



A stem taper equation for *Eucalyptus camaldulensis* in northeast Nigeria

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Received: 23 January 2025 / Accepted: 26 April 2025 / Published online: 13 September 2025

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Abstract *Eucalyptus (Eucalyptus camaldulensis* Dehnh.) is an important exotic species in northern Nigeria commonly used for poles and timber. Sustainable management of this resource would require quantifying its volume. Stem taper equations are one of the main and most efficient methods for estimating stem volume to any merchantable limit of a species. There is currently no taper equation for *Eucalyptus* species in Nigeria. Therefore, this study developed taper equations for *E. camaldulensis* in northern Nigeria. Data for this study were obtained from a private plantation in Jalingo Local Government Area, Taraba State, Nigeria. 68 trees were felled and sectioned into 1-m bolt across the stem to a merchantable limit of 5 cm, which were used as the fitting dataset. An additional 22 trees were felled and used to validate the taper equations for stem volume estimation.

The online version is available at <http://link.springer.com>.

Corresponding editor: Shuxuan Li.

Supplementary Information The online version contains supplementary material available at <https://doi.org/10.1007/s11676-025-01919-7>.

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Seven taper equations were initially fitted to the dataset using nonlinear least squares. The best taper equation was then refitted using a nonlinear mixed-effects approach and calibrated using diameters of one to five sections from the butt end. The taper equations were numerically integrated to obtain the stem volume, which was compared with empirical volume equations. The result shows that the Kozak (Can J For Res 27(5): 619–629. 10.1139/x97-011, 1997) equation, which included eight parameters, provided the best fit for predicting section diameters for under and over bark. The mixed-effects taper equation (NLME-TE) explained most stem diameter variations in the fitting dataset (pseudo- R^2 : 0.986–0.987; RMSE: 0.547–0.578 cm) without substantial residual trends. The validation showed that the prediction accuracy of the integrated NLME-TE improved as the number of sectional diameter measurements increased, with at least a 35% reduction in volume estimate error. For practical implementation, two calibration sectional diameter measurements taken from the butt end per tree are recommended. This approach would reduce measurement effort and cost while improving model performance.

Keywords *Eucalyptus* plantation · Stem volume · Nonlinear least squares · Mixed-effects · Response calibration

Introduction

Eucalyptus camaldulensis Dehnh. is a fast-growing evergreen tree species in the Myrtaceae family. Although this species is naturally found in Australia (Boland et al. 1984), its ability to thrive in diverse climatic conditions has led to its introduction and cultivation in various parts of the world. *E. camaldulensis* was introduced to Nigeria in 1918 as a

timber species and a cover crop, and it has since become one of the most planted exotic tree species in Nigeria, especially in the northern region (Wimbush 1963; PTPU 1988). Its prevalence in forest plantations in northern Nigeria is due to its rapid growth, versatile wood, suitability for afforestation and land reclamation programs, and notable use for windbreaks and shelterbelts (Adegbehin et al. 1988; Otegbeye and Samarawira 1991; Jaiyeoba 2001). *E. camaldulensis* wood has a wide range of uses, including its timber for construction, furniture, veneer, plywood, poles, pulpwood and firewood (Watanabe et al. 2009; Ekhuemelo et al. 2017; Abubakar et al. 2019). However, it remains underutilized in this region (Ekhuemelo et al. 2017), with its plantation stands often poorly managed and some continually degraded (Abubakar et al. 2019).

Forest inventory is important for sustainable forest management as it provides information for forest planning, resource evaluation and decision-making for current and future management scenarios (Burkhart and Tomé 2012). Accurate information on tree stem volume, as well as its structural assortments based on the variation in stem form profile, are essential for managing forest growth and yield effectively, especially given the possibility of alterations in the merchantable limits depending on the specific intended products and utilization standards (Burkhart and Tomé 2012; Krisnawati 2016). Stem taper equations are one of the primary and most precise methods for estimating stem volume to any merchantable limit (Kozak 2004; Krisnawati 2016). Taper describes the rate at which the stem diameter decreases as height increases from the tree's base to its tip, thereby defining its form (Schreuder et al. 1993; Kozak 2004; Burkhart and Tomé 2012). Taper equations are used for obtaining accurate tree diameters (both over bark and under bark) at any height along the stem, which are then used to estimate specific log volumes of any length and total tree volume (Kozak and Smith 1993; Kozak 2004; Kublin et al. 2008; Salekin et al. 2021). Likewise, the equations can predict tree heights at specific diameters along the stem (Fonweban et al. 2011; Salekin et al. 2021).

Given the importance of taper equations in forest management, many have been developed and studied for different commercial species worldwide. Studies like Salekin et al. (2021) and McTague and Weiskittel (2021) provided reviews of global tree stem taper modeling, in which they compiled tree stem taper models published for a variety of species, like pine (Subedi et al. 2011; Corral-Rivas et al. 2017), spruce (Adamec et al. 2019), oak (Tarp-Johansen et al. 1997; Zheng et al. 2017), Eucalyptus (Bi 2000) and teak (Lanssanova et al. 2019), among others. However, most available taper functions have been developed for species or forests in other continents, with Africa constituting only about 4% of existing taper functions (Salekin et al. 2021). Though taper equations exist for eucalyptus forests, most of

the existing functions have been fitted for Eucalyptus in Australia—the species' predominant natural range, New Zealand and Brazil (Da Silva et al. 2006; Maraseni et al. 2007; Salekin et al. 2021; Boczniewicz et al. 2022). Only a few models are found for the tree species in Cameroon (Fonweban 1999) and South Africa (Morley and Little 2012). Furthermore, applying taper equations has been very limited in Nigeria, with only one known explicit taper study for *Gmelina arborea* Roxb. in southwestern Nigeria, appropriately using sectional diameter measurements (Alo et al. 2011). Despite the recent and anticipated expansion of Eucalyptus plantation cover in northern Nigeria, driven by its preference for afforestation in the nation's Savanna region, no taper equation for Eucalyptus species currently exists in Nigeria (Adegbehin et al. 1988; Watanabe et al. 2009; Kurowska et al. 2022). Therefore, developing taper equations to accurately fit local forest data is essential.

Taper equations exhibit varying forms and complexity. According to Salekin et al. (2021), taper equations could be parametric or non-parametric, with the former category being the standard and most common form comprising a couple of static and complex equations. Many studies have adopted various taper equations, such as static polynomials of varying orders (Kozak et al. 1969; Da Silva et al. 2006), segmented polynomials (Max and Burkhart 1976; Adamec et al. 2019), sigmoid (Hada 1958; Ormerod 1973) and trigonometric (Bi 2000; Pukkala et al. 2019). A more notable type is the variable-exponent (or broadly, variable-form), which better accounts for variation in tree form along the stem (Newnham 1992; Kozak 2004; Heiðarsson and Pukkala 2011). Other complex taper counterparts are compatible equations (Demaerschalk 1972; Zheng et al. 2017), which integrate taper functions with volume equations, whole-bole systems (Demaerschalk and Kozak 1977) and dynamic taper equations (Muhairwe 1994). Regardless, it is worth noting that the taper functions vary in terms of accuracy, with no unanimous adequate equation or theoretical explanation for stem form and taper variation for all species, stands and site conditions (Newnham 1992; Oluwajuwon et al. 2025). One major advantage of taper equations is that they can be used to implicitly derive stem volume through either analytical or numerical integration. They also support closed-form solutions or numerical methods for invertibility when predicting height at a given diameter and can be optimized for predicting specific variables (i.e., diameter or volume) to meet various forest management needs.

Taper equations are developed using a tree's section diameters, which represent its longitudinal structure and capture tree-level variations. To ensure reliable inference, it is essential to utilize advanced modeling techniques like the mixed-effects modeling method. This approach accounts for the hierarchical structure of the data, reducing bias in standard errors and improving their reliability compared to

models that ignore such dependencies. Mixed-effects models incorporate both fixed effects, which represent the average relationship across the entire sectional diameter dataset or a distinct group (like an average plot or tree), as well as the random effects, which allow for group-specific variations within the dataset (Cao and Wang 2015; Lanssanova et al. 2019; Bronisz and Zasada 2020). However, studies have argued that mixed-effects models may not always demonstrate the highest possible accuracy and predictive ability, and accounting for model random parameters/effects may not always be practical for taper modeling (Warner et al. 2016; Bronisz and Zasada 2020). Two alternatives have been proposed and adopted: estimates from ordinary or nonlinear least squares and fixed-effect estimates of the mixed-effects model, albeit with divergent trends of comparative performances and errors in predicting section diameters (Warner et al. 2016; Pukkala et al. 2019; Bronisz and Zasada 2020).

This study generally aims to develop stem taper equations that provide the best predictions for section diameters and volume for *Eucalyptus* plantations in the northeast of

Nigeria by: (1) fitting different candidate taper equations to the section diameters (both over bark and under bark) for the species using nonlinear least squares; (2) refitting the best taper equation with mixed-effects approach to account for inter-tree variation, and (3) comparing the numerically integrated derived stem volume from the taper equations and the predicted volume from empirical volume equations.

Materials and methods

Data collection

The study was carried out on a 2.4-ha private plantation of *Eucalyptus* (*Eucalyptus camaldulensis*) located in the Jalingo Local Government Area of Taraba State, Nigeria, between latitudes 8.960° N to 8.961° N and longitudes 11.380° to 11.381° E (Fig. 1). The harvested trees used for this research were grown from seeds. The *Eucalyptus* plantation was established primarily to secure the land to avoid encroachment

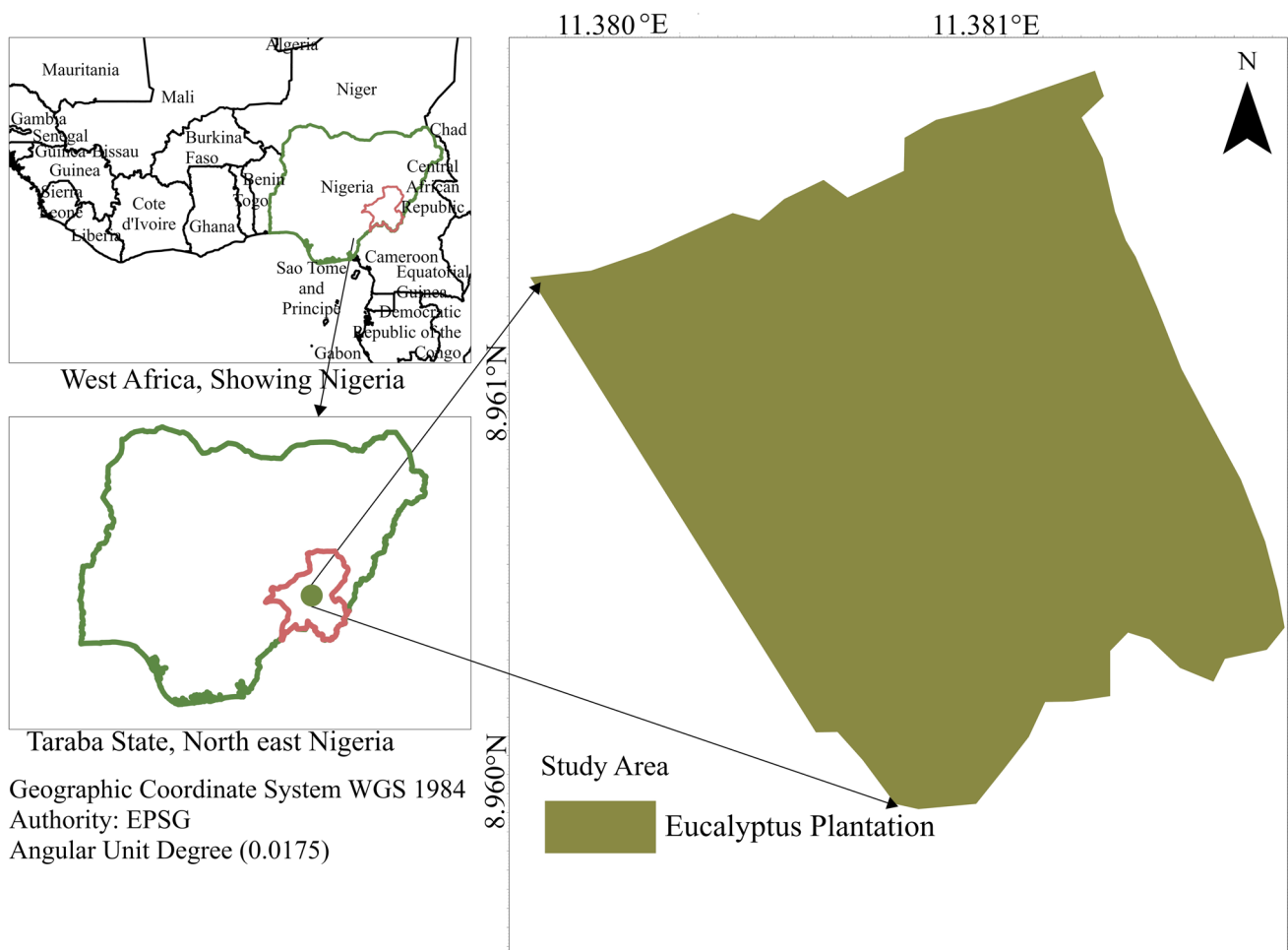


Fig. 1 Map of the plantation located in Taraba State, Nigeria

by the surrounding farmers. The secondary purpose was to generate income from timber harvesting. Before felling, a reconnaissance survey of the plantation stands was conducted to assess the distribution of tree diameters, ensuring a suitable number of trees were sampled across diameter classes. Sixty-eight trees were initially felled in January 2024 and sectioned into 1-m bolt across the stem of each tree. Diameter measurements at each 1-m interval were recorded to the nearest 0.1 cm with a diameter tap for both under and over bark readings, and heights were recorded to the nearest 0.1 m. A summary of the fitting data is presented in Table 1. The diameter over bark was measured along the stem using a diameter tape. Over bark diameter measurements were taken along the stem using a diameter tape, starting at 0.1 m from the base and continuing at 1-m intervals up to the merchantable height, defined by a 5 cm top diameter. Two measurements were obtained for the bark thickness at an angle of 90°. To measure bark thickness, two readings were taken at 90° angles using a bark gauge, which measures half the bark thickness. The average bark thickness was doubled and subtracted from the over bark diameter to calculate the diameter under bark. Another set of 22 trees was also felled in July 2024 using the same procedure above. The dataset was used to validate the taper equation.

Taper equations

This study adopted several taper equations, which include diameter at breast height (DBH) and heights as predictor variables. The selected equations offer a range of mathematical approaches, including variable-exponent, segmented, trigonometric and variable-form models, to predict stem taper under and over bark. Since no single equation works on every species, our study adapted several equations from

recent studies (Bronisz and Zasada 2020; Morrone and Green 2024; Saud et al. 2024). Among the equations evaluated were Eq. 1 (Kozak 1997), Eq. 2 (Kozak 1988), Eq. 3 (Bi 2000), Eq. 4 (Muhairwe 1999), Eq. 5 (Newberry and Burkhart 1986), Eq. 6 (Demaerschalk 1972), Eq. 7 (Lee et al. 2003). Some of the equations have been applied previously to Eucalyptus species in Australia (Muhairwe 1999; Bi 2000).

The taper equations were fitted using the Levenberg–Marquardt nonlinear least squares (NLS) algorithm in the ‘*minpack.lm*’ R package (Elzhov et al. 2023) for under and over bark. The best taper equation for the eucalyptus was determined using mean absolute percentage error (MAPE, %; Eq. 8), root mean square error (RMSE, cm; Eq. 9), percentage root mean square error (percentage of RMSE; Eq. 10), Bayesian Information Criterion (BIC; Eq. 11), and pseudo coefficient of determination (pseudo- R^2 ; Eq. 12) obtained with the ‘*fitstat*’ function in ‘*EMAR*’ package (Ogana and Corral-Rivas 2023). The smaller the MAPE, RMSE, percentage of RMSE, BIC and the higher pseudo- R^2 , the better the taper equations. We used relative rank (Poudel and Cao 2013) to decide on the best taper equation for estimating upper stem diameter under and over bark. All analyses were performed in the R environment (R Core Team 2023).

$$\begin{aligned} \text{T1: } d_{ij} = & \left(b_1 DBH_i^{b_2} H_i^{b_3} \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right) \right)^{b_4} \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right)^{0.1} \\ & + b_5 RH^4 + b_6 \arcsin(1 - \sqrt{RH}) + b_7 \left(\frac{1}{e^{H_i}} \right) + b_8 DBH_i \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right) + \varepsilon_{ij} \end{aligned} \quad (1)$$

$$\text{T2: } d_{ij} = \left(b_1 DBH_i^{b_2} \right) b_3^{DBH_i} X^{b_4 RH^2 + b_5 \ln(RH + 0.001) + b_6 (DBH_i / H_i)} + \varepsilon_{ij} \quad (2)$$

Table 1 Summary statistics of the harvested sample trees by 4 cm diameter classes

	DBH class (cm)	Ind.	DBH (cm)			Height (m)		
			Mean	Min	Max	Mean	Min	Max
Fitting data	9–13	18	11.8	9.8	13.0	9.9	5.8	14.2
	13–17	17	15.1	13.1	16.9	13.8	9.8	18.2
	17–21	27	18.8	17.2	20.6	14.2	10.3	17.8
	21–25	4	22.5	21.4	23.8	17.3	15.2	18.2
	25–27	2	26.5	26.1	26.9	14.5	11.9	17.1
	Total	68	9.8	16.5	26.9	5.8	13.1	18.2
Validation data	9–13	3	11.2	10.7	11.5	11.9	11.7	12.0
	13–17	8	15.4	13.6	16.9	14.1	13.1	15.1
	17–21	3	18.5	17.7	19.1	15.8	15.3	16.4
	21–25	8	22.9	21.2	24.2	17.1	11.8	19.4
	Total	22	18.0	10.7	24.2	15.1	11.7	19.4

DBH: diameter at breast height; Ind.: number of harvested trees.

$$\text{T3: } d_{ij} = DBH_i \left(\frac{\ln \sin(\pi/2 RH)}{\ln \sin(\pi/2 t)} \right)^{b_1 + b_2 \sin(\pi/2 RH) + b_3 \cos(3\pi/2 RH) + b_4 ((\sin(\pi/2 RH))/RH) + b_5 DBH_i + b_6 RH \sqrt{DBH_i} + b_7 RH \sqrt{H_i}} + \varepsilon_{ij} \quad (3)$$

$$\text{T4: } d_{ij} = \left[b_1 DBH_i^{b_2} (1 - RH) \left(b_3 RH + b_4 RH^2 + \left(\frac{b_5}{RH} \right) + b_6 RH^3 + b_7 DBH_i + b_8 \left(\frac{DBH_i}{H_i} \right) \right) \right] + \varepsilon_{ij} \quad (4)$$

$$\text{T5: } d_{ij} = b_1 DBH_i \left(\frac{H_i - h_{ij}}{H_i - 1.3} \right)^{b_2} + \varepsilon_{ij} \quad (5)$$

$$\text{T6: } d_{ij} = b_1 DBH_i^{b_2} (H_i - h_{ij})^{b_3} H_i^{b_4} + \varepsilon_{ij} \quad (6)$$

$$\text{T7: } d_{ij} = b_1 DBH_i^{b_2} (1 - RH)^{b_3 RH^2 + b_4 RH + b_5} + \varepsilon_{ij} \quad (7)$$

where, $b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8$ are parameters of the taper equation; d_{ij} is expected upper stem diameter (cm) at a specified height h_{ij} (m) of section j on tree i ; H_i and DBH_i are height (m) and diameter at breast height over bark (cm) of tree i ; t is the relative breast height ($1.3/H_i$); RH is the relative height: $RH = h_{ij}/H_i$; $X = (1 - \sqrt{RH})/(1 - \sqrt{t})$, ε_{ij} is the error term [$\varepsilon_{ij} \sim N(0, \sigma^2)$]; $i = 1, 2, \dots, n$; $j = 1, 2, \dots, n_i$. n_i is the number of observations (sections) of the i th tree.

$$MAPE = \left(1/n \left(\sum_{i=1}^n \sum_{j=1}^{n_i} \frac{|d_{ij} - \hat{d}_{ij}|}{d_{ij}} \right) \right) \times 100 \quad (8)$$

$$RMSE = \sqrt{\frac{1}{(n-1)} \left(\sum_{i=1}^n \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2 \right)} \quad (9)$$

$$\%RMSE = \sqrt{\frac{1}{(n-1)} \left(\sum_{i=1}^n \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2 \right)} \times 100 \quad (10)$$

$$BIC = n_i \ln \left(\sum_{i=1}^n \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2 \right) + p \ln n_i \quad (11)$$

$$pseudoR^2 = 1 - \left(\left(\sum_{i=1}^n \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2 \right) / \sum_{i=1}^n \sum_{j=1}^{n_i} (d_{ij} - \bar{d})^2 \right) \quad (12)$$

where, n is the number of observations; d_{ij} and $\hat{d}_{ij}$ represent observed and predicted sectional diameters, respectively, of

the j th section on the i th tree; and $\bar{d}$ is the mean sectional diameter.

Nonlinear mixed-effects taper equation (NLME-TE)

The taper data are longitudinal due to multiple sectional measurements per tree; thus, using least squares to model such data would violate its underlying assumption of independence. We formulated a mixed-effects taper equation for the *Eucalyptus* species by including tree-level random effects to address this challenge. The general expression of the NLME-TE is given by Pinheiro and Bates (2000) (Eq. 13):

$$d_{ij} = f(X_{ij}; \theta_i) + \varepsilon_{ij}; \text{ with } \theta_i = \psi_i \beta + Z_i b_i \quad (13)$$

where, d_{ij} is the diameter of section j on tree i ; X_{ij} represents a covariate vector; θ_i vector $r \times 1$; r is the parameters of the selected taper equation; β is a vector of the fixed parameters; b_i vector of the random parameter; ψ_i ($r \times p$) and Z_i ($r \times q$) represent the dimensional matrix for fixed and random effects, for a given tree i ; p and q are the numbers of fixed and random parameters, respectively. The random effect is assumed to have a joint multivariate normal distribution with a mean equal to zero and a positive-definite variance-covariance matrix $D[b_i \sim N(0, D)]$ for all values of i . The ε_{ij} is the random error with a mean equal to zero and a homoscedastic variance [$\varepsilon_{ij} \sim N(0, \sigma^2)$]. Heteroscedasticity in the residual variance was stabilized with the power variance-stabilizing function (Eq. 14):

$$\text{var}(\varepsilon_{ij}) = \sigma^2 \hat{d}_{ij}^{2\delta} \quad (14)$$

where, σ and δ are parameters to be estimated, respectively; $\hat{d}_{ij}$ is the predicted diameter of the j th section on the i th tree. We used the restricted maximum likelihood of the 'nlme' package (Pinheiro and Bates 2023) to fit the NLME-TE. Only the best taper equation selected with the relative rank was refit with a nonlinear mixed-effect approach. Different fixed parameters and random effects combinations were fitted to the fitting dataset. The smallest AIC determined the

best combination of fixed parameters and random effects, and whether convergence was achieved (Ogana et al. 2023).

Though the NLME-TE can be used to predict section diameters of a tree when information on the tree-level random effects is unavailable, the estimate will be less accurate. However, with response calibration, the random effects could be predicted and included in the NLME-TE, to improve accuracy. We used the empirical best linear unbiased prediction (EBLUP) technique (Eq. 15) (Vonesh and Chinchilli 1997) to estimate the random effect parameter ($\hat{b}$).

$$\hat{b} = \hat{DZ}'(\hat{R}_i + Z_i\hat{D}Z_i')^{-1}\hat{\epsilon}_i \quad (15)$$

where, $\hat{b}$ is a vector of the tree-level random effects; Z_i is the design matrix of the random part, obtained by taking the partial derivative of the NLME-TE with respect to the fixed-effect parameters to which the random effect parameters were added. $\hat{D}$ represents the estimated variance–covariance matrix of the tree random effect. $\hat{R}_i$ is the estimate of the error matrix, derived from $\hat{\sigma}^2\hat{d}_{ij}^2I_i$; where I_i is the identity matrix; $\hat{\epsilon}_i$ represents the residual vector obtained from the difference between the observed sectional diameter and values predicted using the NLME-TE, including only the fixed effect. For details about estimating the random effect parameters of a mixed-effect model, see Mehtätalo and Lappi (2020). Fit statistics and residual graphs were used to assess the quality of the NLME-TE. Since measured section diameters are required to estimate the random effect parameters, we used measurements from one to five section diameters per tree at a 1-m interval, for response calibration. Sample sections were purposively selected from the large butt end upward along the tree for the practical application of the NLME-TE, particularly for a standing tree. Random effect parameters were predicted using the measured one to five sectional diameters per tree from the independent data. Additionally, we evaluated the scenario where the NLME-TE was not calibrated, using only the fixed-effect component.

Stem volume

The observed under and over bark volumes for each sampled tree were computed as the sum of all 1-m sectional volumes of the respective tree stems. Following common practices in taper and form studies (e.g., Bronisz and Zasada 2020; Oluwajuwon et al. 2024), Smalian's formula (Eq. 16) was employed to compute the section-wise volumes.

$$V_i = \sum_{j=1}^{n_i} \left(\frac{g_{j1} + g_{j2}}{2} \right) \times L_{ij} \quad (16)$$

where, V_i represents the observed stem volume (m^3) of tree i ; g_{j1} and g_{j2} are the cross-sectional areas at the upper and lower ends of the individual sections (m^2); and L_{ij} is the length of the section j (m). To determine the most suited empirical volume equation for estimating the stem volume under and over bark, four standard equations were evaluated (Eqs. 17–20). These equations represent some of the most used forms in volume studies (e.g., Burkhart and Tomé 2012; Dávila-Molina et al. 2022). The stem volume equations were evaluated using MAPE, RMSE, percentage of RMSE, BIC and the pseudo- R^2 .

The best empirical volume equation was used to perform a comparative assessment of the volume predictions generated by our integrated taper equation. We numerically integrated the taper equations fitted with NLS and mixed-effect to derive the stem volume for under and over bark.

$$\text{mbined variable (V1)} : V_i = b_0 + b_1 DBH_i^2 H_i + \epsilon_i \quad (17)$$

$$\text{Constant form factor (V2)} : V_i = b_1 + DBH_i^2 H_i + \epsilon_i \quad (18)$$

$$\text{Logarithmic (V3)} : V_i = b_1 DBH_i^{b_3} h^{b_3} + \epsilon_i \quad (19)$$

$$\text{Honer transformation variable (V4)} : V_i = \frac{DBH_i^2}{b_0 + b_1 H_i^{-1}} + \epsilon_i \quad (20)$$

where, b_0, b_1, b_2, b_3 , are model parameters; V_i expected stem volume of tree i at a given diameter at breast height (DBH_i) and stem height (H_i); ϵ_i is the error term [$\epsilon_i \sim N(0, \sigma^2)$].

Results

Fitted taper equations

Fit statistics for the seven nonlinear taper equations developed for estimating the upper stem diameter of Eucalyptus in northeast Nigeria are shown in Tables 2 and 3. All parameters of the under bark taper equations had low standard errors and were significantly different from zero ($p < 0.05$), except T4 (Eq. 4) where b_8 was not significant. Similarly, all parameters in the T1, T5, T6 and T7 over bark taper equations were significant. However, the parameters b_4 and b_8 in T2 and T4, respectively were not significant. Parameters b_4 and b_5 were also not significant in the T3 over bark equation. The standard errors of the parameter b_1 in T4 for under and over bark were relatively large, which showed some degree of uncertainty in the estimate. The fit statistics (MAPE, RMSE, percentage of RMSE, BIC and pseudo- R^2) showed that T1 (Kozak 1997, Eq. 1) outperformed other taper equations for under and over bark. The RMSE of the

Table 2 Parameter estimates for the taper equations fitted with NLS (under and over bark). Numbers in parentheses are standard error estimates of each parameter. Numbers with asterisks are not significant at 5%

Parameter estimates									
Model name	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	
Under bark	T1	0.205 (0.049)	1.442 (0.113)	-0.181 (0.070)	0.148 (0.012)	-4.733 (0.289)	3.427 (0.418)	7.548 (1.571)	0.135 (0.010)
	T2	0.962 (0.184)	0.938 (0.010)	1.007 (0.005)	0.103 (0.024)	-0.147 (0.013)	0.201 (0.012)	-	-
	T3	-2.438 (0.197)	0.801 (0.010)	0.182 (0.019)	1.577 (0.117)	-0.005 (0.002)	0.120 (0.022)	-0.042 (0.015)	-
	T4	224.629 (88.004)	0.999 (0.019)	-9.103 (0.885)	13.265 (1.420)	5.251 (0.371)	-7.906 (0.814)	0.011 (0.002)	5.99E-05* (0.024)
	T5	0.905 (0.004)	0.572 (0.008)	-	-	-	-	-	-
	T6	1.300 (0.068)	0.943 (0.025)	0.571 (0.008)	-0.622 (0.032)	-	-	-	-
	T7	1.461 (0.063)	0.907 (0.014)	0.629 (0.134)	-1.680 (0.196)	1.548 (0.076)	-	-	-
Over bark	T1	0.318 (0.066)	1.313 (0.101)	-0.156 (0.063)	0.125 (0.009)	-4.763 (0.273)	3.391 (0.396)	9.192 (1.662)	0.138 (0.009)
	T2	1.346 (0.253)	0.848 (0.098)	1.007 (0.005)	0.038* (0.022)	-0.125 (0.013)	0.197 (0.011)	-	-
	T3	0.393 (0.181)	-0.474 (0.091)	-0.072 (0.017)	0.007* (0.106)	-0.002* (0.002)	0.098 (0.019)	-0.040 (0.013)	-
	T4	160.863 (63.179)	0.939 (0.019)	-8.316 (0.885)	12.157 (1.409)	4.677 (0.372)	-7.262 (0.804)	0.010 (0.002)	0.029* (0.022)
	T5	0.980 (0.005)	0.489 (0.007)	-	-	-	-	-	-
	T6	1.672 (0.087)	0.843 (0.025)	0.486 (0.007)	-0.492 (0.032)	-	-	-	-
	T7	1.930 (0.082)	0.835 (0.014)	0.447 (0.127)	-1.409 (0.189)	1.377 (0.075)	-	-	-

Table 3 Fit statistics for the taper equation fitted with NLS

	<i>Model name</i>	<i>p</i>	MAPE	RMSE	Percentage of RMSE (%)	<i>pseudo-R</i> ²	BIC	Rank Sum
Under bark	T1	8	12.818	1.375	12.641	0.919	5347	5.040*
	T2	6	13.120	1.404	12.910	0.916	5397	8.550
	T3	7	13.790	1.485	13.650	0.906	5573	18.310
	T4	8	12.803	1.383	12.718	0.918	5366	5.810
	T5	2	14.968	1.626	14.953	0.888	5814	34.880
	T6	4	14.974	1.623	14.926	0.888	5823	34.780
	T7	5	13.261	1.418	13.041	0.914	5420	10.230
Over bark	T1	8	9.771	1.299	10.502	0.929	5174	5.000*
	T2	6	11.055	1.559	12.605	0.897	5714	18.460
	T3	7	10.452	1.522	12.306	0.902	5648	15.930
	T4	8	10.758	1.535	12.414	0.900	5682	17.030
	T5	2	13.690	1.844	14.913	0.857	6196	35.000
	T6	4	13.510	1.811	14.647	0.861	6156	33.340
	T7	5	11.346	1.579	12.770	0.895	5746	19.770

The asterisks mean the best equations based on the smallest rank.

Table 4 Parameter estimates and fit statistics of equation (T1) fitted with NLME (NLME-TE) for diameter under and over bark. Numbers with asterisks are not significant at 5%

Components	Parameters	Under bark		Over bark	
		Estimate	SE	Estimate	SE
Fixed part	b_1	0.362*	0.235	0.255	0.128
	b_2	0.993	0.338	1.190	0.255
	b_3	0.044*	0.218	-0.090*	0.162
	b_4	0.039	0.007	0.0426	0.006
	b_5	-2.918	0.115	-2.876	0.227
	b_6	6.833	0.531	7.024	0.393
	b_7	0.927*	5.200	11.526	2.881
	b_8	0.081	0.011	0.072	0.011
Random part: Tree	$\sigma_{u_1}^2$	4.134 ²		0.255 ²	
	$\sigma_{u_2}^2$	12.004 ²		2.818 ²	
	$\sigma_{u_3}^2$	0.078 ²		0.080 ²	
	$\text{cor}(u_1, u_2)$	-0.882		-0.278	
	$\text{cor}(u_1, u_3)$	-0.819		-0.365	
	$\text{cor}(u_2, u_3)$	0.509		-0.698	
	σ^2	0.224		0.190	
	δ	0.396		0.482	
	MAPE (%)	4.336		3.778	
	RMSE (cm)	0.547		0.578	
Fit indices	Percentage of RMSE (%)	5.026		4.675	
	pseudo- R^2	0.987		0.986	
	BIC	3335		3409	

T1 ranged from 1.299–1.375 cm and accounted for at least 91.9% (pseudo- R^2 : 0.919–0.929) of the variation in upper stem diameters. It had the lowest relative ranks (Table S1). The taper equation T4 performed relatively well, but only for estimating upper stem diameter under bark. T5 had the worst results for estimating the upper stem diameter under and over bark. Based on the relative rank sum of the taper equations for estimating under bark, the ranking order was: T1 > T4 > T2 > T7 > T3 > T6 > T5. For over bark, the ranking order was: T1 > T3 > T4 > T7 > T2 > T6 > T5.

Nonlinear mixed effects (NLME-TE)

Using the NLME approach for the best taper equation (T1) improved the accuracy and precision in estimating the section diameters under and over bark (Eqs. 21–22, Table 4). The RMSE and MAPE ranged from 0.547 to 0.578 cm and 3.778% to 4.336%, respectively, and accounted for at least 98.6% (pseudo- R^2 : 0.986–0.987) of the variation in the section diameters. However, some of the parameters of the

NLME-TE, particularly for diameter under bark, were not significant ($p > 0.05$). The optimal NLME-TE for diameter under bark was achieved when parameters $b_6 - b_8$ were used as random (Eq. 21) (Table S2). For the over bark taper equation, using b_5 , b_6 and b_8 as random parameters yielded the best performance (Eq. 22) (Table S3). Additionally, an assessment of the residuals of the prediction equations showed homoscedastic variance across predicted section diameters, relative diameters (d/DBH) and relative heights (h/H), with about 99% of the residuals falling between -2.5 to 2.5 cm (Fig. 2). The taper equation showed a tendency to overestimate the larger section diameters.

$$\hat{d} = \left(b_1 DBH^2 H^{b_3} \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right) \right)^{b_4} \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right)^{0.1} + b_5 RH^4 + (b_6 + u_1) \arcsin(1 - \sqrt{RH}) + (b_7 + u_2) \left(\frac{1}{DBH e^H} \right) \quad (21)$$

$$+ (b_8 + u_3) DBH \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right) \hat{d} = \left(b_1 DBH^2 H^{b_3} \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right) \right)^{b_4} \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right)^{0.1} + (b_5 + u_1) RH^4 + (b_6 + u_2) \arcsin(1 - \sqrt{RH}) + b_7 \left(\frac{1}{DBH e^H} \right) + (b_8 + u_3) DBH \left(\frac{1 - \sqrt{RH}}{1 - \sqrt{t}} \right) \quad (22)$$

where, u_1 , u_2 , u_3 are the random parameters. Other parameters were previously defined.

Integrated taper and volume equations

The numerically integrated mixed-effect taper equation (NLME-TE) gave a better approximation of the stem volumes compared to the empirical volume equations (V1 to V4) in both under and over bark (Table 5). However, the V1 to V4 equations were better than the integrated taper fitted with NLS (T1). In the case of the general performance of the empirical volume equations, the logarithmic equation (V3) performed relatively better than V1, V2 and V4, as shown by the fit statistics (MAPE, RMSE and percentage of RMSE) for the fitting dataset. The combined variable volume equation (V1) also performed relatively well, particularly for volume under bark, but one of the parameters was not significant ($p > 0.05$). The constant form factor equation (V2) performed poorly for both under and over bark.

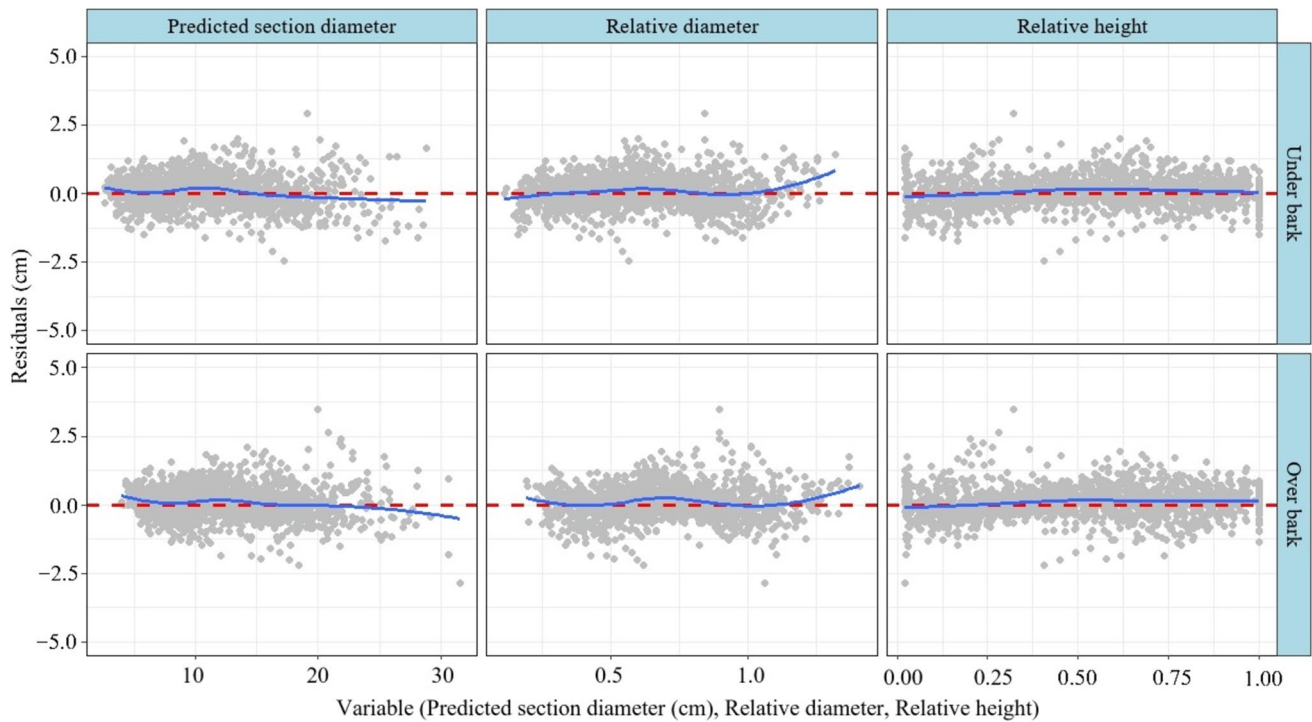


Fig. 2 Scatter plots of residuals versus predicted section diameter, relative diameter, and relative height for diameter under and over bark. The horizontal dashed red lines represent the expected mean

residual (i.e. $E(\epsilon_{ij}) = 0$) and the solid blue lines were obtained with a local regression (LOE)

Table 5 Parameter estimates and fit statistics of the empirical volume equations. Numbers with asterisks are not significant at 5%

Name		b_1	b_2	b_3	MAPE	RMSE	Percentage of RMSE (%)	pseudo- R^2
Under bark	V1	0.006*	3.13E-05	—	9.225	0.015	13.452	0.950
	V2	—	3.26E-05	—	9.835	0.016	13.766	0.948
	V3	5.09E-05	1.889749	0.957	9.256	0.015	13.446	0.950
	V4	378.455*	25,955.89	—	9.464	0.016	13.575	0.949
	T1	Numerical integrated taper			14.043	0.017	14.995	0.938
	NLME-TE	Numerical integrated taper			6.841	0.008	6.866	0.987
Over bark	V1	0.0158	3.68E-05	—	6.239	0.014	9.492	0.971
	V2	—	4.01E-05	—	9.517	0.016	11.129	0.960
	V3	9.33E-05	1.692	1.037	5.885	0.013	9.097	0.974
	V4	521.349	18,444.560	—	7.272	0.015	10.450	0.965
	T1	Numerical integrated taper			11.039	0.016	10.927	0.962
	NLME-TE	Numerical integrated taper			6.340	0.008	5.562	0.990

T1: taper equation fitted with NLS; NLME-TE refitted T1 with mixed-effects. Numbers with asterisks are not significant at 5%.

Validation of the Integrated taper-volume equations

The results of the calibrated NLME-TE concerning upper stem diameter estimation are presented in the Appendix (Figs. S1 and S2). Regardless of whether under or over bark, the numerical integration of the calibrated taper equation gave better stem volume estimates with the

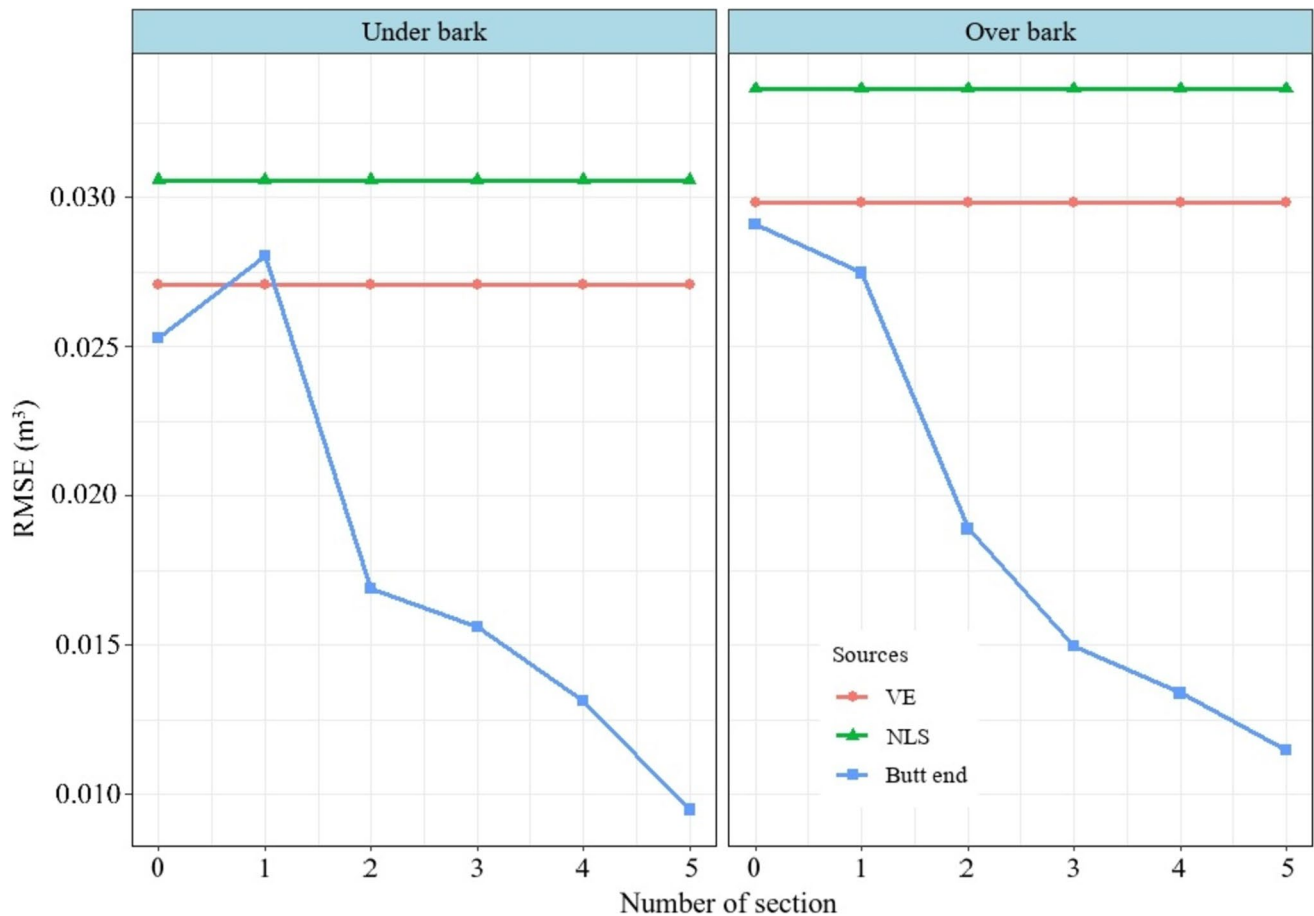


Fig. 3 Performance of sample section diameter measurements required to calibrate the NLME-T1 (Eqs. 21 and 22) to improve stem volume estimation. Note: zero section means without response cali-

bration (i.e., fixed-effect part only). VE: empirical volume equation (V3, Eq. 19); NLS: integrated T1 (Eq. 1)

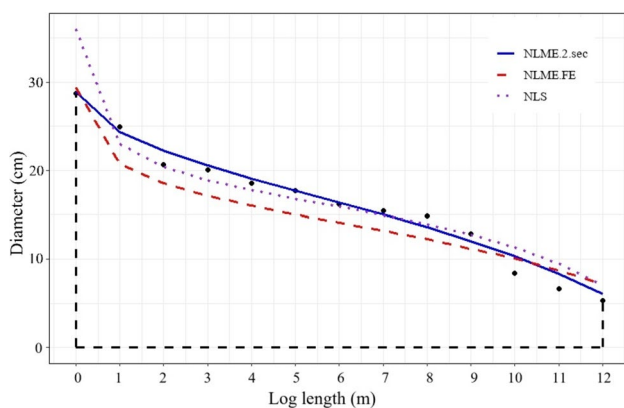


Fig. 4 A stem profile of a felled tree (over bark) with DBH of 23.6 cm, section to 5 cm top diameter (merchantable limit) at a height of 12 m. The black circles are the observed sections; solid blue lines are predictions from the NLME-T1 calibrated with two sectional diameter measurements taken from the butt end; dashed red lines are predictions from the uncalibrated NLME-T1 (fixed-effect part only); dotted purple lines are predictions from T1 fitted with NLS

validation dataset (Fig. 3 and Fig. S3). The analysis of the number of sectional diameter measurements required for response calibration of the NLME-TE to enhance volume estimation revealed that the prediction accuracy of the equation improved with an increase in the number of sections (from 1 to 5 sectional diameter measurements), with two sections being ideal for practical implementation. Without calibration (using only the fixed-effect component), volume estimation was less accurate, especially for over bark measurements. Stem volume estimates from the integrated NLME-TE equation exhibited higher accuracy than those derived from the empirical volume equation (VE). Conversely, the taper equation fitted with nonlinear least squares (T1) produced the least accurate stem volume estimates for under and over bark conditions. The stem profile of a representative tree revealed that T1 poorly predicted the butt end of the stem, which was reflected in the volume estimation (Fig. 4).

Discussion

This study developed taper equations that simultaneously estimate upper stem diameter and volume up to 5 cm top diameter for *E. camaldulensis* plantations in northeastern Nigeria. Given the expected expansion of hectares occupied by *E. camaldulensis* plantations in the region, the taper equations are critical to support effective forest management and utilization strategies.

Among the seven taper equations evaluated in this study, the Kozak (1997) equation (T1) performed best in describing both under and over bark taper for the eucalypts. Similar findings have been reported in other studies. For instance, Bronisz and Zasada (2020) identified the Kozak (1997) taper equation as the best and most effective for predicting section diameters of black locust (*Robinia pseudoacacia* L.) trees, both over and under bark. Likewise, de-Miguel et al. (2012) concluded that the Kozak equation was the best of all 33 taper equations tested for Calabrian pine. The Kozak (1997) equation is one of the most widely adopted and tested variable-form taper models, alongside others such as those developed by Newnham (1992) and Kozak (2004).

Variable-form taper equations are generally preferred for their flexibility and robust performance across diverse tree forms, coupled with the fact that they are relatively straightforward to parameterize (McTague and Weiskittel 2021). Bronisz and Zasada (2020) also emphasized that the Kozak (1997) equation can maintain low intercorrelations among independent variables, which is crucial for its predictive ability. However, Zheng et al. (2017) further argued that equations like Kozak (1997) can sometimes suffer from complexity, making them challenging for practical applications by forest managers. Heiðarsson and Pukkala (2011) ranked the Kozak (1997) equation second, although its performance was comparable to that of the Kozak (2004) Model-02. Nevertheless, McTague and Weiskittel (2021) highlighted a common drawback of variable-form taper models, especially those fitted using the conventional least squares method: they can exhibit moderate to high levels of multicollinearity and require numerical integration for volume estimation. These conditions underline the potential benefits of incorporating random effects through mixed-effects modeling, which can improve the accuracy of upper stem diameter estimations and consequently volume.

The mixed-effect approach (NLME-TE) improved the upper stem diameter prediction accuracy by approximately 8% (e.g., pseudo- R^2 : from 92 to 99%), and at least 55% reduction in the root mean square error. This aligns with findings by Lanssanova et al. (2019), who reported a 5.9–11.9% increase in accuracy when incorporating one or two coefficients/parameters as random effects for teak (*Tectona grandis*), over bark. Other studies (Arias-Rodil et al. 2015; Bronisz and Zasada 2020; Li et al. 2021; Zhang et al.

2021; Saud et al. 2024) have also demonstrated the importance of NLME in improving the prediction accuracy of taper equations. Li et al. (2021) reported a relatively smaller improvement in the predictive performance of the Kozak (2004) Model-02 for larch (*Larix olgensis* A. Henry) stands, with a 1.7% increase in R^2 (from 96.7% to 98.4%) using two random parameter combinations. In our study, the best NLME-TE was achieved by incorporating three parameters as random effects: $\beta_6 - \beta_8$ for under bark diameter estimations, and β_5 , β_6 and β_8 for over bark diameter estimations.

Estimating random effects parameters utilizes additional sectional diameter measurements, albeit in small quantities, to enhance feature prediction. However, Bronisz and Zasada (2020) highlighted the practical challenges of obtaining additional data for random effects prediction in NLME models, and suggested using fixed-effects predictions under the assumption that random effects are zero—“fixed-effects of the elaborated mixed-effects taper model”. Our results showed that using the fixed-effect part of the NLME-TE to estimate stem volume had a slight improvement relative to the Schumacher-Hall volume equation. A similar result was also obtained for black locust in western Poland (Bronisz and Zasada 2020). However, when the NLME-TE was calibrated with two to five additional sectional diameter measurements, it achieved high predictive accuracy. The NLME-TE calibrated with two to five sectional datasets brought about a substantial reduction in prediction error of stem volume by an average of 55% (44–66), 49% (37–61) and 48% (35–61) compared to the Kozak (1997) equation fitted with NLS, Schumacher-Hall volume equation, fixed effects of the NLME-TE, respectively (Fig. 3 and Figs. S1–S3). Similar findings have been reported in other studies, which demonstrated that using diameter measurements at a few specific heights serves as a reliable response calibration for mixed-effects taper models for different tree species. For instance, Adamec et al. (2019) found that calibrating stem taper equations with two diameter measurements minimized bias when predicting the merchantable wood volume of Norway spruce (*Picea abies* (L.) H. Karst.) in the Czech Republic. The Kozak (1997) equation, fitted using NLS, provided the least accurate results for estimating the stem volume of eucalyptus. Additionally, calibrating the NLME-TE with a single sectional measurement is not recommended, especially for under-bark volume estimation.

Depending on the number of sections (two or five) a forest manager opts to include in practical scenarios, our findings indicate that the most feasible approach is to sample from the butt end of the tree, beginning with diameter measurements at the basal or stump height (0–0.3 m) and proceeding upward along the stem. Selecting measurements at extreme or smallest points for calibration has been emphasized in previous studies, though primarily in the context

of height-diameter functions (Bronisz and Mehtätalo 2020; Ogana et al. 2023). In contrast, some studies have recommended calibrating mixed-effects taper models with diameter data from the upper or middle portions of the stem. For example, Adamec et al. (2019) suggested using two upper diameter measurements (specifically at 5 and 7 m) for optimal prediction of merchantable stem volume in standing Norway spruce trees. Similarly, Arias-Rodil et al. (2015) and Zhang et al. (2021) observed that including additional section diameters from the middle of the stem (40%–60% and 30%–40% of the total tree height, respectively) resulted in the greatest improvement in volume predictions for maritime pine (*Pinus pinaster* Ait.) in northwest Spain and Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook.) in southern China. Accurately measuring upper and middle diameters in standing trees can be challenging and costly, especially without access to advanced non-destructive remote sensing technologies such as terrestrial laser scanning, ground-penetrating radar, or terrestrial stereoscopic photogrammetry. These tools are often prohibitively expensive for forest managers and come with technical, operational and methodological challenges. Given these limitations, we recommend obtaining and using sectional diameters (at least two) to calibrate the NLME-TE. This practical and accessible approach facilitates the estimation of standing tree diameters and volumes in eucalyptus forests in northeast Nigeria without requiring tree felling.

Conclusion

To meet the urgent demand for precise and locally applicable taper equations to promote efficient forest management, this study developed and calibrated taper equations for *Eucalyptus camaldulensis* in northeastern Nigeria. The results showed that the mixed-effects modeling approach greatly improved the prediction accuracy for estimating the upper stem diameters and volumes up to the 5 cm merchantable limit. With just a few sectional diameter measurements, it is possible to estimate the upper stem diameter and volume with high precision, thereby reducing measurement effort and inventory costs. For optimal accuracy and practical implementation, two sectional diameter measurements taken from the butt end per tree are recommended. The results of this study not only enhance the scientific understanding of taper dynamics in Eucalyptus species but also provide practical applications for forest owners. However, the taper model developed in this study is best suited for Eucalyptus plantations in areas within the region, with similar site and stand conditions. Additional taper data from other plantations in the region, gathered through future studies, will help enhance the model's suitability and improve its broader applicability and accuracy.

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