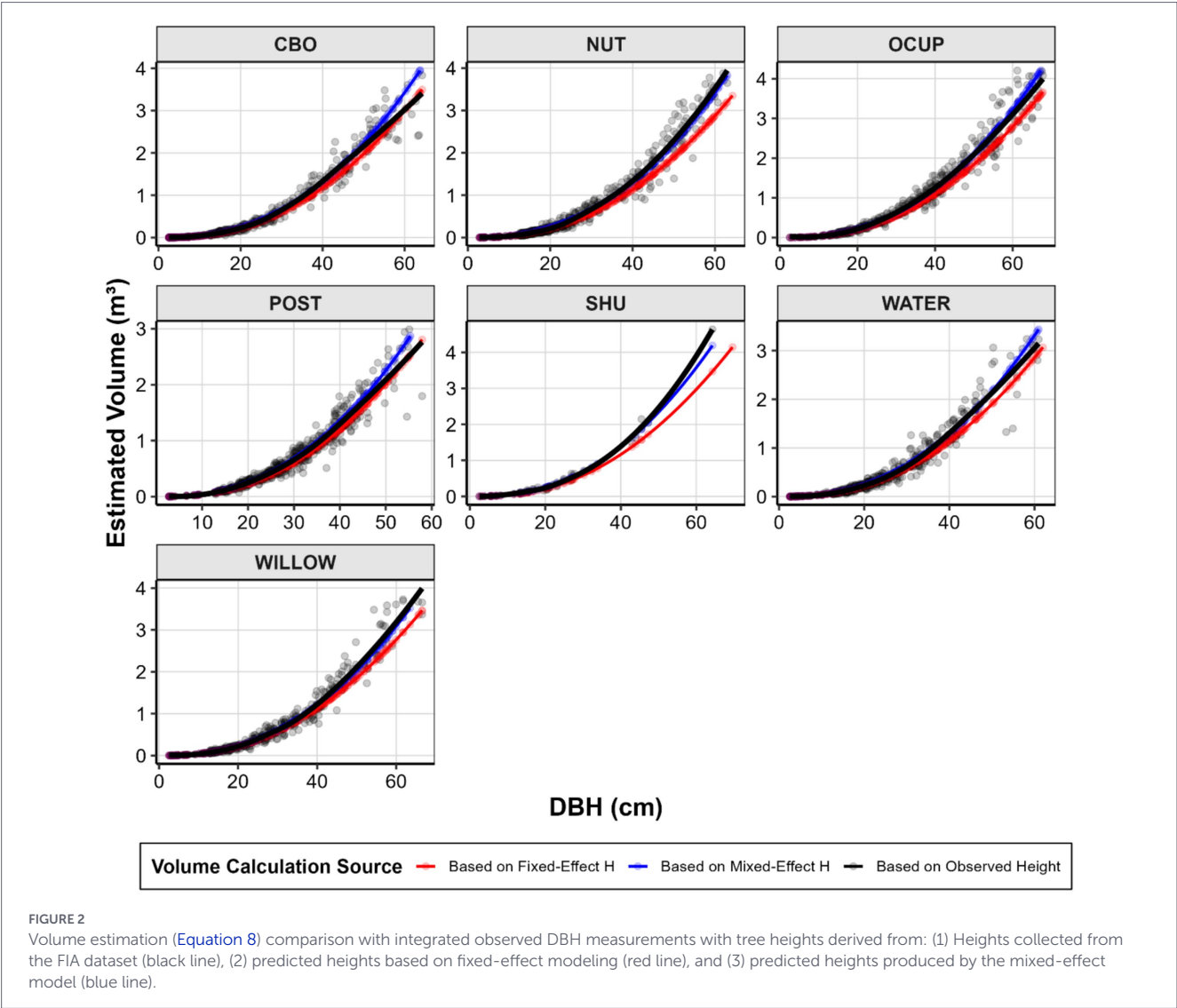


TABLE 9 Descriptive statistics for tree volume (m³) calculated from observed heights, fixed-effects predictions, and mixed-effects predictions across three DBH size classes.

DBH	Volume source	Mean	SD	Max
0–25	Fixed effect	9.73	6.1	27.8
	Mixed effect	10	6.2	28.4
	Observed data	9.86	6.52	33
25–50	Fixed effect	57.4	27.2	144
	Mixed effect	60	29.4	154
	Observed data	60.6	32.3	169
>50	Fixed effect	189	58	364
	Mixed effect	211	73	450
	Observed data	215	72.5	427

predictions. The comparisons were made between the volume derived from the model-predicted heights and the baseline volume derived from the observed heights. MAPE values showed distinct patterns in diameter classes across species (Table 10). Specifically, there were higher prediction errors for diameter classes of 0–10 cm and 60 + cm for cherry bark oak and water oak, whereas a smaller MAPE was observed for mid-range diameter classes across the species.

3.5 Response calibration of the mixed-effects model

Among the tree selection strategies for response calibration, incorporating all tree sizes produced the lowest RMSE (2.849 m) (Figure 3). For single-tree size approaches, selecting a medium-sized tree resulted in the lowest RMSE (2.875 m), compared to small (RMSE = 2.985 m) and large (RMSE = 2.906 m) trees.

TABLE 10 MAPE (%) values across DBH classes (cm) for volume (m³) estimates for oak species.

Species	(DBH 0–10)	(DBH 10–20)	(DBH 20–30)	(DBH 30–40)	(DBH 40–50)	(DBH 50–60)	(DBH 60+)
CBO	18.45	18.49	18.11	11.15	14.24	12.61	18.63
NUT	19.59	21.56	21.2	12.93	14.38	10.11	9.31
OCUP	27.26	19.54	14.49	12.66	10.65	12.06	12.52
POST	14.15	15.18	12.94	15.12	13.34	22.39	32.33
SHU	18.95	18.18	12.32	5.92	5.77	8.33	11.35
WATER	15.76	21.27	21.66	18.38	12.26	25.95	14.54
WILLOW	18.71	18.14	17.85	15.76	12.21	10.97	7.47

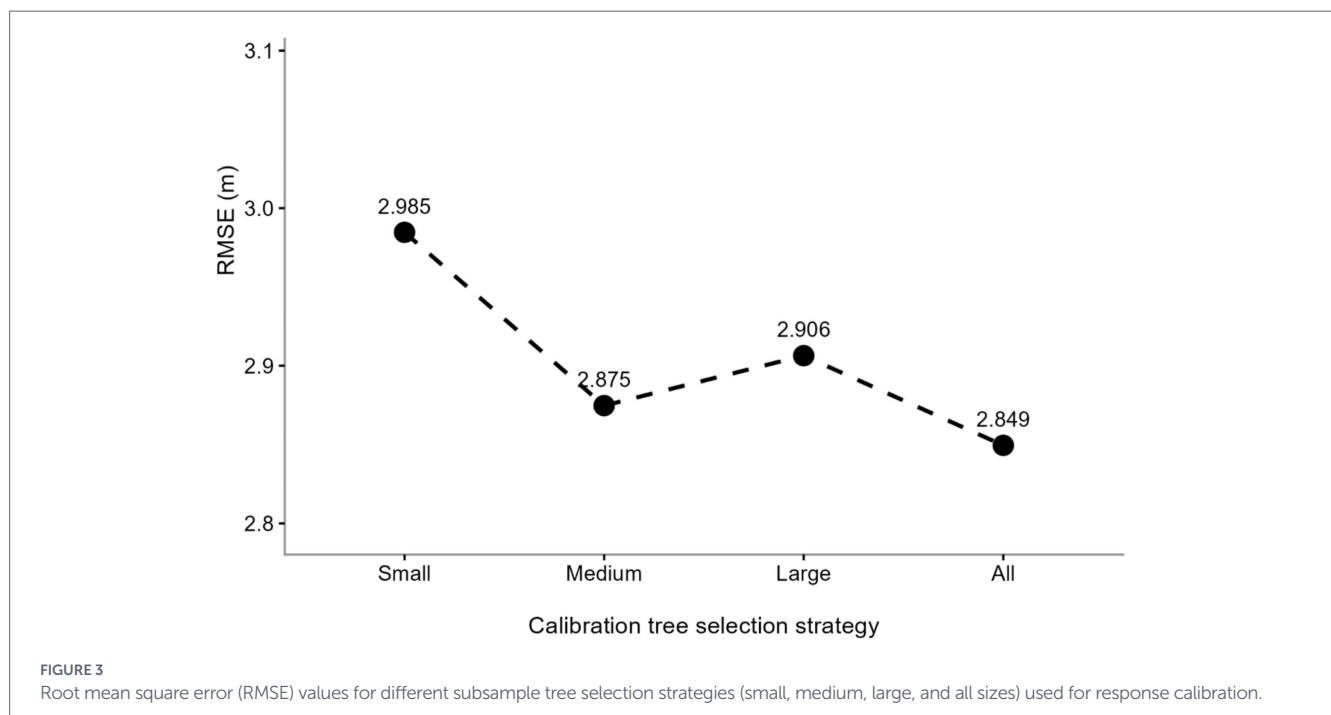


TABLE 11 Comparison of prediction accuracy (RMSE and MAE) between the fixed-effects model and the BLUP-calibrated mixed-effects model across two validation datasets.

Dataset	Model	RMSE(m)	MAE(m)
7 selected plots	Fixed effects	2.8197	2.3120
	Calibrated mixed effect	2.8192	2.3116
Remaining plots	Fixed effects	3.3644	2.6414
	Calibrated mixed effect	3.3640	2.6412

The BLUP-calibrated model reduced prediction errors relative to the fixed-effects model across both test datasets (Table 11). Within the seven calibration plots (excluding the specific trees used for calibration), the BLUP-calibrated model showed an RMSE of 2.8192 m and a mean absolute error (MAE) of 2.3116 m, compared to 2.8197 m and 2.3120 m for the fixed-effects model. In the remaining dataset (excluding the calibration plots entirely), the BLUP-calibrated model produced an RMSE of 3.3640 m and an MAE of 2.6412 m, compared to the fixed-effects model's RMSE of 3.3644 m and MAE of 2.6414 m.

4 Discussion

The H-DBH model remains one of the most widely applied tools in forest biometrics due to its central role in growth and yield systems. Unlike the extensive research on southern pines, H-DBH modeling for mixed hardwoods in the western Gulf region remains comparatively limited, despite the ecological and economic significance of bottomland hardwood ecosystems. Hardwood H-DBH relationships are inherently more variable than those of conifers, reflecting broader species diversity, more heterogeneous stand structures, and varied

disturbance histories. This study aims to develop H-DBH models specifically for oak species in bottomland hardwoods across the LMAV of Arkansas that can be used to guide and understand timelines associated with the planning of restoration and reforestation.

Statistical performance was similar among the top-ranked candidate models. Although Gompertz and Meyer models had slightly better relative rank sum metrics, the three-parameter Weibull model was selected as the base function based on its theoretical flexibility and biological realism. This function represents the nonlinear H-DBH trajectory of oak-dominated stands: height growth during early development, a decreasing growth rate as diameter increases, and asymptotic behavior at maturity. This pattern is consistent with biological realism (Sharma, 2009; Upadhyaya and Khadka, 2024). The theoretical applicability of the asymptotic Weibull function in our study also aligns with Fulton (1999) who successfully modeled 15 hardwood species in neighboring East Texas using asymptotic functions. Fulton argued that non-asymptotic relationships are biologically unrealistic for mature stands due to mechanical support constraints and hydraulic limitations that restrict height growth as diameter increases. It is supported by Koch et al. (2004), noting that non-saturating curves that do not reflect a declining height increment rate lack biological credibility, as height growth typically slows once trees reach canopy position.

The flexibility of the Weibull function, known for modeling diverse diameter distributions (Bailey, 1973), making it especially effective for stands with complex structural layering (Podlaski and Zasada, 2008), such as the oak-dominated bottomland hardwoods analyzed in this study. These findings are consistent with other studies: Long et al. (2022) reported the Weibull function as the best-fitting model among six probability distributions for *Quercus* species in southern China, and Zhang et al. (2014) reached similar conclusions for both nonlinear least squares and Bayesian frameworks. Likewise, Ledo et al. (2016) found the Weibull model superior for tropical forests in Panama and Peru.

The H-DBH relationships in oaks are especially sensitive to variation in competition, site quality, and species-specific growth form. Prior studies show that incorporating stand-level covariates such as stand density index, basal area, and site index can improve model accuracy (Li et al., 2015; Mensah et al., 2018; Sharma and Parton, 2007), though these enhancements often reduce model applicability where such data are unavailable. However, mixed-effects modeling provides an alternative that improves prediction without requiring additional stand-level inputs (Trincado et al., 2007; Zang et al., 2016), and the inclusion of stand covariates within mixed-effects frameworks typically yields only modest further gains (Temesgen et al., 2014). Consistent with these findings, allowing the Weibull parameter b_1 to vary by species, in this study, captured a slightly higher portion of the unexplained variation and enhanced model performance. Species-level differences were evident: red oaks generally exhibited slightly faster height growth with increasing diameter than white oaks. For example, Nuttall oak and Shumard oak showed higher effective slopes (0.027 and 0.028), whereas post oak and overcup oak had lower slopes (0.0255 and 0.0251) (Table 6). These patterns align with Jeffreys (2014), who reported that white oaks in the LMAV grew more slowly than red oaks, partly due to the superior site adaptability of the latter. Accounting for these species-level differences, the mixed-effects

model increased R^2 by 5.68% and reduced RMSE by 1.13%, relative to the fixed-effects formulation. Regarding species-specific performance of the mixed effect model, precision was highest for Nuttall and water oaks, while post and overcup oaks exhibited relatively higher residual errors.

Beyond model form, the finding of this study that mixed-effects models outperform fixed-effects models is consistent with regional studies. Lynch et al. (2005) modeled height-diameter relationships for cherrybark oak (*Quercus pagoda*) in bottomland hardwood sites of East Texas and reported that calibrating a mixed-effects model with local data reduced the Root Mean Square Error (RMSE) by approximately 35% (from 4.39 m to 2.87 m) compared to a fixed-effects global model. Similarly, Fulton (1999) who studied 15 hardwood species in East Texas observed that H-DBH variation was significantly better expressed when site factors were included in the model. These results are further corroborated by Lhotka (2012), who studied water oak (*Quercus nigra*) and other hardwoods in the Piedmont and Coastal Plain regions of Alabama, Georgia, and South Carolina. Lhotka also reported that mixed-effects models provided better predictive ability compared to fixed-effects models, reinforcing the utility of this approach for capturing the interspecific and site-level variation inherent in southern hardwood stands.

To assess the operational performance of fitted models, functional validation through volume estimation was done. The fixed-effects model did not show consistency in volume estimation, wherein it underestimated volume for larger DBH classes. This result is consistent with previous research showing that while different height-diameter models yield identical volume estimates, especially for small trees, predictions diverge for trees of larger DBH classes (Peng et al., 2001). In bottomland hardwood forests, mature oaks account for a major portion of the total stand volume and commercial timber (Kellison and Young, 1997; Lockhart et al., 2010). In this study, the mixed-effects model showed improvements in volume estimations that were consistent with baseline volumes throughout DBH classes. These estimations can inform stand-level planning, harvest scheduling, and thinning operations.

To further enhance the application of the mixed-effects model, response calibration was done. Four calibration subsampling strategies were evaluated. Results showed that utilizing the complete subsample across all three tree sizes yielded the lowest RMSE (2.849 m), with medium-sized trees providing comparable accuracy RMSE (2.875 m). Considering the optimal selection of trees for calibration, previous studies have found the thickest or extreme-sized trees to be most effective (Bronisz and Mehtätalo, 2020). However, our results align with (Ciceu et al., 2020), i.e., medium-sized trees were optimal for response calibration. Furthermore, applying this calibrated model showed reductions in both RMSE and MAE for both datasets used for evaluation. This consistent error reduction is supported by Hall and Bailey (2001), highlighting that predicting and incorporating random effects through localization results in more accurate height estimates.

A key application of H-DBH modeling is to reduce measurement costs by predicting tree height from DBH alone, a benefit in hardwood systems where height measurements are labor-intensive and time-consuming. The H-DBH models developed in this study have direct implications for forest management in BLH ecosystems of the LMAV, as accurate height predictions could help managers and landowners to characterize stand structure. Predicted heights, combined with DBH, are useful to estimate volume and biomass. The mixed-effects model, which provided consistent volume estimates across all DBH classes, including larger trees (DBH > 50 cm), provides useful and fundamental

information for harvest scheduling, thinning operations, and timber volume assessments. The response-calibrated model further enhances operational utility by allowing localized predictions when a small subsample of trees (as few as three per species) is available, making it practical under limited inventory budgets. Moreover, H-DBH models provide a tool for assessing site quality in uneven-aged oak stands (Burkhart and Tomé, 2012a, 2012b). Overall, these models provide a quantitative foundation for sustainable silvicultural decision-making, restoration planning, and biomass stock assessments, and the findings of this study have both scientific and practical relevance for the management of bottomland hardwood forests in the region.

Despite these utilities, several limitations should be acknowledged. First, while H-DBH relationships are known to be influenced by factors such as stand density, localized site conditions, soil properties, and silvicultural history, the FIA dataset utilized in this study lacks detailed covariates for these conditions. Furthermore, the focus of this study was individual tree-level modeling, rather than generalized stand-level modeling, to ensure that the resulting tools remain operational for managers with limited data. Without including stand-level factors, the models inevitably oversimplify some underlying biological processes. While adding a random species effect partially addressed this by capturing species-level variability, future research utilizing distinct experimental plot data could evaluate these covariates to present managers with options for accuracy improvements when broader stand-level data is available.

5 Conclusion

Oak stands within the BLH forests represent highly productive and economically significant natural resources and ecosystems. Forest professionals and landowners require reliable, species-specific tools to project future stand development and inform management decisions. This study developed allometric H-DBH relationship models for major oak species using twelve nonlinear models fitted to a comprehensive FIA dataset, and the results indicated that the Weibull model demonstrated the best performance compared to the other models assessed.

To account for interspecific variability, the Weibull function was extended into a nonlinear mixed-effects framework. The fixed-effects model underestimated height for large trees (DBH > 50 cm), which resulted in their volume underestimation. Predicted heights of the mixed-effects model were consistent with observed heights, and therefore their volume estimations were consistent even for larger diameter classes. Moreover, response calibration utilizing a subsample of trees offered localized predictions of heights that showed a consistent reduction in RMSE compared to the uncalibrated population-average models.

However, height-diameter relationships naturally vary with local environmental conditions and stand dynamics. The models developed in this study only use DBH as an explanatory variable and do not explicitly account for climatic or ecological site factors. Thus, applying these models outside the sampled region or using them with non-FIA datasets should be done with caution. Future work with H-DBH models can focus on expanding model applicability by incorporating additional covariates, such as competition

indices and distance-dependent attributes, and validating performance across broader site conditions.

Data availability statement

Publicly available datasets were analyzed in this study. This data can be found here: data for this study were collected from the U.S. Forest Service FIA database (<https://apps.fs.usda.gov/fia/datamart/datamart.html>).

Author contributions

SB: Data curation, Formal analysis, Methodology, Software, Writing – original draft. NT: Conceptualization, Investigation, Project administration, Resources, Supervision, Writing – original draft. YW: Conceptualization, Methodology, Supervision, Validation, Writing – original draft. DO: Investigation, Resources, Supervision, Writing – review & editing. JG: Supervision, Writing – review & editing.

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Supplementary material

The Supplementary material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/ffgc.2026.1777333/full#supplementary-material>

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