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Selection Method for Height-Diameter Curves and Its Application to the Data of Sugi (*Cryptomeria japonica*) in Japan

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Abstract: This paper focuses on the selection of height-diameter curve (HDC) which characterizes the relationship between tree height and diameter at breast height (DBH) for growth prediction. Tree height and DBH are commonly used variables in monitoring forest growth and predicting its stock. To select the appropriate HDC among multiple candidates, empirical rules or mathematical approaches based on the residual sum of squares have been applied in previous research. In this paper we apply cross-validation (CV) criterion to select an appropriate HDC for the purpose of forecasting. The CV criterion is easy to use because it is based on simple iterative process without any assumptions required on the candidate models for HDC. Not only is CV easy to use, it also evaluates forecast accuracy, which is consistent with the objectives for HDC use. In this paper, we demonstrate the results for analyzing real life data of sugi (*Cryptomeria japonica*) stands in Japan by preparing five candidates of HDC. We also show the results of numerical experiments for verifying the ability of the method introduced in this paper.

Keywords: cross-validation (CV) criterion, height-diameter curve (HDC), model selection

1. Introduction

This paper focuses on the selection of height-diameter curve (HDC) by using the cross-validation (CV) criterion. Height-diameter curve characterizes the relationship between tree height and diameter at breast height (DBH) for growth prediction. Tree height and DBH are commonly used variables in monitoring forest growth and predicting forest stock volume. In DBH measurements, since the horizontally cut disk of a tree is not perfectly circular (because of differences in growth patterns on the mountain side and valley side or competition factor among neighbor trees), the average of the longest and shortest diameter is generally adopted (Laar and Akça 2007). It is well known that stand density has a strong influence on DBH growth along with soil and environmental factors (Ando 1968; Takahashi and Furuya 2014), while tree height is least affected by stand density. The height of upper dominant trees is strongly correlated with volume growth (Inoue 1974; Davis et al. 2001). Therefore, height of specific age and condition is commonly used as site index (Davis et al. 2001). In field measurements of these two variables, DBH can be measured directly, thus providing rather accurate and stable observations. On the other hand, tree height is measured indirectly using measuring instruments like clinometer with trigonometric functions, which could cause large error in tree height with small error in angle. Since height and DBH are from the same tree, they are associated with each other through many factors. With this thought, the idea of predicting tree height from DBH has been developed by constructing a mathematical model with DBH as the independent variable and height as the dependent variable.

The relationship between height and DBH is nonlinear rather than linear, which is described by HDC (Pretzsch 2009). From a practical viewpoint, a relatively simple nonlinear function is preferred. In practice, an appropriate HDC is determined by paired data of [DBH, height] from sampled trees, and the determined HDC is then applied to the entire DBH-measured trees to predict the average height. Such approach can be used to develop volume estimation from data that omits some height measurements by using a two-variable volume equation (Burkhart et al. 2018).

In this paper, we focus on the selection procedure for suitable or optimal HDC among candidates. Selecting an optimal HDC is an important issue in the initial phase of HDC analysis, because an inappropriate HDC setting tends to mislead the result in the subsequent detailed HDC analysis. In selecting the optimal HDC, previous studies have used goodness of fit to select the best HDC among

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the empirical candidate of HDCs (Nagumo and Minowa 1990; Pretzsch 2009). Another research, Matsumura (2003), attempted to select HDC based on the residual information such as the residual sum of squares (RSS).

The information criterion (IC) is widely used for selecting the appropriate model in regression analysis (Konishi and Kitagawa 2008). The main use for IC is to choose the optimal combination of independent variables from multiple candidates of independent variables. With regards to applying IC to select HDC, we are not selecting a variable, but rather a function. From the perspective of selecting a function in the forestry field, Kamo and Yoshimoto (2013) have developed the method to select an optimal growth function among candidates. The problem with applying the IC to function selection is due to difficulty of explicitly constructing a penalty element (e.g. the number of variables in Akaike's IC (AIC) (Akaike 1973)) that reflects the complexity of the model. Regarding this issue, Harvich and Tsai (1989) proposed the corrected version of AIC (AICc) which can be applied to nonlinear models. However, there are two difficulties to apply AICc to HDC, one is the several mathematical assumptions that must be met, and another is that AICc is not mainly contributing to forecasting accuracy. Thus, we apply the cross-validation (CV) criterion (Stone 1974), which minimizes forecasting error without the requirement of any assumption. In the CV criterion, instead of defining the penalty term for model complexity explicitly, model selection is preformed based on jackknife estimation. Furthermore, since CV criterion evaluates forecast accuracy, it is suitable to the objective for HDC application.

Regarding the selection of HDC, besides the effect that selecting the appropriate function in the initial phase of HDC analysis leads to correct results or interpretation, a clustering ability of classification by HDC is expected when different HDC patterns are mixed. For the clustering of functions, previous research papers exist on growth functions, for example, Yanagihara et al (2008) proposed clustering based on the estimated parameters of the growth function for each sample. Elsewhere, Kamo (2019) proposed a statistical process that combines growth function selection and parameter clustering by k-means (MacQueen 1967). By regressing these techniques on the growth function to HDC, we show the necessity for function selection by CV criterion as a 1st step in HDC analysis. As for real life data analysis, the statistical procedure introduced in this paper is applied to data of sugi (*Cryptomeria japonica*) in Japan (Yoshimoto et al. 2012). Furthermore, results from numerical experiments will be shown to verify the validity of our method.

2. Selection of Height-Diameter Curve

Let y_i represent tree height and x_i represent DBH of sample i $(i = 1, 2, ..., n)$. The nonlinear regression model with y_i as the dependent variable, x_i as the independent variable, and a HDC $f_k(\cdot)$ $(k = 1, 2, .., K)$ is described as:

$$E[y_i] = f_k(x_i). \quad [1]$$

The CV criterion is used to select the best HDC, among K candidate of HDCs. Let $\hat{y}_{i[-i],k}$ be the estimator of y_i which is estimated based on the function $f_k(\cdot)$ by using the sub-dataset excluding the i-th sample from the total data (so called jackknife estimate). Then the CV criterion of k-th function is defined as:

$$\mathrm{CV}_k = \sum_{i=1}^{n} \left(y_i - \hat{y}_{i[-i],k}\right)^2. \quad [2]$$

For equation [2], $f_{\hat{k}}(\cdot)$ satisfying $\hat{k} = \arg\min_k \mathrm{CV}_k$ is regarded as the best HDC. In this paper, the following five HDCs were adopted as candidates:

[3] Inverse function (e.g. Wykoff et al 1982) : $f_1(x) = \alpha + \dfrac{x}{\beta + \gamma x}$,

[4] Henricksen function (Henricksen 1950) : $f_2(x) = \beta + \gamma \log x$,

[5] Näslund function (Näslund 1937) : $f_3(x) = \alpha + \dfrac{x^2}{(\beta + \gamma x)^2}$,

[6] Stoffels function (Stoffels and Soest 1953) : $f_4(x) = \alpha + \beta x^\gamma$,

[7] Trorey function (Trorey 1932) : $f_5(x) = \alpha + \beta x - \gamma x^2$.

The parameters α, β, and γ in each function determine the properties of the function. The parameter α in equations [3, 5, 6, 7] is fixed at $\alpha = 1.2$ which corresponds to breast height above ground level in Japan (Laar and Akça 2007). The functions [3, 4, 5, 6] are monotonic. The functions [3, 5] have upper bounds as $x \to +\infty$ ($\alpha + \frac{1}{\gamma}$ in [3] and $\alpha + \frac{1}{\gamma^2}$ in [5]). Table 1 summarizes the characteristics of candidate functions [3, 4, 5, 6, 7], and Figure 1 shows the example of HDCs for some parameters.

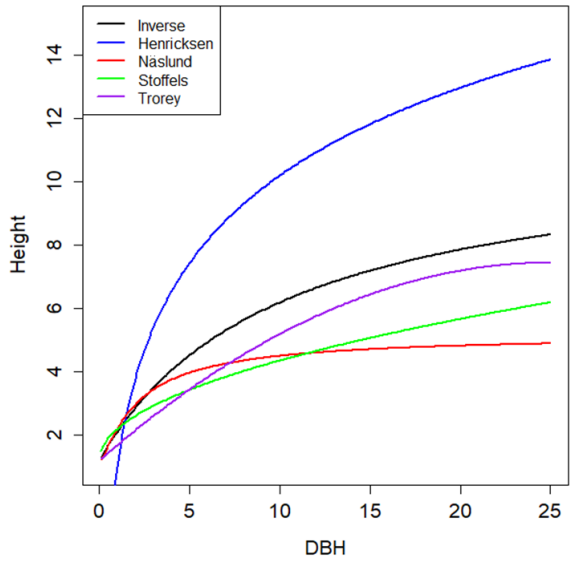


Figure 1. Five types of HDC.

The behavior of five different HDCs:
Inverse: $y = 1.2 + \frac{x}{1+0.1x}$, Henricksen: $y = 1 + 4\log x$, Näslund: $y = 1.2 + \left(\frac{x}{0.5+0.5x}\right)^2$, Sroffels: $y = 1.2 + x^{0.5}$
Trorey: $y = 1.2 + 0.5x - 0.01x^2$

Table 1. Five candidates of HDC.

Name	Function form	Differentiation		Character	
		1st	2nd	Monotone	Upper bound
Inverse	$\alpha + \frac{x}{\beta+\gamma x}$	$\frac{\beta}{(\beta+\gamma x)^2}$	$\frac{-2\beta\gamma}{(\beta+\gamma x)^3}$	○	○
Henricksen	$\beta + \gamma \log x$	$\frac{\gamma}{x}$	$-\frac{\gamma}{x^2}$	○	
Näslund	$\alpha + \left(\frac{x}{\beta+\gamma x}\right)^2$	$\frac{2\beta x}{(\beta+\gamma x)^3}$	$\frac{\beta-3\gamma x}{(\beta+\gamma x)^4}$	○	○
Stoffels	$\alpha + \beta x^\gamma$	$\beta\gamma x^{\gamma-1}$	$\beta\gamma(\gamma-1)x^{\gamma-2}$	○	
Trorey	$\alpha + \beta x - \gamma x^2$	$\beta - 2\gamma x$	-2γ		

The five candidates of HDCs and their characteristics. Variable x is the independent variable (DBH) and α, β and γ are parameters. Since we fix $\alpha = 1.2$, the unknown parameters are β and γ only.

3. Data Analysis

3.1. Real life data

In this section, we demonstrate the results of applying the method of HDC selection, which is described in the previous chapter. The data represents real life sample of 30 sugi stands collected in the Kyushu region of Japan (see Yoshimoto et al. 2012). The target of HDC analysis is mature trees that have attained a certain age of maturity and those for which height measurement is difficult. We then used observations which were 10 years old or older. The sample with the lowest DBH and height at age 10 were both in the same sample (ID=127 in Table 3), with a DBH of 5.58 and a height of 5.30.

A scatterplot of height against DBH for all samples, and the five estimated HDCs are shown in Figure 2 (the estimated parameters, the values of CV criterion, and RSS are shown in Table 2). The best HDC determined by the CV criterion is Näslund, and it is shown in red.

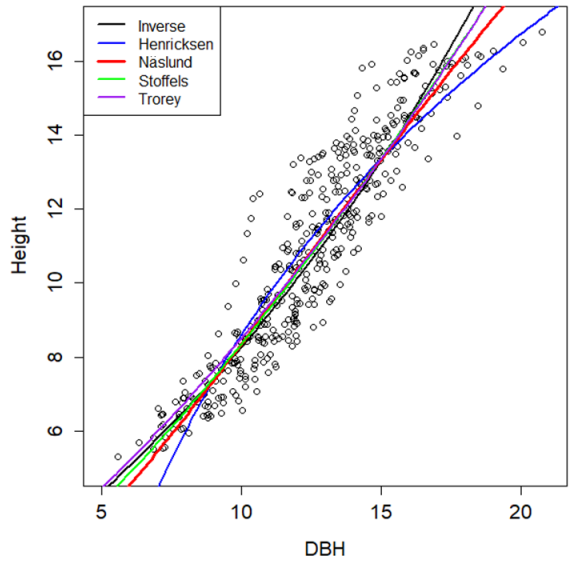


Figure 2. Observed plot and estimated HDCs.

The parameters estimated for each HDC are shown in Table 2. The Näslund, which is shown in red, is selected as the best HDC by the CV criterion.

Table 3 shows the results of estimating the optimal HDC for each sample based on the RSS and the CV criterion [2]. Among the 30 samples, four samples had different selection between RSS and CV criterion. The 30 samples were assigned to each of the five candidates of which 21 were Inverse, 1 was Henricksen, 2 were Näslund, 4 were Stoffels, and 2 were Trorey by CV criterion. The DBH-height overplots for each sample classified to each function are shown in Figure 3, and the classification results are shown in Table 3.

Table 2. Estimated parameters, RSS and CV criterion for five functions.

Function	Parameters		RSS	CV
	β	γ		
Inverse	1.76767	-0.03532	559.888	566.569
Henricksen	-18.44059	11.74597	561.782	567.524
Näslund	2.61020	0.11293	495.196	499.240
Stoffels	0.34864	1.31208	510.816	515.119
Trorey	0.57258	-0.01586	518.222	524.495

Estimated parameters, RSS and CV criterion estimated from 30 samples.

Table 3. Classification of samples.

Tree ID	HDC		Growth cluster	
	RSS	CV	Function	Cluster
7	Trorey	Stoffels	Korf	1
12	Inverse	Inverse	Korf	1
15	Inverse	Inverse	Korf	1
31	Inverse	Inverse	Levacovic-1	3
37	Inverse	Inverse	Korf	1
52	Inverse	Inverse	Korf	1
59	Stoffels	Stoffels	Korf	2
62	Trorey	Trorey	Korf	2
72	Trorey	Inverse	Korf	2
73	Henricksen	Henricksen	Korf	2
76	Trorey	Trorey	Korf	2
77	Stoffels	Stoffels	Korf	2
78	Inverse	Inverse	Korf	2
83	Näslund	Näslund	Korf	1
86	Inverse	Inverse	Korf	2
87	Inverse	Inverse	Levacovic-3	3
93	Inverse	Inverse	Korf	1
96	Inverse	Näslund	Korf	1
100	Inverse	Inverse	Korf	1
102	Inverse	Inverse	Korf	1
104	Inverse	Inverse	Korf	1
107	Inverse	Inverse	Korf	1
111	Inverse	Inverse	Korf	1
116	Inverse	Inverse	Levacovic-1	3
120	Trorey	Stoffels	Korf	1
126	Inverse	Inverse	Levacovic-3	3
127	Inverse	Inverse	Levacovic-1	3
131	Inverse	Inverse	Korf	1
133	Inverse	Inverse	Levacovic-3	3
135	Inverse	Inverse	Levacovic-1	3
Total	Inverse: 21 Trorey: 5 Stoffels: 2 Näslund: 1 Henricksen: 1	Inverse: 21 Stoffels: 4 Näslund: 2 Trorey: 2 Henricksen: 1	Korf: 23 Levacovic-1: 4 Levacovic-3: 3	1: 15 2: 8 3: 7

Results for the selection of the HDC by RSS and CV criterion, together with results for the selection of the growth function presented in Kamo (2019).

3.2. Numerical experiments

The validity of the method proposed in this paper is verified by numerical examinations. In the numerical examinations, we can know the true model and therefore calculate the probability of a correct selection. We prepare two true models as follows. One is Näslund, which was selected in the whole data set in the real life data analysis (see Figure 2), and the other is Inverse, which was selected most often in the individual analyses (see Figure 3). These are set as $1.2 + \frac{x^2}{(0.2+0.8x)^2}$ for Näslund, and $1.2 + \frac{x}{0.4+0.01x}$ for Inverse. The 10 observation points are set as $\{5, 7, 9, ..., 23\}$ in each sample, and the error term is set as distributing to $N(0, \sigma^2)$. We control the standard deviation (SD) σ as $\{0.1, 0.2, ..., 1.0\}$, then calculate the probability of the correct selection by creating 1,000 virtual samples at each SD. Figure 4 shows the result for the frequency of the HDC selection. The solid lines show the results for true HDC, while black and red indicate the results from RSS and CV criterion, respectively. Although, the percentage of correct responses declines along as SD increases, selection by CV criterion is better than RSS.

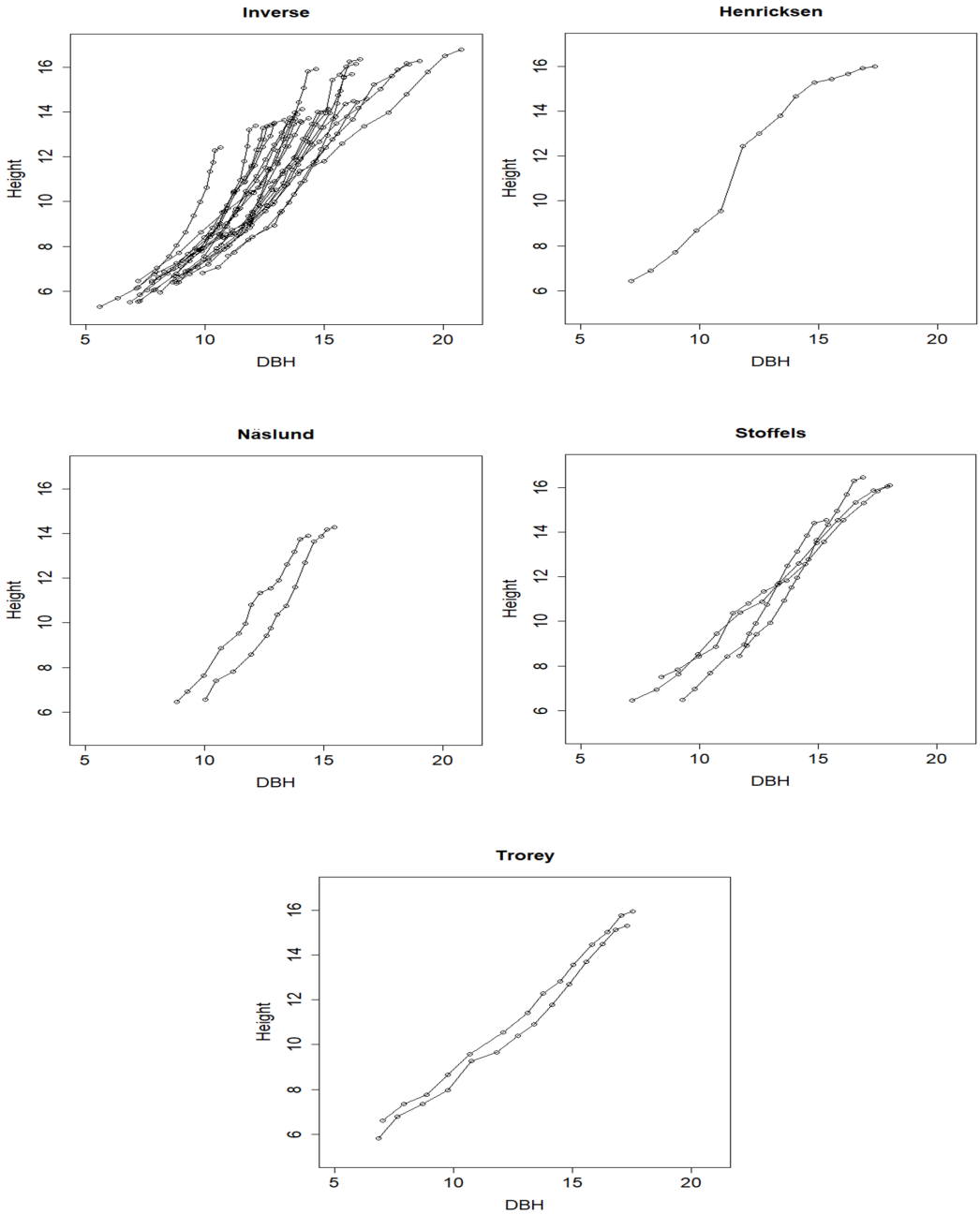


Figure 3. Classification by cross-validation criterion.

Five different HDCs were applied to 30 samples, and the results were classified according to the best HDC. There were 21 Inverse, 1 Henricksen, 2 Näslund, 4 Stoffels, and 2 Trorey.

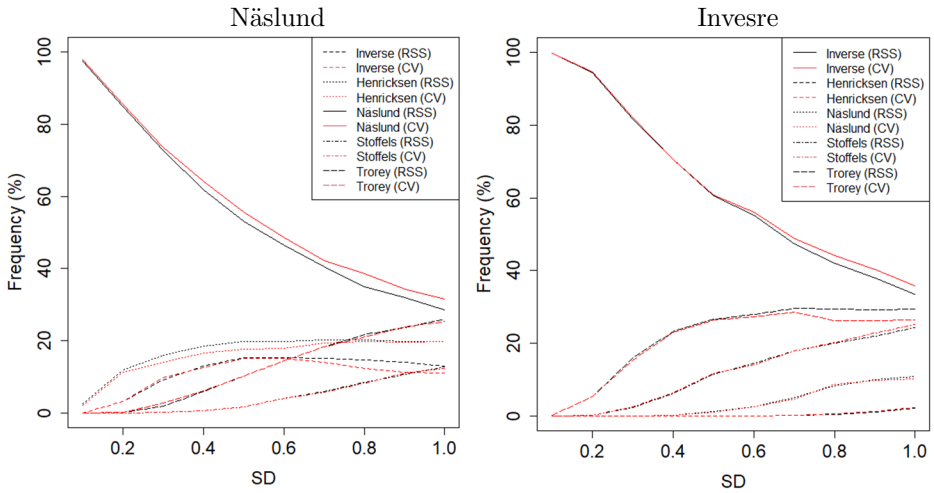


Figure 4. Probability of the selection for HDC.

The figures show the frequency of selected HDCs, expressed as a broken line, with 1,000 virtual samples generated for each SD which is 0.1 increment from 0.1 to 1.0. The true HDC is set as Näslund in the left-hand figure and Inverse in the right-hand figure. The solid lines represent the selection probability for the true HDC and the dashed lines represent one for the other HDCs. In each situation, black represents the selection result by RSS and red by CV criterion.

4. Discussion

In this paper, we focus on the selection of HDC by applying CV criterion and we present the results using real life data and numerical experiments. In previous research, the selection of HDC has often been made a priori or empirically. On the other hand, Matsumura (2003) and Levedev et al. (2020) proposed a selection method based on RSS-type criterion. It is known that RSS-type criterion is not suitable as an indicator of model comparison, because it cannot evaluate the complexity of the model. We then presented a statistical procedure for selecting the best HDC by applying the CV criterion in this paper. The reason for applying CV criterion is as follows. For AIC, Hurvich and Tsai (1989) proposed the AICc which can be applied to nonlinear models. Then, the application of this criterion can be considered as another option instead of the CV criterion. However, to apply the AICc it is necessary to check whether these assumptions (normality and homoscedasticity assumption in the error term) are valid for HDC analysis. Since it is not certain whether these assumptions for AICc are fulfilled in the HDC analysis, we apply CV criterion in this paper. Once such issues are resolved, the results obtained by AICc can be compared with those based on CV criterion, and the HDC selection process can be brushed up in future studies. Another approach for HDC analysis is semiparametric method such as spline (Ruppert et al. 2003). The main objective of this method is to interpolate data. Then, if the structural relationship between tree height and DBH is of concern in the growth-mature phase, it could become useful candidates in addition to HDCs. Since the oldest tree age in the data used in this paper was still 23 much before its maturity, only parametric models were considered as candidates.

As for the real data analysis, Figure 2 shows the fitted HDC for the entire data. Näslund was selected as the best model, which has been frequently used in the analysis of even-aged plantation forests in Japan. Takata (1958) evaluated the high reproducibility of HDC based on the Näslund, while Kajihara (1965) used it to estimate tree height for the adjustment of stand inventory table of sugi. Nishizono et al. (2014) also used the Näslund to analyze regional differences in the height growth of sugi. Furthermore, the Näslund has been used to develop measurement techniques for sugi stands using high-resolution satellites (Hirata et al. 2009), LiDAR (Nakajima et al. 2008), and UAV (Ota et al. 2017). The results shown in Figure 2 supports the validity of the Näslund for describing the relationship between DBH and tree height of sugi stands.

Next, we attempted to apply our method to each sample. Figure 3 shows the results of the classification into five patterns, based on the selection of HDC for each sample. Although no clear characteristics were observed to explain the differences among the groups, the growth slope in many

of the samples seemed to be classified as Inverse. Note that the deaccelerated growth in the last parts of the observed data may be caused by truncation of the data at rather earlier stage of growth. The samples classified as Henricksen and Näslund are characterized by slow tree height growth relative to DBH growth in the initial stage, which is equivalent to the early stage of the sigmoid curve. Especially the observed interval for DBH is very short in the sample classified as Näslund. The samples classified as Stoffels and Trorey had a strong linear trend throughout the observation period. Focusing on the frequency of the classified numbers, the Inverse accounted for the greatest majority (i.e. 70%). This can be interpreted as samples classified by the same HDC, but some of them were assigned to other groups most likely by errors. As shown in Figure 2, the results reveal that Näslund was selected as the best HDC from the entire sample using the CV criteria, while Inverse was selected as the best for each sample. Further investigation is needed to clarify any cause of this discrepancy from both the forestry and mathematical viewpoints.

In Kamo (2019), the results for age-dependent DBH growth function clustering were introduced by using the same data. The results are shown together in Tables 3 and 4. Table 3 shows the classification for each sample based on HDCs and Table 4 shows the agreement between HDC classification and growth clustering. No significant relation was found between age-dependent growth and HDC (p=0.059 by Fisher testing for independence to Table 4), but the results was analogous to the existence of a relation, as the p-values are close to the significance level of 0.05.

Table 4. Agreement for selection between HDC and growth function.

		Growth cluster			Total
		1	2	3	
HDC	Inverse	11	3	7	21
	Henricksen	0	1	0	1
	Näslund	2	0	0	2
	Sofffels	2	2	0	4
	Trorey	0	2	0	2
Total		15	8	7	30

As shown in Figure 4, the ability of CV criterion to select HDC was verified by numerical experiments. In the numerical experiments, the true model can be known and the probability of selecting the true model can be calculated. Here, two types of true model, Näslund and Inverse, were prepared, and for ten patterns of SD such as $\{0.1, 0.2, ..., 1.0\}$, 1,000 virtual observations were generated by random numbers. The frequency for selecting true HDC based on RSS and CV criterion was then generated. The probability of selecting the true model, which is denoted by solid line in Figure 4, decreases as the SD increases, which means that the randomness elements increase. The probability of selecting the true model was slightly higher for CV criterion (3.0% for Näslund and 2.5% for Inverse at SD =1) rather than RSS, which indicates the usefulness of CV criterion in this issue.

Although this theme is still under development, we can conclude with the following description of practical problems and future possibilities, for advancing this analysis. The first problem is the enrichment of candidates. As with the growth function, various hypothetical functions have been proposed for the HDC, and by adding these to the candidates, a wide variety of choices can be made. In fact, Zeide (1993) covered 12 types of candidate growth functions, and for the selection of optimal growth function from them, Kamo (2019) prepared the process to consolidate the candidates. When we have rich candidates, we have to note that increasing the number of candidates has the disadvantage of excessive increase in the number of groups. The second problem is that this method cannot solve the situation where the relations between tree height and DBH are not identified when there are different groups mixed within the same HDC. In such a situation, a possible solution is to unify the functions and apply a clustering method such as the k-means method (MacQueen 1967) to the estimated parameters (Yanagihara et al. 2008), because the parameters are considered to control the property of HDC. The third problem is a situation where the function or parameter changes gradually, depending on external factors affecting the HDC. In such a situation, a possible solution will be to introduce the concept of varying coefficient (e.g., Satoh and Yanagihara, 2010), which introduces a regression structure to the parameters, or the changing point theory for the

sudden change (e.g. Ninomiya and Yoshimoto 2004). These problems are common to those in growth analysis. On the other hand, HDC can be considered as a type of taper function (e.g. Kozak, 1998), whose theories have been developed for a long time. There is therefore the possibility that the key to solve the issues for HDC belongs in growth analysis or taper function theory. So, we need to investigate the several topics of these areas which are closely related to HDC. Although the above problems and possibilities for further development remain, the idea of selecting the HDC based on a data-driven policy, other than empirical rules, is rather statistically appropriate with less bias on selecting HDC.

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Disclosure Statement

The authors declare there are no conflicting interests.

Data Availability

Data are available on request for research purposes.

Author Contribution

Conceptualization: KK, TT, MT
Funding acquisition: MT
Investigation: MT, NM
Methodology: KK, TT
Project administration: KK
Supervision: n/a
Writing - original draft: KK
Writing - review & editing: TT, MT, NM

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