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Original Article

Regional differences in stem form between southern and northern red spruce (*Picea rubens* Sarg.) populations

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Abstract

Red spruce (*Picea rubens* Sarg.) is a commercially and ecologically important conifer species that primarily exists at northern latitudes of eastern North America and extends southward following the Appalachian Mountains into North Carolina and Tennessee. Due to a warming climate and human-caused disturbances, only fragmented, sky-island populations remain at the highest peaks of the southern Appalachians where their habitat continues to be threatened. While they have been recognized for the rare wildlife habitat they provide in the region, these populations remain understudied. This work aimed to examine differences in stem form between the northern and southern populations of red spruce and to provide additional quantitative methods for managing red spruce stands through providing regionally fitted stem taper models. First, we examined differences in stem form using two methods: a sectional rate of change in diameter and a region variable added to the Kozak (2004) Model 02 taper model. The sectional taper comparison showed significant differences ($P < .05$) in taper rate throughout the stem that were most pronounced below breast height and above the midpoint. The nested model comparison also showed a significant difference after performing a likelihood ratio test. These results agreed that significant differences in stem form between the two populations exist and supported the idea that localized taper models would provide the best results. Next, we evaluated four stem taper models for their ability to predict upper stem diameters and total volume in southern Appalachian red spruce: a quadratic polynomial, a segmented, a variable exponent, and a geometric model. Fitting data came from a 1992 study across three southern Appalachian spruce-fir sites. Validation data came from a new dataset collected in 2022 at Unaka Mountain, Tennessee. Based on past studies and our results, we found that the Kozak (My last words on taper equations. *For Chron* 2004;80:507–15. https://doi.org/10.5558/tfc80507-4.) Model 02 variable exponent and the Max and Burkhart (Segmented polynomial regression applied to taper equations. *For Sci* 1976;22:283–9.) segmented polynomial models performed equally well. The choice of the final model should depend on the users' objectives and practical limitations (i.e. programming ability, availability of fitting data, error tolerance).

Keywords: stem; southern; spruce; taper; Appalachia; comparison

Introduction

Red spruce (*Picea rubens* Sarg.) is a long-lived, shade-tolerant conifer native to the cool, temperate regions of eastern North America (White and Cogbill, 1992; Nowacki et al., 2010). During the peak of the last Wisconsin glaciation, red spruce dominated much of the Appalachian Mountains and eastern North America. As the climate warmed, red spruce retreated north and upslope with montane populations left to the highest peaks with suitable climates (White and Cogbill, 1992). Today, the bulk of red spruce's range extends from the southern parts of the New Brunswick, Quebec, and Ontario provinces, south into central New York and northeast Pennsylvania, and east to the coasts of Maine and Massachusetts. From there, it follows the peaks of the Appalachian Mountains south to western North Carolina and eastern Tennessee (Blum, 1990) (Fig. 1).

The red spruce population can be split into three geographic regions: the northern, from the Adirondack Mountains northeastward; the central, primarily in West Virginia; and the southern, in the mountains of southwestern Virginia, western North Carolina, and eastern Tennessee (Rentch et al., 2007; Nowacki et al., 2010; White et al., 2012). Due to climatic habitability and past logging practices, the central and southern populations exist as montane forests on the peaks of the Appalachian Mountains (Rentch et al., 2007; Walter et al., 2017). These montane spruce-fir cover types are rare in the southeastern USA and provide unique ecosystem services. By 1937, it was estimated that the range of red spruce was reduced to only 10% of its original extent in the central and southern Appalachians, although the high peaks in the South may have been less severely impacted (Korstian, 1937; Saunders, 1979; P. S. White and Cogbill, 1992). According to Forest Inventory and Analysis (FIA) and Landsat satellite data collected as part of the Southern Appalachian Assessment in 1996, montane spruce-fir forests cover 36 400 out of 15 million ha (0.2%) of vegetative land cover in Southern Appalachia (SAMAB, 1996). Importantly, these southern spruce-fir forests contain some of the only examples of old-growth spruce-fir remaining after 19th- and early 20th-century logging.

The stem form of trees can be described as the geometric shape of the stem and is affected by multiple factors (Schneider, 2019).

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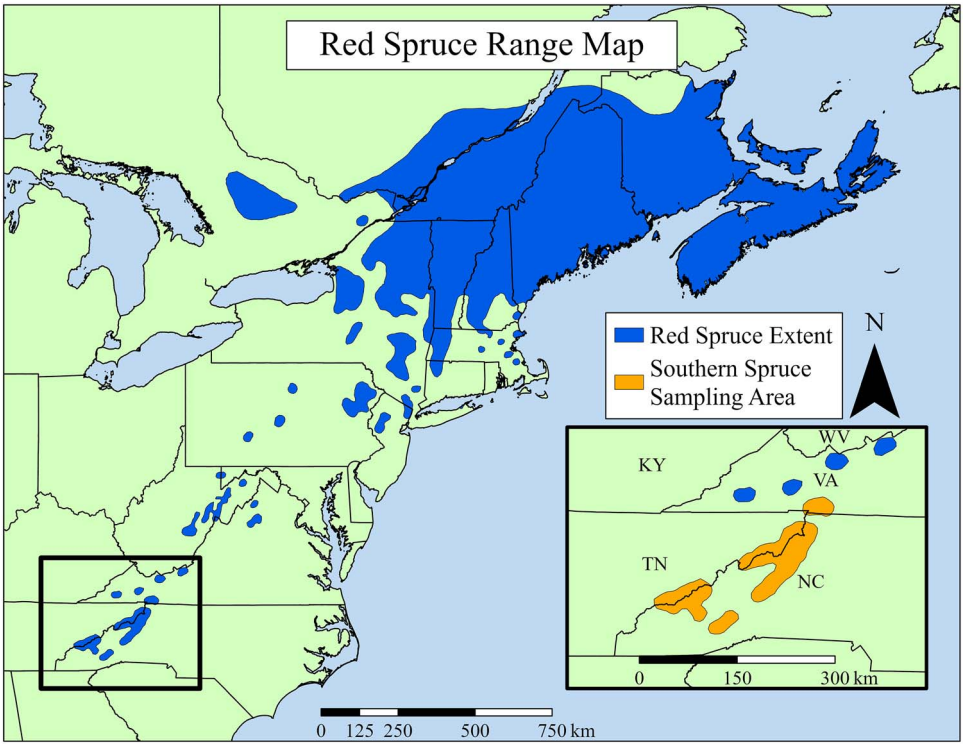


Figure 1. Red spruce range map with inset highlighting the southern population sample areas.

While trees have generally been divided into three segments (a neiloid base, paraboloid middle, and conic top), the proportions and eccentricities of these shapes will vary according to species and environmental factors (Max and Burkhart, 1976; Burkhart and Tomé, 2012). Many of these factors can be interrelated, and determining which factors change stem form in which way can be difficult. For example, stand density affects the live-crown ratio and within-stand wind exposure, all of which affect the stem diameter and taper (Valinger, 1992; Zhang et al., 1996; Mitchell, 2000; Leites and Robinson, 2004). While the causes of stem form differences are important to consider, equally important is having methods of identifying these differences among populations. With such methods, managers could identify the effects of silvicultural treatments on characteristics like the allocation of biomass in stems for wood or carbon volume estimation. These methods could also highlight when a localized stem taper model would be necessary for a particular population.

Stem taper models quantify stem form by estimating the diameter of a tree (inside or outside bark) at a given height along the stem (Burkhart et al., 2019). Using these models, it is possible to determine the diameter of the tree at any point along the stem or to determine the height at which a certain diameter occurs. With this, users can calculate the volume of a log of any length and at any stem position for a specific tree by integrating the taper model with respect to height. Additionally, these models can be used to classify stem forms across a range of growing conditions. These classifications can describe how close a tree is to some ideal shape or to identify conditions that affect growth patterns (MacFarlane and Weiskittel, 2016).

There have been a variety of approaches to taper model development. The simplest form of taper model uses a linear regression approach. These can range from the Gray (1956) model of $d^2 = a + bH$ (d = diameter at height H) to quadratic models like the Kozak et al. (1969) form or higher-order polynomial models as seen in Gordon (1983). Linear models have the advantage of being easy to fit, continuous, and algebraically integrable and invertible. On the other hand, they often require constraints to behave logically, might be overfitted, and often do not extrapolate well (McTague and Weiskittel, 2021). Ormerod (1973) developed an alternative approach by applying a sigmoid curve to estimate tree taper. This type of model is flexible and more biologically interpretable than a variable exponent model. It is also algebraically invertible and integrable. It can, however, sometimes be noncontinuous or inconsistent in its behavior (McTague and Weiskittel, 2021).

An alternative method of taper modeling is to divide the stem into sections and define a regression function for each section to estimate the changes in the stem. A widely used example is the Max and Burkhart (1976) segmented polynomial model which divides the tree into three sections: a neiloid bottom, a parabolic middle, and a conic top. These models are biologically consistent in their behavior and algebraically invertible and integrable; however, they are sometimes noncontinuous and often rely on nonlinear estimation to determine join points (McTague and Weiskittel, 2021). Variable exponent taper functions, such as the Bi (2000) and Kozak (2004) models, have been used to model complicated stem forms. These use a changing exponent to describe the stem shape continuously as the diameter changes from ground to tip. These types of models tend to have less bias and more precision in their taper predictions compared to segmented models (Kozak, 2004; Rojo et al., 2005; Li and Weiskittel, 2010). They are highly flexible in performance across various species, but due to their nonlinear nature, parameter estimation can be a challenge. Additionally, they require numerical integration and iteration for volume and height prediction (McTague and Weiskittel, 2021). Previous studies have shown that the Kozak (2004) equation performs very well in comparison to other models for red spruce (Li and Weiskittel, 2010) and other conifer species (Corral-Rivas et al., 2007; Sakici et al., 2008; Schröder et al., 2015; Tang et al., 2017; Hussain et al., 2021). The Max and Burkhart (1976) model has generally performed well but is worse than the Kozak (2004) model in

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similar studies (Sakici et al., 2008; Schröder et al., 2015; Tang et al., 2017; Hussain et al., 2021). The Kozak et al. (1969) and Ormerod (1973) models are much simpler while still delivering good results (Martin, 1981; Parresol et al., 1987; Sakici et al., 2008; Tang et al., 2017).

Multiple approaches have been used to compare stem forms of tree populations. Amateis and Burkhart (1987) compared populations of loblolly pine using simple volume and taper models. F-tests were performed on the slope coefficients of simple taper models to show differences in the populations. Other studies have expanded on this methodology with additional model forms. McTague and Weiskittel (2021) recommended using the Kozak (2004) variable exponent models and adding variables as necessary to the exponent function to test the inclusion of additional predictors. A likelihood ratio or an F-test between the original and expanded models could then determine if the added variable explained significantly more of the variance than the original model. This was demonstrated in Li et al. (2012) by the inclusion of a variable for planted and naturally regenerated stands and in Li and Weiskittel (2010) by the inclusion of crown variables. Similarly, Sharma and Zhang (2004) used a compatible taper model to show differences in taper were caused by differences in stand density. While many fitted taper model parameters are species-specific, having useful methods to compare stem forms among populations is essential for determining when a newly fitted taper model is necessary (Poudel et al., 2020; Ozcelik et al., 2023).

Due to the costs associated with land management, managers interested in restoring or preserving red spruce in southern Appalachia will require reliable quantitative tools to predict wood volume for harvesting or carbon accounting purposes. Small changes in the way biomass or volume is calculated can have large impacts when applied across large acreages, so it is important to use up to date and accurate models (Weiskittel et al., 2015). The objective of this study was to analyze red spruce stem taper data using a variety of models and methods to test the differences in stem form between the northern and southern red spruce populations. In doing so, we provide end users with useful taper models for better managing southern Appalachian red spruce.

Materials and Methods

Data sources

Most of the tree taper data for northern and southern spruce populations used in this study came from previous datasets (Honer, 1965; Vicary et al., 1984; Nicholas, 1992; Gilmore and Seymour, 1996; Maguire et al., 1998; Meyer, 2005). The northern data consist of 1710 trees from populations in Maine, USA; New Brunswick, CA; and Ontario, CA. Samples from Maine were collected by Vicary et al. (1984), Gilmore and Seymour (1996), Maguire et al. (1998), and Meyer (2005). Samples from New Brunswick were collected by Li et al. (2012). Samples from Ontario were collected by Honer (1965). The southern data preceding this study consist of 44 trees from populations in southwest Virginia, eastern Tennessee, and western North Carolina collected by Nicholas (1992). All data were sampled from felled trees using similar techniques where diameter measurements were collected at intervals of one-tenth total height or approximately every 1–2 m. Only trees above 4.9 cm at diameter at breast height (DBH) were used for analysis.

New data were collected in summer of 2022 at a 7.28-ha pure red spruce stand on Unaka Mountain, TN, at an elevation of 1578 m. A preliminary forest inventory of 10 sample plots at the time of data collection estimated a basal area of 63.33 m^2ha^{-1} and a mean dominant height of 17.4 m. Sample trees were felled as part of a gap expansion and understory release restoration project. These trees were selected due to their location above existing regeneration and not for any particular characteristics. Outside-bark diameter and bark thickness were measured at 1-m intervals (starting at ground level) along the main bole of felled trees using calipers and a bark gauge. Additional diameter measurements were collected at stump height, breast height (1.37 m), and Girard form class height (5.27 m). Total height was also recorded for each tree. Sectional volumes for each tree were calculated with Smalian's formula ($V = L * \frac{A+a}{2}$ where V is section volume, L is section length, A is the cross-sectional area of the bottom of the section, and a is the cross-sectional area of the top of the section) using the diameters at each measured point as the upper or lower section diameters (Burkhart et al., 2019). Although Smalian's formula can slightly overestimate the tree tip compared to using a conical volume formula, we felt the difference was trivial given the section was less than 1 m in length with diameters under 4 cm. Individual sections on each tree were summed for the total volume. Forty-six trees were measured for taper from the Unaka site giving a total of 90 trees in the southern population dataset. Summary statistics for the northern and southern data are presented in Table 1.

Stem form comparison between populations

To test for sectional differences in taper between the northern and southern populations, individual trees were divided into quarters by finding the diameter measurement closest to each quarter of the total height. Quarter points (Q1 through Q4) were counted from the ground up (e.g. Q1 was the measured point closest to 25% of total tree height). Trees used in this comparison were at least 9.5-m tall so that the lowest quarter point would not be within 1 m of DBH. Trees without a measurement within 1 m of each of the quarter points were dropped from the dataset. We also removed trees with a stump diameter smaller than DBH and those with their lowest measurement point at DBH. This left 1194 trees from the northern population and 75 trees from the south. To account for the large difference in the number of measured trees between the populations, 100 subsamples of the northern data were randomly selected for hypothesis testing which equaled the size of the southern data ($n = 75$).

Sectional stem taper was calculated using a linear slope: change in diameter (in centimeters) divided by the change in height (in meters). We tested for taper differences between the populations within the stump-to-DBH section and within each quarter section above that. These sectional tapers were compared using one-sided Wilcoxon tests due to the non-normality of the data. A one-sided test was chosen for two reasons: first, the mean DBH of the southern population was larger than the mean DBH for the northern trees for a given height. It would follow, then, that the mean taper for the southern trees must be greater than the northern trees' mean taper to reach the same height. Second, a one-sided test could evaluate that hypothesis with more power than a two-sided test, somewhat making up for the Wilcoxon test having less power than a parametric test. The alternative hypothesis for these tests was that the southern trees' taper was less than the northern trees' taper. Because these values were negative, a lesser value indicated a more extreme taper. Example code used for sectional taper calculations is provided in the supplementary materials (Supplementary Figure A4).

To test for regional differences between the populations using a taper model, a region dummy variable was added to the Kozak (2004) Model 02 variable-exponent model. The full dataset of

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Table 1. Summary of the northern and southern datasets.

Source	No. of trees	Measure	Mean	Range
North	1710	DBH (cm)	15.13	(4.90, 50.00)
		Total height (m)	13.04	(3.90, 25.90)
South	90	DBH (cm)	25.97	(5.84, 41.91)
		Total height (m)	16.40	(3.63, 23.80)

Table 2. Full and reduced versions of the Kozak (2004) model with and without a region dummy variable ($R = 1$ for north region, $R = 0$ for south region) where d_i = diameter at measurement height h_i, D = diameter at breast height, and H = total height.

Model	Form
Kozak (2004) Model 02 with region variable	$d_i = b_0 D^{b_1} H^{b_2} X_i^{b_3 Z_i^4 + b_4\left[\frac{1}{e^{\frac{D}{H}}}\right] + b_5 X_i^{0.1} + b_6\left[\frac{1}{D}\right] + b_7 H^{Q_i} + b_8 X_i + b_9 R}$ where $X_i = \left[1.0 - \left(\frac{h_i}{H}\right)^{\frac{1}{3}}\right] \Big/ \left[1.0 - p^{\frac{1}{3}}\right]$ $Q_i = 1.0 - \left(\frac{h_i}{H}\right)^{\frac{1}{3}}$ $p = \frac{1.3}{H}$
Kozak (2004) Model 02 without region variable	$d_i = b_0 D^{b_1} H^{b_2} X_i^{b_3 Z_i^4 + b_4\left[\frac{1}{e^{\frac{D}{H}}}\right] + b_5 X_i^{0.1} + b_6\left[\frac{1}{D}\right] + b_7 H^{Q_i} + b_8 X_i}$ where $X_i = \left[1.0 - \left(\frac{h_i}{H}\right)^{\frac{1}{3}}\right] \Big/ \left[1.0 - p^{\frac{1}{3}}\right]$ $Q_i = 1.0 - \left(\frac{h_i}{H}\right)^{\frac{1}{3}}$ $p = \frac{1.3}{H}$

northern and southern trees greater than 4.9 cm at DBH was used for model fitting. The Kozak (2004) model was chosen due to its fitting performance and predictive ability in previous studies of red spruce taper, as well as its ability to easily incorporate new parameters for hypothesis testing (Li and Weiskittel, 2010; Li et al., 2012). The full and reduced model forms are presented in Table 2. Because tree taper data violate the assumptions that residuals are independent and homoscedastic, individual tree random effects were included in the models for the b_0 and b_5 terms as demonstrated in Li et al. (2012). The models were fit using maximum likelihood estimation, and a likelihood ratio test was performed to compare the full and reduced forms. Fitting and testing were performed in R using the Stats package and the NLME package (Pinheiro et al., 2021; R Core Team, 2021).

Southern data model fitting

Four existing taper models were fit and tested for their ability to predict upper stem diameters and total volumes of the southern red spruce samples: a quadratic model (Kozak et al., 1969), a segmented polynomial model (Max and Burkhart, 1976), a geometric model (Ormerod, 1973), and a variable-exponent model (Kozak, 2004). We selected these models because they cover a variety of model types and complexities. The models were fit in their original published forms (Table 3) using only fixed effects, unlike in the regional comparison that included individual tree random effects. The estimated total volume was calculated by dividing each tree into 100 sections and integrating the fit taper functions numerically from the ground level to the tree tip. This estimated volume was then compared to the Smalian's formula–derived total volume. The Nicholas (1992) data were used for model fitting while the Unaka data were treated as a validation dataset. After validation, the southern datasets were combined to produce the final model parameter estimates. One tree from the Nicholas (1992) dataset was removed due to an assumed measurement error. The Kozak et al. (1969) quadratic form was fit using ordinary least squares with the Stats package in R, and nonlinear models (all other forms) were fit using the damped least squares (Levenberg–Marquardt) method with the minpack.lm package in R (R Core Team, 2021; Elzhov et al., 2022). The example code for fitting each model is provided in the supplementary materials (Supplementary Figures A1-A3).

Model accuracy was assessed using the coefficient of determination (R^2), mean absolute bias (MAB), and root mean squared error (RMSE) statistics for diameter and total volume (Table 4). In addition to comparisons among the models using these statistics, an overall accuracy index provides a useful mode of comparison. A similar method to the one demonstrated in Goodwin (2009) termed the Mean Summary Statistic 6 (MSS6) takes the MAB, RMSE, and 1-R^2 for the diameter and volume predictions and divides them by the mean of each test statistic across all models. The MSS6 index is the average of those scaled statistics for each model. The scaling factor (test mean) was included for comparison across the models (Table 9). The average MSS6 index across all models is 1 and an MSS6 index of 0 would indicate a perfect fit. Indices below 1 are better than the average of all models, and above 1 are worse.

Results

Stem form comparison

Table 5 shows the ranges of results after subsampling the northern tree data 100 times and comparing those subsamples to the southern data. When comparing DBH/Height, the southern trees were significantly wider at BH ($P < .05$) than the northern ones. At all sections of the tree from BH to the tip, the southern trees had a significantly greater taper ($P < .05$) than the northern trees. The trees in both populations were least tapered in the second quarter of the stem and most tapered in the stump section and

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Table 3. Model forms to be tested in this study, where d_i = diameter at measurement height h_i, D = diameter at breast height, and H = total height.

Model	Form
Kozak et al. (1969)	$\frac{d_i^2}{D^2} = b_0\left(\frac{h_i}{H}-1\right)+b_1\left(\frac{h_i^2}{H^2}-1\right)$
Kozak (2004) Model 02	$d_i = b_0 D^{b_1} H^{b_2} X_i^{b_3\left[\frac{h_i}{H}\right]^4+b_4\left[\frac{1}{e^{\frac{1}{D}}}\right]+b_5 X_i^{0.1}+b_6\left[\frac{1}{D}\right]+b_7 H^{Q_i}+b_8 X_i}$ $where\ X_i = \left[1.0-\left(\frac{h_i}{H}\right)^{\frac{1}{3}}\right]/\left[1.0-p^{\frac{1}{3}}\right]$ $Q_i = 1.0-\left(\frac{h_i}{H}\right)^{\frac{1}{3}}$ $p = \frac{1.3}{H}$
Max and Burkhart (1976) Model 4	$\frac{d_i^2}{D^2} = b_0\left(\frac{h_i}{H}-1\right)+b_1\left(\frac{h_i^2}{H^2}-1\right)+$ $b_2\left(b_4-\frac{h_i}{H}\right)^2 I_1+b_3\left(b_5-\frac{h_i}{H}\right)^2 I_2$ $where\ I_1 = 1\ if\ \frac{h_i}{H} \le b_4, = 0\ otherwise$ $I_2 = 1\ if\ \frac{h_i}{H} \le b_5, = 0\ otherwise$
Ormerod (1973)	$d_i = D\left[\frac{H-h_i}{H-k}\right]^{b_0}$ $where\ k = height\ at\ D$

Table 4. Equations for the R^2, MAB, and RMSE where y_i and $\hat{y}_i$ are the actual and predicted values for the ith observation out of n total observations and $\bar{y}$ is the mean.

Statistic	Formula
R^2	$R^2 = 1-\frac{RSS}{TSS}$ where $RSS = \sum_{i=1}^{n}(y_i-\hat{y}_i)^2$ and $TSS = \sum_{i=1}^{n}(y_i-\bar{y})^2$
MAB	$MAB = \frac{1}{n}\sum_{i=1}^{n}\lvert y_i-\hat{y}_i\rvert$
RMSE	$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n}}$

Table 5. Ranges of means (cm/m), W-statistics, and P-values after 100 subsamples of northern trees' sectional tapers were compared to those of southern trees. The number of significant results ($P < .05$) out of the total is presented. Q1 refers to the measurement nearest to the lowest quarter of total height, Q2 refers to the second quarter point, and so on.

	South mean (cm/m)	North mean range (cm/m)	W-stat range	P-value range	No. significant
DBH/Height	1.5994	1.0717	504	<0.0001	100/100
		1.2157	1151	<0.0001	
Stump to DBH	−4.3111	−2.6182	3962	<0.0001	100/100
		−1.6077	4547	<0.0001	
DBH to Q1 taper	−1.3793	−1.2357	3262	<0.0001	100/100
		−0.9646	4005	0.0457	
Q1 to Q2 taper	−1.0559	−0.7660	4117.5	<0.0001	100/100
		−0.6500	4617	<0.0001	
Q2 to Q3 taper	−1.8019	−1.3284	4071	<0.0001	100/100
		−1.1032	4697	<0.0001	
Q3 to Q4 taper	−2.6817	−2.1258	4427	<0.0001	100/100
		−1.8647	5012	<0.0001	

top quarter. According to these tests, the southern trees tended to be wider at BH and tapered more sharply throughout the length of the stem than the northern trees (Fig. 3).

Region variable comparison

The full and reduced versions of the Kozak (2004) model produced nearly identical results in terms of fit statistics, but using a likelihood ratio test on the Kozak (2004) models showed that including the region variable resulted in a significant increase ($P < .05$) in variance explained by the model (Table 6). The parameter estimates for the full and reduced form are shown in Table 7.

Model fit analysis

Fit statistics for diameter and volume estimation were calculated based on the fitted models' performance in the fitting (Table 8) and validation (Table 9) data sets. In the fitting dataset, the Kozak (2004) model had the best performance in all tests for both diameter and total volume estimation. The Ormerod (1973) model had the second-best results for all total volume estimation categories and the fourth-best diameter R^2. The MAB and RMSE for the total volume predictions by the Ormerod (1973) model were both within 0.004 m^3 of the Kozak (2004) results. The MAB for the diameter predictions were third lowest behind the Max and Burkhart (1976)

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Table 6. Likelihood ratio and statistical fit test results comparing the full and reduced versions of the Kozak (2004) model. Headers indicate degrees of freedom (df), coefficient of determination (R^2), mean absolute bias (MAB), root mean squared error (RMSE), log likelihood (logLik), likelihood ratio (L. Ratio), and *P*-value. The variance (s^2) of the random effect parameters (b_1 and b_6) are also presented.

Model	df	R^2	MAB	RMSE	logLik	L.Ratio	*P*-value	b_1 s^2	b_6 s^2	Residual s^2
Full	5	0.9909	0.4334	0.6988	−23253.81			0.0005	0.0050	0.5540
Reduced	4	0.9909	0.4339	0.6996	−23255.92	4.2136	0.0401	0.0005	0.0049	0.5550

Table 7. Parameter estimates and standard errors of the full and reduced Kozak (2004) models.

Model	Full		Reduced	
Parameter	Estimate	SE	Estimate	SE
b_0	0.7362	0.0073	0.7361	0.0073
b_1	1.0025	0.0038	1.0026	0.0038
b_2	0.1040	0.0063	0.1040	0.0063
b_3	0.6863	0.0066	0.6833	0.0064
b_4	−0.3067	0.0297	−0.3203	0.0287
b_5	0.5623	0.0120	0.5492	0.0103
b_6	−0.4915	0.0846	−0.4825	0.0841
b_7	−0.0093	0.0005	−0.0092	0.0005
b_8	0.0056	0.0055	0.0077	0.0055
b_9	−0.0151	0.0072		

Table 8. R^2 subtracted from 1, MAB, and RMSE in centimeters for diameter and cubic meters for volume. Results are based on the fitting dataset. The best values are in bold.

Model	Kozak et al. (1969)	Kozak (2004)	Max and Burkhart (1976)	Ormerod (1973)
Diameter: 1—R^2	0.0483	**0.0365**	0.0406	0.0537
Diameter: MAB	1.5100	**1.2422**	1.3505	1.5047
Diameter: RMSE	2.0482	**1.7809**	1.8784	2.1592
Volume: 1—R^2	0.0282	**0.0105**	0.0222	0.0122
Volume: MAB	0.0298	**0.0214**	0.0268	0.0248
Volume: RMSE	0.0562	**0.0342**	0.0498	0.0369

model, while the RMSE was fourth lowest. The segmented polynomial model had the second-best results for diameter prediction and third best for volume prediction. The Kozak et al. (1969) model was third best in diameter R^2 and RMSE but fourth in the remaining categories. The range of MAB and RMSE values for volume estimates were all within 0.01 and 0.02 m^3, respectively. The range of the MAB and RMSE values for diameter estimation was ∼0.3 cm each.

All the models performed worse to varying degrees when applied to the validation dataset (Table 9). The Kozak (2004) model once again had the best results for predicting diameter but was only third best in all categories for predicting total volume. The Max and Burkhart (1976) model had the second best results for diameter prediction and total volume MAB and the best results for total volume R^2 and RMSE. The quadratic model had the fourth-best taper prediction results but the second-best total volume R^2 and RMSE results. It had the best MAB results for volume prediction. The Ormerod (1973) model had the largest drop in performance between the fitting data and the validation data for total volume estimation. It had the third-best taper prediction statistics and the fourth-best total volume predictions. The MAB and RMSE for diameter differed by ∼0.3 cm among the models compared. The range of MAB and RMSE values for total volume estimates was just over 0.02 m^3.

Scaling the validation data results by dividing by the test means to produce the MSS6 statistics left the same rankings for each statistic (Table 10). The segmented polynomial model had the best MSS6 rank of 0.8758. All its fit statistics were better than the mean (less than 1). While its diameter results were second best overall, this was apparently compensated for by its performance in total volume prediction. The Kozak (2004) model had the best diameter prediction results and third-best total volume prediction results that led it to have the second-best MSS6 rank (0.8813) which was very close to the top score. All of its statistics were also better than the mean. The quadratic model had the third-best MSS6 score. Its diameter estimation statistics were fourth best and above the mean, but its total volume prediction statistics were first and second overall and much better than the mean. The Ormerod (1973) model was very close to the mean in all categories and placed fourth in the MSS6 score.

Analysis of the residuals showed that most models had similar variances (Fig. 4). The models appeared unbiased in their predictions. The only model that had an outlying point was the Kozak (2004) model (Fig. 4d–f). Variances did appear to follow a distribution corresponding with the height along the bole. In all models, the variance was the widest around the base. Residual variance narrowed in the 10%–25% of relative height range and then slowly increased as height increased. This appeared to show that the

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Table 9. R^2 subtracted from 1, MAB, and RMSE in centimeters for diameter and cubic meters for volume. Test mean is the average of each statistic across all models. Results are based on the validation dataset. The best values are in bold.

Model	Kozak et al. (1969)	Kozak (2004)	Max and Burkhart (1976)	Ormerod (1973)	Test mean
Diameter: 1—R^2	0.0777	**0.0514**	0.0543	0.0755	0.0647
Diameter: MAB	1.8920	**1.4125**	1.4491	1.7423	1.6240
Diameter: RMSE	2.4732	**2.0123**	2.0677	2.4388	2.2480
Volume: 1—R^2	0.0677	0.0685	**0.0653**	0.1144	0.0790
Volume: MAB	**0.0423**	0.0461	0.0430	0.0684	0.0500
Volume: RMSE	0.0673	0.0677	**0.0661**	0.0875	0.0722

Table 10. Fit statistics results for each model after dividing by the test mean as a scaling factor. Statistics are from the validation results in Table 9. The best score is in bold.

Model	Kozak et al. (1969)	Kozak (2004)	Max and Burkhart (1976)	Ormerod (1973)
Diameter: 1—R^2	1.2005	**0.7941**	0.8389	1.1665
Diameter: MAB	1.1650	**0.8698**	0.8923	1.0729
Diameter: RMSE	1.1002	**0.8952**	0.9198	1.0849
Volume: 1—R^2	0.8572	0.8674	**0.8268**	1.4486
Volume: MAB	**0.8468**	0.9229	0.8609	1.3694
Volume: RMSE	0.9328	0.9383	**0.9161**	1.2128
MSS6	1.0171	0.8813	**0.8758**	1.2258
Rank	3	2	**1**	4

variances, while equally distributed around 0, were not constant. This heteroskedasticity violates the assumptions of least squares estimation used for fitting the models. Each of the residual plots showed right skewing to varying degrees. The Kozak et al. (1969) and Ormerod (1973) models showed only slight skewing and appeared to be mostly normally distributed (Fig. 4c and l). The Kozak (2004) and Max and Burkhart (1976) models were more heavily tailed than the others to the point of violating normality (Fig. 4f and i).

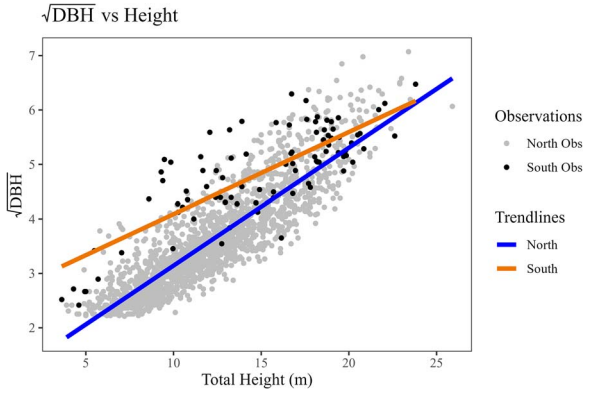


Figure 2. The square root of DBH predicted by total height of the northern and southern tree samples.

Discussion

Stem form comparison

Based on the results of these tests, it appears that the spruce trees in the southern Appalachians are tapered differently than those from northern populations. In general, the sampled southern trees were taller and wider at DBH than the sampled northern trees. However, larger stems should not be conflated with differences in stem form, since they may be proportional to each other. For that reason, we tested for sectional differences in stem form as well as statistical differences in the stem taper model. The results showed that there were statistically significant differences in stem form between the populations.

To account for population level differences, we compared the ratio of DBH to total height so that small trees would not be compared to large trees (Fig. 2 and Table 5). This comparison showed that a hypothetical tree of 15 m in height would be ~7.5-cm wider at BH in the south versus in the north on average (a difference of 0.5 cm/m). It would follow then that southern trees would have more taper along the stem to start from a wider point and reach the same top height. This was confirmed to be the case through each section of the stem (Table 5). Above DBH, the differences in taper appeared to increase with the relative height. The first two quarters had a difference of 0.2–0.4 cm/m, while the fourth quarter had a difference of 0.6–0.8 cm/m. The difference was most pronounced below BH, with the southern stump sections averaging 1.7–2.7 cm/m more taper than the northern stump sections. This changing difference showed that the trees were tapered differently throughout the stem and were not just proportionally different.

The other test used nested models to examine if adding a parameter for region improved the model fit. If so, that would indicate that the trees needed to be modeled differently depending on the region. A likelihood ratio test using the Kozak (2004) model showed a significant difference in explained variance when a region variable was added (Table 6). This model is well suited for hypothesis testing regarding stem form due to its exponent function that allows additional fitting parameters to easily be added on to the function (McTague and Weiskittel, 2021). This was demonstrated in Li et al. (2012) to show differences in stem form between planted and natural stands of red spruce and in Li and Weiskittel (2010) to test the addition of crown variables. A caveat of our result is that other model forms may produce different

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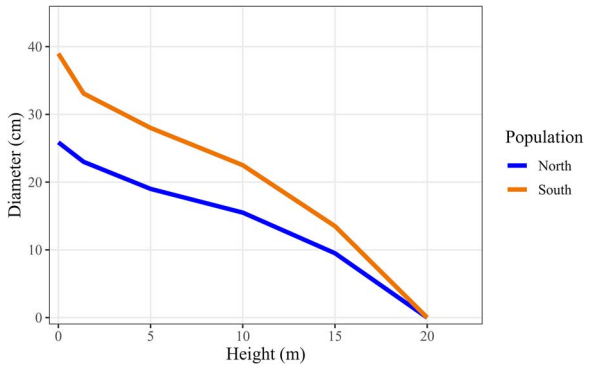


Figure 3. Graphical representation of the sectional tapers (see Table 5) of the northern and southern populations using a hypothetical 20-m-tall tree. The North example has a DBH of 23.0 cm, and the South example has a DBH of 33.1 cm. The medians of the mean sectional tapers were used for the northern spruce data.

levels of significance depending on their flexibility. A follow-up study could test more model forms to see if there is a consensus among multiple models.

Both tests showed evidence of differences in stem form between the southern and northern spruce populations. The sectional taper tests indicated that the biggest differences in stem form occurred near the stump and above the midpoint of the stem, where the southern trees began to taper more quickly than the northern trees. This could indicate that the southern trees prioritize radial growth in the lower part of the stem over height growth, staying wider through the midpoint and then quickly tapering to the tip. Tree age could affect tree height–diameter ratio, but age was not collected in our samples. Due to the shade-tolerant nature of red spruce, age in a natural stand would likely be an unreliable predictor of tree size characteristics (Franceschini and Schneider, 2014). The southern trees had a more pronounced root flare as well, suggesting a need for stabilization at their base. The allocation of more biomass in the lower part of the stem could be important for users interested in merchantable volume since the lower part of the stem contains the most valuable product classes (Burkhart et al., 2019). Differences in stem form may be important for individual tree biomass estimation as well, since the stem can account for up to 70% of a tree's total biomass (Peichl and Arain, 2007).

The cause of these form differences was not examined directly in this study. Genetic variation could be one reason, as these populations have been separated for millennia (White and Cogbill, 1992). Another reason the trees exhibit taper differences may be related to wind firmness. The sampled southern spruce populations exist entirely above 1500 m, while the spruce-fir tree line reaches a maximum of 1480 m at 44° N and decreases with increasing latitude (Cogbill and White, 1991; Nicholas, 1992). Because the southern populations exist exclusively at high altitudes, they experience strong winds more frequently than lower-altitude northern populations (Wiser, 1998). Having a wider root flare and allocating more mass in the lower half of the stem both support the idea that these southern trees prioritized stabilization. Wind-related mechanical stress has been shown to increase root biomass and the root-mass-to-shoot-mass ratio in other conifer species (Nicoll and Ray, 1996; Stokes et al., 1997; Danjon et al., 2005). Tree selection may have contributed to differences, as these trees were selected due to their location above existing spruce saplings. It is possible that the small gaps that allowed those saplings to germinate allowed more space for growth of the canopy trees (Amateis and Burkhart, 2012). However, gaps were expanded beyond the bounds of the saplings, so a diversity of horizontal structure was accounted for in the samples. Additionally, nutrient and water availability have been studied as possible stem form influences (Snowdon and Waring, 1990; Wiklund et al., 1995; Crous et al., 2009). Zhang et al. (2002) recommended that the combination of all climatic, environmental, and vegetation factors plays an important role in determining differences is height-diameter relationships among different ecoregions. These considerations may be worth investigating in future studies of southern red spruce populations.

The proposed methods appeared to be an effective way of testing differences in stem form across tree sizes and regions. Future studies could break down the trees into smaller sections or test overlapping sections. Ideally, a more in-depth stem form study would use measurements at identical proportions along the stem. This study used approximations for the quarters of the stem due to different sampling protocols across datasets. Because of that, the quarter points were not exactly at 25%, 50%, or 75% of the total height for each tree, and this likely was a source of error in the comparisons. We could have used linear or quadratic interpolation, but that would also be a source of error, and so we decided to use the nearest measurements. Furthermore, the large sample size differences, nonrandom sample selection, and lack of normality within the sample sets necessitated using a less powerful, nonparametric comparison test. With a larger sample of the southern population, a more powerful, parametric test might be used to detect smaller differences in form class. Still, the results presented here support the case that these tree populations grow differently and should be treated as such when modeling their stem forms. Future studies should investigate some of the reasons for these differences, whether those are environmental or genetic.

Model fits

Past studies on red spruce stem taper models in other regions have shown that the Kozak (2004) Model 02 model was the best fit compared to other tested models (Li and Weiskittel, 2010). This study agreed that the model performs very well as it had a very close second overall rank. The segmented polynomial (Max and Burkhart, 1976) model performed best overall and had taper and volume statistics that were either leading or very close to the leader in those categories. The quadratic model (Kozak et al., 1969) was third best overall and, considering it is a linear model, had fit statistics that were comparable to the nonlinear forms, including the top two volume predictions from the validation data. In our opinion, these three models would be the most useful from a practical standpoint depending on the user's needs.

Kozak (2004) Model 02

The Kozak (2004) model had the best fit statistics of the four models tested for diameter predictions and was a very close third in total volume prediction for the validation dataset (Table 9). Li and Weiskittel (2010) found the Kozak (2004) model to give the best predictions for diameter and second best volume predictions of red spruce. Hussain et al. (2021) found this model to perform second best overall of the eight tested models for predicting volume and taper in Korean spruce (*P. koraiensis* Nakai) in northeast China. Our results support those findings. The MAB (1.24 cm) and RMSE (1.78 cm) for the diameter predictions of the fitting dataset (Table 8) were both ~0.5–0.75 cm higher than the values reported in two previous studies using this model for red spruce diameter predictions (Li and Weiskittel, 2010; Li et al., 2012). Our goodness-of-fit results for diameter prediction were similar to

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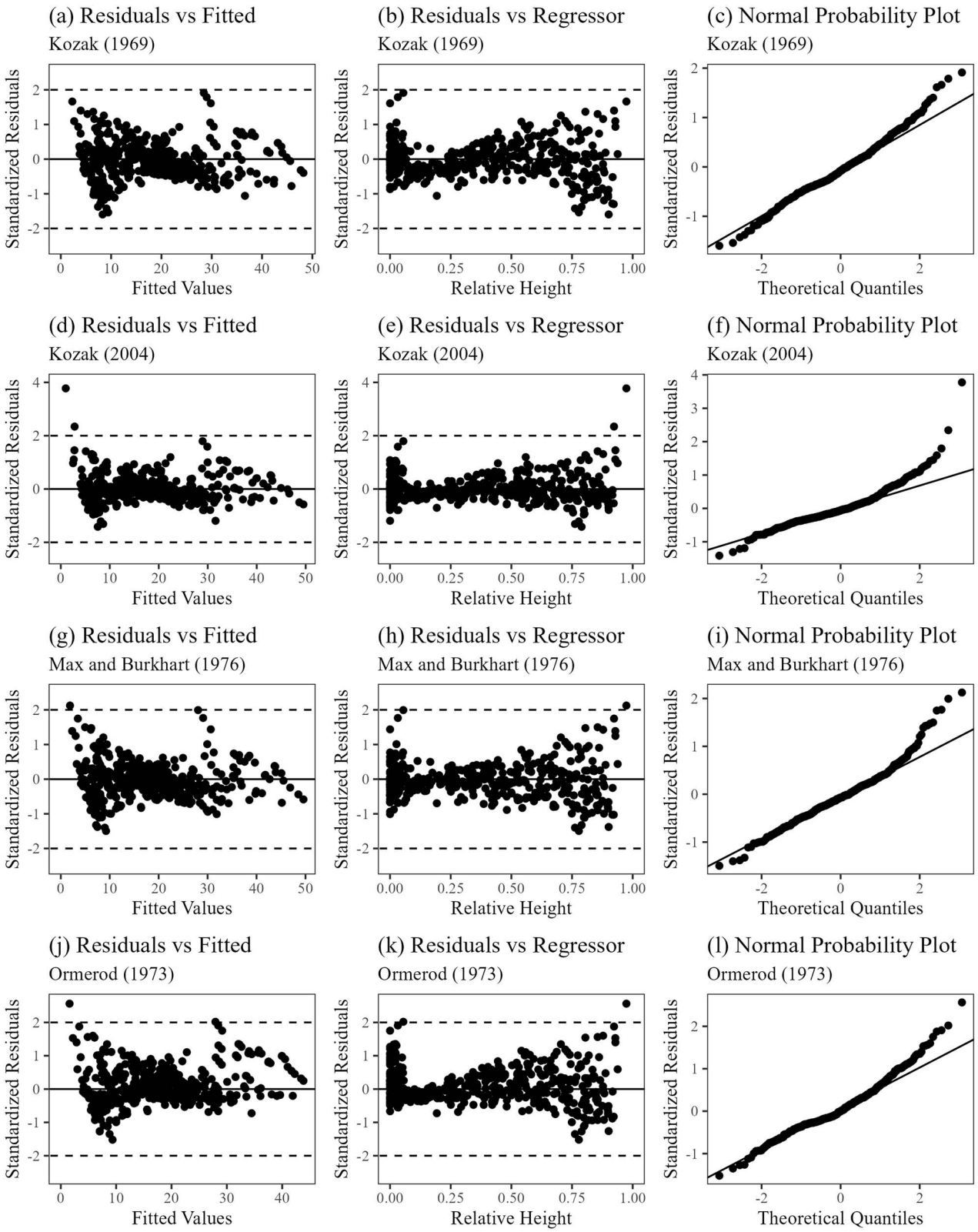


Figure 4. Residual and normal probability plots for each model.

those found using this model for other conifer species. Schröder et al. (2015) reported an RMSE of 1.80 cm for slash pine (*Pinus elliotii* Engelm.) in Brazil, Crecente-Campo et al. (2009) found RMSE values between 0.92 and 1.59 cm using 11 parameters for *P. sylvestris* (L.) in Spain, and Corral-Rivas et al. (2007) found RMSE values between 1.40 and 1.60 cm across five pine species in Durango, Mexico. A comparison of the fitting data statistics was used because not all the aforementioned studies mentioned using a validation dataset. The volume MAB (0.02 m^3) and RMSE (0.03 m^3) of the fitting data were both ~0.01–0.02 m^3 higher

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than the results reported in those studies. The reason for this discrepancy is likely that those authors used at least one of the following corrections: a mixed effects model incorporating the individual tree random effects, an autocorrelation error structure, or a weighted variance whereas we used fixed effects only. With our dataset of 43 trees for fitting (46 for validation), convergence was unachievable when including the correlation and variance terms. Furthermore, while we were able to incorporate individual tree identities as a random effect for the Kozak (2004) model, we could not do so for all the models tested for varying reasons that will be explained. For those reasons, all of the models were fit using only fixed effects to keep the model comparisons as fair as possible. With a larger dataset in the future, it may be possible to incorporate all three corrections to produce the best possible form of this model. Even without accounting for autocorrelation and multicollinearity, the estimates remain unbiased and accuracy only suffers slightly (Kozak, 1997). While this would present problems for formal hypothesis testing on the viability of specific predictors, that is not what was examined here. This model was fit using only the fixed effects, unlike in the population comparison test that incorporated random effects.

While the Kozak (2004) model had the second-best statistical performance (Table 10), it does have weaknesses. Achieving convergence for highly parameterized, nonlinear models is not guaranteed or may be time-consuming when it is applied to large datasets (Goodwin, 2009). This model cannot be analytically integrated or inverted to calculate volume or height to a specified diameter directly. Its advantage is that it is very flexible for estimating stem forms of various species and tree sizes, given that the variable exponent allows the curve shape to continuously change with the stem height. The validation results showed that the model continued to perform best at predicting the diameters of new data and was very close to the best at predicting volumes. In addition to its flexible form, a likely reason for its ability to accurately predict diameters is that it was optimized on predicting diameter directly and did not need to be transformed like the segmented and quadratic models. Its output did need to be transformed for volume estimation where it still performed almost as well as the Max and Burkhart (1976) and Kozak et al. (1969) models. Still, multiple taper model comparison studies have shown the Kozak (2004) model to be one of the best diameter predictors across various conifer species and regions (Corral-Rivas et al., 2007; Crecente-Campo et al., 2009; Li and Weiskittel, 2010; Schröder et al., 2015; Doyog et al., 2017). We would recommend this model for users who are most interested in diameter estimates. It will perform well at predicting volume but requires numerical integration techniques that may be overly complicated or computationally demanding.

Max and Burkhart (1976) Model 4

The segmented polynomial (Max and Burkhart, 1976) model had the best MSS6 score of the tested models (Table 10). It had the second-best results for diameter and first- or second-best results for volume prediction of the validation data (Table 9). The MAB and RMSE values for diameter estimates of the fitting data (Table 8) were ~0.8 and 0.9 cm higher, respectively, than those reported in Li and Weiskittel (2010). The MAB and RMSE values for total volume estimates were ~0.01 and 0.02 m^3 higher, respectively. We found similar results when compared to studies testing this model on other species: Coble and Hilpp (2006) found MAB values of 0.89 cm and 0.013 m^3 for loblolly pine (*P. taeda*) in east Texas, and Hussain et al. (2020) reported MAB values of 0.18 cm and 0.004 m^3 for Korean spruce. Brooks et al. (2008) tested the model on three conifer species in Turkey and found absolute biases for diameter prediction of 0.31, 0.41, and 0.78 cm. Their volume MAB results were 0.0006, 0.0012, and 0.0021 m^3. It should be noted that Brooks et al. (2008) and Hussain et al. (2020) simultaneously fit diameter and volume, rather than deriving volumes from the diameter-prediction form. When compared to other models, Hussain et al. (2021) found this model to predict diameters for Korean spruce in China slightly worse than Kozak (2004) but third best of the eight models tested. Li and Weiskittel (2010) found that this model was the poorest stem volume predictor of the 10 models they tested for red spruce in the northeastern USA. While past comparisons have shown a clear advantage of the Kozak (2004) model in diameter and volume prediction for other species and regions, our results found the two models to perform nearly equally.

An individual tree random effect was applied initially, but the results produced negative squared diameter estimates for some trees that could not be transformed into usable diameter estimates or used for logical volume estimates. Modeling the autocorrelation structure, as performed in Rojo et al. (2005), was also attempted but this resulted in nonconvergence. Using the fixed-effect parameter estimates only from the mixed-effect form compensated for this, but produced MAB and RMSE results that were worse than fitting the model using only fixed effects. This was supported by de-Miguel et al. (2012) who also found that using the fixed effects portion of a mixed model produced worse estimates than using the model fitted with fixed effects only. In another study, incorporating those corrections was tested for this model specifically, and only small improvements (1%) in diameter prediction were found (Williams and Reich, 1997). For those reasons, only the fixed effects were used for model fitting.

This model, like the Kozak (2004) model, must be fit nonlinearly and comes with the associated computational drawbacks. There are fewer parameters to be estimated than in the variable-exponent model (six versus nine), which should make fitting easier. This model has the advantages of being both integrable and invertible, so a user could directly calculate total or sectional volume and height to a desired diameter. The parameters are also simple to interpret, with two join points representing the relative height where the quadratic segments 'activate' (I changes from 0 to 1) and change the shape of the curve from neiloid to paraboloid to conic. While caution should always be taken when extrapolating with regression models, extrapolation is sometimes necessary depending on a user's data. This model performed better at predicting size classes outside of the fitted data than the Kozak (2004) model in a study on loblolly pine (Yang and Burkhart, 2020). It is optimized to predict squared diameter, which means volume predictions do not require a transformation of the response and the bias that incurs (Yang and Burkhart, 2020). This model has been extensively tested and has provided good results across many conifer species (Martin, 1981; Coble and Hilpp, 2006; Brooks et al., 2008; Doyog et al., 2017; Hussain et al., 2020). In general, it performed nearly equally to the Kozak (2004) model for southern Appalachian red spruce. We would first recommend this model to those who are most interested in volume prediction. We would also recommend this model to users who are interested in both taper and volume prediction but do not wish to use numerical integration or iteration.

Kozak et al. (1969)

The quadratic polynomial model by Kozak et al. (1969) was the only linear model tested and performed third best overall. This model was fit without random effects for the same reasons as

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the Max and Burkhart (1976) model. Because this model was conditioned to pass through the origin at the maximum tree height, the random intercept term for the individual tree effects slightly altered that point for some trees resulting in negative values at the tip. Attempting to model the correlation structure produced the same type of errors. When the model was tested Sitka spruce (*P. sitchensis*) in the original Kozak et al. (1969) study, the MAB of diameter estimates ranged from 0.82 cm (interior) to 2.63 cm (coastal) whereas our MAB was 1.51 cm. Our results appeared to fall in line with the Kozak et al. (1969) study which also used fixed effects only. Martin (1981) found diameter MAB values ranging from 0.82 to 1.67 cm when testing the model on various Appalachian hardwoods. His volume estimates ranged from 0.007 to 0.014 m^3. Sakici et al. (2008) reported diameter MAB and RMSE values of 2.01 and 2.74 cm, respectively, for Bornmullerian fir (*Abies nordmanniana* subsp. *bornmulleriana)* in Turkey and found that this model performed worst of the four models tested in our study. Our results continued to fall short when compared with studies that incorporate autocorrelation and mixed effect structures as in Diéguez-Aranda et al. (2006).

While this model did not provide the same level of accuracy and flexibility as a variable exponent or segmented model when predicting diameter, it performed well at predicting volume. Some of the problems stemming from its strict form can be seen in the residual plot, where the model seemed to consistently overestimate diameters in the 15%–25% height range and underestimate them from 50% to 70% (Fig. 4b). Depending on the user's needs, an RMSE of 2.47 cm for diameter could be acceptable when considering this model is linear (and thus more stable and less computationally intensive than nonlinear fits), integrable, and invertible. We would recommend this model for users interested in volume prediction exclusively or those who require a simple model form that fits into existing systems. Those interested in using it for taper may wish to refit the model after optimizing the response for diameter instead of squared diameter.

Ormerod (1973)

Finally, the Ormerod (1973) geometric model had the fourth best statistical performance of the group. The validation fit statistics for diameter estimates were slightly better than the quadratic model, but the volume statistics were worse. Compared to Kozak et al. (1969), the differences in MAB and RMSE were <0.2 cm in the case of diameter estimates. The MAB and RMSE of the volume estimates were both ~0.02 m^3 higher than the Kozak (2004) results. Martin (1981) reported diameter MAB values from 0.79 to 1.39 cm using this model for Appalachian hardwoods. Parresol et al. (1987) found diameter MAB values from 1.96 to 2.01 cm for bald cypress (*Taxodium distichum* (L.) Rich.) in Louisiana. The model ranked last for predicting diameter in both of those comparison studies. Sakici et al. (2008) used the model to estimate diameters on Bornmullerian fir in Turkey and found an MAB of 1.78 cm and an RMSE of 2.59 cm. Our validation results were better than those (Table 9), but that study also had Ormerod (1973) ranked below Kozak (2004) and Max and Burkhart (1976). They did find Ormerod (1973) performed slightly better than Kozak et al. (1969). Tang et al. (2017) found Ormerod (1973) to perform slightly worse in diameter prediction than Kozak et al. (1969) for *Betula alnoides* in south China. In addition to the fit statistics, it showed some bias in its estimates at the lower 25% of the stem, underestimating the base and then overestimating the area just above that (Fig. 4k). While the model can be integrated for volume and inverted for height prediction, we would not recommend this model for practical use. The other nonlinear models produce better results in all categories, and the quadratic model had nearly the same diameter statistics and much better volume statistics.

This analysis could be improved in a few ways. For one, Kozak and Smith (1993) suggested evaluating models for height prediction in addition to diameter and volume prediction. Other studies have also performed analyses of diameter prediction accuracy at height intervals (usually tenths) along the stem to test where models perform best (Kozak et al., 1969; Max and Burkhart, 1976; Fang et al., 2000; Kozak, 2004; Diéguez-Aranda et al., 2006; Goodwin, 2009). This leads to an analysis of how well models predict merchantable or sectional volumes (Fang et al., 2000; Brooks et al., 2008; Goodwin, 2009). Studies have also divided data into size classes to see how well models perform on small or large trees (Kozak and Smith, 1993; Diéguez-Aranda et al., 2006). These methods would provide additional useful metrics of comparison, but we felt our methods were sufficient given the small number of models being evaluated. Another model we could have included is the Clark et al. (1991) segmented model. This model has proven to be reliable in predicting diameter and stem volume in a variety of conifer species (Figueiredo-Filho et al., 1996; Sakici et al., 2008; Li and Weiskittel, 2010; Hussain et al. 2021) and is amenable to nested model testing (Li and Weiskittel, 2010). However, it requires a diameter measurement at 5.3 m. We chose to use the Kozak (2004) model so that we would not have to interpolate or predict this measurement on some trees but not others, which we felt would lead to a fairer comparison. This analysis was not meant to be an exhaustive study on all taper models but to examine examples from multiple model types using simple or well-cited examples. There are many other existing taper models that could be tested as part of a larger study (Rojo et al., 2005; Diéguez-Aranda et al., 2006; Li and Weiskittel, 2010; de-Miguel et al., 2012).

In general, our bias and error statistics were higher than those found in the literature. As stated earlier, some of this is due to the model fitting process where only fixed effects were used. When compared to studies that used fixed effects only, our goodness-of-fit results were about equal to the literature (Kozak et al., 1969; Parresol et al., 1987; Sakici et al., 2008), although this comparison is not perfect as model performance is species dependent (Schröder et al., 2015). In our opinion, the most likely source of error came from the small sample size with which to fit the models compared to past studies. Other studies have used samples ranging from hundreds (203 in Rojo et al., 2005) to thousands (53 608 across 38 species in Kozak, 2004) of trees. A larger sample size could result in improved fit statistics as well as better convergence results, but felled tree data for red spruce in the southeastern USA are difficult to obtain. With that in mind, the models studied here performed adequately and should still be useful to users interested in the information they can provide.

Conclusion

The results of this study show that there are likely differences in the stem form of red spruce between its northern and southern populations. The fact that the southern trees were wider at DBH for a given height than the northern trees suggests that these trees must have different tapers, and our tests supported this hypothesis. The strongest evidence for stem form differences occurred in the stump and the upper half of the stem, suggesting southern red spruce are prioritizing radial growth and stabilization near the base of the tree over height growth. The reasons for this were not investigated directly, but we theorized this could be an adaptation to high winds experienced in the montane environment where

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Table 11. Parameter estimates and standard errors of the Kozak et al. (1969), Kozak (2004) Model 02, and Max and Burkhart (1976) models after fitting with the full southern population data.

Model	Kozak et al. (1969)		Kozak (2004)		Max and Burkhart (1976)	
Parameter	Estimate	SE	Estimate	SE	Estimate	SE
b_0	−1.9170	0.0396	0.8762	0.0374	−5.3259	1.4380
b_1	0.6777	0.0328	0.9981	0.0148	2.6193	0.8071
b_2			0.0493	0.0161	−3.1566	0.7632
b_3			0.7267	0.0380	35.6625	4.1672
b_4			−0.3799	0.0806	0.7484	0.0460
b_5			0.6215	0.0358	0.1240	0.0081
b_6			−1.2537	0.4688		
b_7			−0.0104	0.0023		
b_8			−0.0962	0.0218		

southern Appalachian spruce exist. These results support the notion that it is better to use localized taper models than general ones (McTague and Weiskittel, 2021). We present parameter estimates for two localized taper models that would work well: the Kozak (2004) Model 02 variable exponent model and the Max and Burkhart (1976) segmented model (Table 11). These parameters were estimated using the full southern population dataset after we found both models validated well. Both models must be fit using nonlinear approaches, but both are accurate and have been shown to effectively characterize stem taper in previous studies on red spruce (Li and Weiskittel, 2010; Li et al., 2012). For a simple, linear model that also performs well at predicting volume, we present the parameters for the Kozak et al. (1969) quadratic model as well (Table 11).

Supplementary data

Supplementary data are available at *Forestry* online.

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Author contributions

Steven Morrone (Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing—original draft) and Patrick Green (Conceptualization, Funding acquisition, Methodology, Project administration, Resources, Supervision, Writing—review & editing)
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Data availability

The data underlying this article will be shared on reasonable request to the corresponding author.

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