Morphological Analogues in Grand Unified Theory Pedagogy: An Instructional Proposal for Teaching SO(10) Symmetry Breaking Using *Nephelium lappaceum* (Rambutan) as a Classroom Model

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(Dated: February 2025)

Abstract

Teaching grand unified theories (GUTs) presents a persistent challenge in undergraduate physics education due to the high level of abstraction involved in group theory, symmetry breaking, and energy-scale separation. This paper proposes an interdisciplinary pedagogical framework that integrates principles from physics education, embodied cognition, and botanical morphology to support conceptual understanding of SO(10) grand unified theory.

Using the morphological structure of *Nephelium lappaceum* L. (rambutan) as a physical analogue, the paper establishes a structured mapping between the fruit's tripartite radial organization (exocarp, aril, and seed) and the decomposition of the 16-dimensional spinor representation under the symmetry breaking sequence $SO(10) \rightarrow SU(5) \times U(1)$. Particular attention is given to the pedagogical representation of the right-handed neutrino and the see-saw mechanism through everyday embodied experience.

The proposed instructional design is intended for future implementation as an instructor-led classroom demonstration. This work does not report empirical classroom data but outlines a structured pedagogical model that may be evaluated using delayed conceptual recall measures in subsequent studies.

This paper does not propose new physical theory. Its contribution is pedagogical: it illustrates how culturally familiar, low-cost physical analogues can help ground abstract theoretical structures in accessible classroom practice.

PACS numbers: 12.10.Dm, 12.60.Jv, 01.40.Fk, 01.55.+b

I. Introduction

Grand unified theories (GUTs) occupy a central place in modern theoretical physics, offering a coherent framework that unifies the fundamental interactions of the Standard Model within a single symmetry structure. Among these, SO(10) grand unification is particularly attractive due to its elegant accommodation of all fermions within a single spinor representation and its natural incorporation of the right-handed neutrino. Despite their conceptual importance, GUTs remain challenging to teach at the undergraduate level, as they require engagement with abstract mathematical formalisms such as Lie group representations, symmetry breaking cascades, and energy-scale separation.

Traditional instructional approaches often rely heavily on formal mathematical exposition, which can exceed the cognitive readiness of students encountering these ideas for the first time. Recent work in embodied cognition suggests that abstract scientific concepts become more accessible when learners can anchor them to concrete, sensorimotor experiences [1,2]. Physical manipulation of familiar objects has been shown to support meaning-making by activating perceptual and experiential representations alongside symbolic reasoning [3].

In this study, we propose a pedagogical framework that employs a culturally familiar botanical object, *Nephelium lappaceum* L. (rambutan), as an embodied analogue for the symmetry structure and decomposition of SO(10) grand unified theory. The distinctive tripartite morphology of the fruit offers a structured physical representation that can be mapped onto the decomposition of the 16-dimensional spinor representation. At Malaysian wet markets, this pedagogical apparatus is available for approximately RM5 per kilogram, a price point that compares favorably with traditional physics demonstration equipment.

The goal of this work is not to suggest any physical correspondence between botanical systems and particle physics, but to examine whether carefully framed analogical mapping can function as effective cognitive scaffolding. The paper proceeds as follows: Section II outlines the theoretical background of SO(10) grand unification. Section III describes the botanical morphology and establishes the formal analogue mapping. Section IV presents the proposed instructional design and evaluation framework. Section V discusses implications and limitations..

II. Theoretical Foundations

A. SO(10) Grand Unified Theory

SO(10) grand unification, proposed independently by Fritzsch and Minkowski [4] and Georgi [5] in 1975, provides a mathematically economical framework in which all fermions of a single generation, including the right-handed neutrino, are embedded within a single 16-dimensional spinor representation. The Lie algebra so(10) has dimension 45, corresponding to the antisymmetric 10 × 10 matrices. The generators satisfy the commutation relations:

$$[M_{ij}, M_{kl}] = i(\delta_{ik}M_{jl} - \delta_{il}M_{jk} - \delta_{jk}M_{il} + \delta_{jl}M_{ik}) \qquad (1)$$

where $M_{ij} = -M_{ji}$ are the generators with $i, j = 1, ..., 10$.

B. Spinor Decomposition and Symmetry Breaking

Under the maximal subgroup SU(5) × U(1)χ, the 16-dimensional spinor representation of SO(10) decomposes as:

$$16 \rightarrow 10_1 \oplus \bar{5}_{-3} \oplus 1_5 \qquad (2)$$

where subscripts denote U(1)χ charges. The particle content assignment is: **10** contains $(u, d)_L$, u_R^c, e_R^c; $\bar{\mathbf{5}}$ contains $(\nu, e)_L$, d_R^c; and **1** contains ν_R (right-handed neutrino singlet).

The breaking of SO(10) to the Standard Model proceeds through multiple stages:

$$SO(10) \rightarrow[M_{GUT}]\ SU(5) \times U(1)\chi \rightarrow[M_I]\ SU(3)C \times SU(2)L \times U(1)Y \rightarrow[M_{EW}]\ SU(3)C \times U(1)EM \qquad (3)$$

where $M_{GUT} \approx 10^{16}$ GeV is the grand unification scale and $M_{EW} \approx 246$ GeV is the electroweak scale.

C. Renormalization Group and Scale Dependence

The scale dependence of gauge couplings is governed by the renormalization group equation:

$$\mu\,(dg_i/d\mu) = \beta_i(g_i) = (b_i/16\pi^2)\,g_i^3 + O(g_i^5) \qquad (4)$$

where g_i denotes the gauge coupling and μ is the renormalization scale. The one-loop beta function coefficients for the Standard Model are $b_1 = 41/10$, $b_2 = -19/6$, $b_3 = -7$.

D. The See-Saw Mechanism

The see-saw mechanism provides an explanation for the smallness of observed neutrino masses. The effective light neutrino mass is given by:

$$mv \approx m^2D / MR \qquad (5)$$

where m_D is the Dirac mass and M_R is the large Majorana mass of the right-handed neutrino. The right-handed neutrino is essential for theoretical completeness but 'decouples' at low energies, a conceptually challenging aspect that motivates our pedagogical approach.

III. Botanical Morphology and Analogical Mapping

A. Taxonomic Classification

Nephelium lappaceum L. belongs to the family Sapindaceae (soapberry family), order Sapindales. The genus Nephelium comprises approximately 22 species native to Southeast Asia. The species epithet lappaceum derives from the Latin lappa (bur), referencing the spinous fruit surface [6].

B. Fruit Morphology

The *Nephelium lappaceum* fruit exhibits a distinctive tripartite radial structure:

Exocarp (Outer Layer): The outer pericarp layer, approximately 2-3 mm thick, bears numerous fleshy spines (spinterns) of length 10-20 mm. Spintern density is approximately 200-400 per fruit, distributed quasi-uniformly over the surface.

Mesocarp/Aril (Intermediate Layer): The edible portion consists of a translucent to white fleshy aril of thickness 5-8 mm, with mass fraction approximately 40-50% of total fruit mass [7].

Seed (Central Core): A single oblong seed of length 20-30 mm occupies the central position. The seed is typically discarded during consumption, a behavioral pattern of considerable pedagogical significance.

C. Formal Analogue Mapping

We establish the formal correspondence between *Nephelium lappaceum* morphology and SO(10) structure. Let R denote the radial coordinate from the fruit center, defining three morphological zones:

$$\Omega_1 = \{r : 0 \leq r < R_{seed}\} \quad [\text{Seed/Singlet Zone} \rightarrow 1] \qquad (6)$$

$$\Omega_2 = \{r : R_{seed} \leq r < R_{aril}\} \quad [\text{Aril Zone} \rightarrow \bar{5}] \qquad (7)$$

$$\Omega_3 = \{r : R_{aril} \leq r < R_{ext}\} \quad [\text{Exocarp Zone} \rightarrow 10] \qquad (8)$$

This mapping allows students to interpret the algebraic decomposition $16 \rightarrow 10 \oplus \bar{5} \oplus 1$ in spatial terms, where components occupy distinct and ordered regions within a unified structure.

D. The See-Saw Mechanism and Seed Disposal

A particularly elegant aspect of the correspondence concerns the see-saw mechanism. The behavioral pattern of *Nephelium lappaceum* consumption mirrors the theoretical treatment of the right-handed neutrino: the seed is essential for botanical completeness yet routinely

discarded. Just as the right-handed neutrino is 'integrated out' in low-energy effective theory, the seed is 'integrated out' in the consumption process. This provides a memorable pedagogical anchor for the conceptually difficult see-saw mechanism. Culturally familiar objects can provide durable cognitive anchors for abstract theoretical ideas.

IV. Proposed Instructional Design and Evaluation Framework

A. Instructional Context (Proposed)

The proposed instructional design is structured as an instructor-led classroom demonstration grounded in principles of embodied cognition and the author's professional experience as a Special Education Educator. It is intended for undergraduate teaching contexts in which students may encounter grand unified theories for the first time and find representation decompositions and energy-scale reasoning conceptually demanding.

The design emphasizes conceptual understanding of unification, representation decomposition, and the conceptual status of heavy degrees of freedom in effective theories, while maintaining mathematical consistency appropriate for classroom delivery.

B. Demonstration Protocol

The proposed instructional sequence is structured as follows:

Step 1 (Unified Phase):
Present intact *Nephelium lappaceum* to represent the SO(10)-symmetric phase at energy scales above M_{GUT}.

Step 2 (Symmetry Breaking):
Remove the exocarp while introducing the symmetry breaking sequence
$SO(10) \rightarrow SU(5) \times U(1)$.

Step 3 (Representation Differentiation):
Display the internal organization of aril and seed to map onto the decomposition
$16 \rightarrow 10 \oplus \bar{5} \oplus 1$,
emphasizing ordered differentiation within an originally unified structure.

Step 4 (Effective Theory):
Discuss lower-energy accessibility and experimentally observable physics using the edible portion as an intuitive anchor.

Step 5 (Singlet Discussion):
Present the remaining seed and discuss the conceptual role of the right-handed neutrino as theoretically essential yet effectively decoupled at low energies.

The analogy is explicitly bounded and does not imply physical equivalence between botanical systems and particle physics.

C. Evaluation Design (Prospective)

Conceptual retention may be evaluated in future implementations using delayed recall approaches, such as short-answer conceptual questions administered approximately one week after instruction. Comparative designs, including analogy-supported and lecture-only sessions, could be employed to examine differences in explanation structure and conceptual organization.

Qualitative analysis of student reasoning in future studies may explore whether learners spontaneously reference structural components of the analogue when describing the decomposition $16 \rightarrow 10 \oplus 5 \oplus 1$. Such evidence would help determine whether the model functions effectively as a cognitive scaffold.

The present work does not report empirical classroom data. Instead, it outlines a structured pedagogical proposal intended for systematic evaluation in subsequent research.

V. Discussion

A. Pedagogical Significance

It is essential to emphasize that this framework represents a serious pedagogical proposal grounded in established principles of embodied cognition, not merely a humorous novelty. Carefully designed physical analogues may help reduce conceptual barriers in theoretical physics education by providing structured spatial and experiential anchors for abstract representations.

This approach is consistent with Barsalou's perceptual symbol systems theory [3], which posits that abstract concepts are cognitively represented through modal simulations grounded in sensorimotor experience. The *Nephelium lappaceum* framework is intended to activate these representational pathways and support deeper conceptual organization.

B. Implications for Inclusive Education

This work contributes to the recognition that effective physics pedagogy must address diverse learning needs. Traditional approaches emphasizing formal exposition may inadvertently exclude learners who benefit from embodied engagement.

Low-cost, culturally accessible physical analogues may broaden conceptual access to advanced theoretical material, particularly in educational contexts with limited instructional resources.

C. Limitations

The correspondence presented is strictly analogical. The fruit does not contain gauge bosons, fermions, Higgs fields, or symmetry groups. Dimensional limitations are significant: SO(10) involves transformations in abstract ten-dimensional space, whereas the fruit exists in three spatial dimensions. The pedagogical value lies in cognitive scaffolding rather than physical modeling.

VI. Conclusion

This paper has outlined an instructional proposal that uses a structured morphological analogy to support teaching key concepts in SO(10) grand unified theory. The tripartite organization of *Nephelium lappaceum* offers a simple, manipulable scaffold for discussing representation decomposition ($16 \rightarrow 10 \oplus \bar{5} \oplus 1$), hierarchy, and the conceptual role of the right-handed neutrino within effective descriptions.

The central contribution of this work is pedagogical rather than empirical. It proposes a culturally grounded instructional model that may help students organize and interpret high-level theoretical structures through accessible classroom reasoning.

Future work should implement formal evaluative designs, such as controlled comparative studies and longer follow-up measures, to assess learning outcomes systematically. More broadly, this study underscores the value of pedagogical creativity,

especially in resource-limited contexts, for making advanced physics ideas teachable without diluting their conceptual integrity.

Acknowledgments

The author thanks the rambutan vendors of Malaysian pasar malam for reliable specimen procurement. Special appreciation is extended to students in special education contexts who continually inspire creative instructional approaches. This work reflects the belief that complex scientific ideas can be communicated through carefully framed, culturally familiar analogies in inclusive educational settings.

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Appendix A: Mathematical Notes and Formal Consistency

This appendix is included to demonstrate formal consistency with standard treatments of grand unified theories, without extending beyond the pedagogical scope of the main text. No new physical claims are introduced here.

A.1 Spinor Representation and Decomposition

In SO(10) grand unified theory, all fermions of a single generation are embedded in the 16-dimensional spinor representation constructed from the Clifford algebra Cl(10). Under the maximal subgroup SU(5) × U(1)χ, this representation decomposes as

$$16 \rightarrow 10 \oplus \bar{5} \oplus 1,$$

where the representations correspond to the usual Standard Model fermion assignments, including the right-handed neutrino as a singlet. This decomposition is standard and may be found in canonical references on grand unification.

A.2 Renormalization Group Scale Dependence

The scale dependence of gauge couplings is governed by the renormalization group equation

$$\mu \, (dg_i/d\mu) = \beta_i(g_i),$$

where g_i denotes the gauge coupling and μ is the renormalization scale. At one-loop order, $\beta_i(g_i) = (b_i/16\pi^2)\, g_i^3 + O(g_i^5)$, with coefficients b_i determined by the particle content of the effective theory.

A.3 See-Saw Mechanism

The see-saw mechanism for neutrino mass generation is summarized by the effective relation $m\nu \approx mD^2/MR$, where mD is the Dirac mass term and MR is the Majorana mass of the right-

handed neutrino. In low-energy effective descriptions, the heavy right-handed neutrino is integrated out, motivating the pedagogical analogy discussed in the main text.

These expressions are included solely to anchor the educational framework within established theoretical physics. Detailed derivations are standard and may be found in the literature.