

On Leaf BRDF Estimates and Their Fit to Microfacet Models

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Abstract—Remote sensing provides high accuracy/precision for quantifying forest biophysical parameters needed for ecological management. Although the significant impact of bidirectional scattering distribution functions (BSDFs) on remote sensing of vegetation is well known, current forest metrics derived from sensor data seldom take leaf BSDF into account, and despite the importance of BSDF effects, leaf directional scattering measurements are almost nonexistent. Previous studies have been limited in the spectral coverage and resolution of observed electromagnetic radiation and lacked models to interpolate all source-sensor angles beyond measurements. This study captured deciduous broadleaf bidirectional reflectance distribution functions (BRDFs) from the visible through shortwave infrared spectral regions (350–2500 nm) and accurately modeled the BRDF for extension to any illumination angle, viewing zenith, or azimuthal angle. We measured biconical directional reflectance factor of leaves from three species of large trees, Norway maple (*Acer platanoides*), American sweetgum (*Liquidambar styraciflua*), and northern red oak (*Quercus rubra*). We then fit the data through nonlinear regression to physical, microfacet BRDF models, resulting in normalized root-mean-square errors of less than 8%, averaged across all wavelengths (excluding low signal-to-noise spectral regions). We extracted leaf physical parameters, including the index of refraction and a relative physical roughness from the microfacet models delineating the three species. The implications for forestry remote sensing are important, as rigorous models to represent leaves allow for the creation of more accurate forest scenes for radiative transfer modeling. Such accuracy enables higher fidelity sensor evaluations and data processing algorithms.

Index Terms—Bidirectional reflectance distribution function (BRDF), bidirectional scattering distribution function (BSDF), goniometer, hyperspectral, leaf optical properties, remote sensing, spectroradiometer.

I. INTRODUCTION

CONSIDERING leaves as anisotropic scatterers versus idealized Lambertian scatterers has implications in physics-based image generation, vegetation remote sensing, and plant physiology studies. Specifically, in computer-based simulations of remote sensing scenes, vegetation canopy models often assume scattering that follows the idealized Lambertian case or that obeys a generic bidirectional scattering distribution function (BSDF) model, i.e., BSDF consisting of the

directional reflectance, bidirectional reflectance distribution function (BRDF), and the directional transmittance, bidirectional transmittance distribution function (BTDF). However, we contend that single-leaf BSDF estimations from laboratory measurements are key to understanding light interactions within vegetation canopies. One important realization is that the BSDF is a theoretical quantity dealing with infinitesimals and cannot be directly measured [1], [2]. Throughout this article, all references to BRDF measurements are in fact estimates from biconical directional reflectance factor (BCRF) measurements. The BCRF as an estimate of the bidirectional reflectance factor (BRF) is related to the BRDF via $BRF = \pi BRDF$.

Access to leaf directional reflectance measurements is limited, with the Institute de Physique du Globe de Paris' [3], [4] being the only known public resource of this type of data, and no known leaf directional transmittance measurements. However, many current remote sensing simulations generally assume that both transmitted and reflected light follow the Lambertian assumption [5], [6]. The accuracy of these assumptions and the effect on simulation results are currently unknown.

Bousquet *et al.* [4], for example, made leaf-level directional reflectance measurements and BRDF estimates for wavelengths 480–880 nm for laurel, European beech, and hazel leaves. The study separated leaf BRDF into diffuse and specular components. Their analysis suggested that the specular component of the BRDF remains largely invariant with wavelength, with some deterioration in the near-infrared. They fit the BRDF model developed by Cook and Torrance [7] to the specular component, whereas for the diffuse component, they simply fit a Lambertian coefficient, controlling the scale of the diffuse radiance. The model was extended to include the PROSPECT model by Jacquemoud and Baret [8], in order to characterize the diffuse BRDF component.

Biliouris *et al.* [9] also made leaf directional reflectance measurements on European beech leaves. As described in [10], they used the compact laboratory spectrogoniometric device (CLab-SpeG) to measure leaf BCRF as an estimate of BRF. BCRF measurements were collected for a total of 60 beech leaves, with a different leaf for each of 12 data collections at five different light source-zenith positions. Each data collection consisted of spectral measurements from 400 to 2500 nm. They inverted the empirical Rahman–Pinty–Verstraete (RPV) model [11] to fit the BCRF at each wavelength and each light source-zenith angle. Because the model was separately fit for each light source measurement angle, it is unclear how to generalize the results

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for those light source angles not measured. Generalizing data to angles beyond those measured is essential because *ray tracing simulations of remote sensing imagery depend critically on accurate characterization of the angular distribution of scattered light*.

Greiner *et al.* [12] is one of the few studies that analyzed the entire leaf BSDF. The authors studied both maple and cottonwood leaves and generalized the BSDF to all illumination angles. Their results showed that larger zenith angles result in an increased specular component. Using a laser wavelength of 1064 nm, they repeated each measurement for four different leaves and averaged these readings in order to minimize the impact of leaf-to-leaf variability. The BSDF components were divided into diffuse and specular components, modeling the diffuse component with a polynomial fit while modeling the reflected specular component with a modified Rayleigh distribution. Measurements showed that the transmission component was largely diffuse and well modeled with a second-order polynomial. Even though their generalized model could address any zenith illumination and viewing angle, the study did not consider azimuthal angles outside of the principal plane (vertical plane containing both the sample and illumination source). The authors mentioned this drawback and suggested an extension beyond the in-plane measurements, which assumes azimuthal symmetry and constrains the specular component. Such an extension would require many assumptions, and strays from an accurate rendition of true leaf optical properties. In addition, all results were normalized by the associated principal plane BSDF measurement, thereby making it difficult to correlate to the actual BSDF.

There have been studies of leaf scattering properties [4], [9], [12]–[16], but as previously mentioned, these have fallen short in addressing the required information for insertion into remote sensing models. Previous studies have been limited in spectral coverage and resolution and lacked models to interpolate to all angles beyond those of actual measurements. We recognize the importance of both leaf BRDF, and BTDF as applicable to remote sensing data, but focused this study on leaf BRDF. Therefore, the objectives of this article are as follows.

- 1) Assess a method to accurately and precisely measure leaf BRDF spectrally, from the visible through shortwave infrared (SWIR) spectral domains.
- 2) Evaluate a BRDF model to extend results to any illumination and viewing zenith or azimuthal angle.
- 3) Assess characteristics of broadleaf (deciduous) BRDF and differences between evaluated species.

In order to accomplish the objectives of this study, we measured hyperspectral directional reflectance optical properties from the visible through SWIR spectral regions for broadleaf trees, specifically Norway maple, American sweetgum, and northern red oak. We assessed specific features observed in the measurements while making comparisons to previously published measurements.

Other important questions addressed include: how valid is the assumption of spectral independence of Bousquet *et al.* [4], and in particular, how constant is the specular component beyond the NIR; how diffuse is leaf reflectance beyond the NIR; and which BRDF model can be generalized for insertion into radiative

transfer models while still providing accurate results? Answering these questions will contribute to the scientific body of the literature by advancing our understanding of light-vegetation interactions.

II. MEASUREMENT METHODS

The goniometer of the Rochester Institute of Technology–Two (GRIT-T) [17] was used to make the leaf biconical reflectance measurements for this study. GRIT-T is a second-generation goniometer, designed and built in the Chester F. Carlson Center for Imaging Science at the Rochester Institute of Technology. The instrument was designed to be field-portable to measure hemispherical-conical reflectance and biconical reflectance in both field and laboratory environments. The system is fully automated, with high precision in pointing, spectral resolution, and signal. A laser range finder allows for “target-plane tracking,” to maintain the observation direction pointing toward the same location on the target to within 10 mm. This is accomplished with three independent degrees of freedom, along an azimuth base ring, through a pointing arm, and by a rotating sensor head. The instrument maintains accuracy within 0.2° in both azimuth and zenith. Moreover, the mechanical design limits shadowing effects. The onboard sensors are Analytical Spectral Devices (ASD), Inc., FieldSpec 4 Hi-Res spectroradiometers, capturing the spectral range between 350 and 2500 nm, with 2151 spectral channels in 1-nm intervals (resampled from 3-nm resolution in the VIS/NIR and 8-nm resolution in the SWIR). GRIT-T has been extensively used to capture a variety of BRDF data both in the field and in the laboratory [18]–[21]. Complete details of the system are outlined in Harms *et al.* [17].

We collected leaf measurements from three species of large eastern USA deciduous trees, namely Norway maple (*Acer platanoides*), American sweetgum (*Liquidambar styraciflua*), and northern red oak (*Quercus rubra*). The leaves were gathered from three trees in August/September 2019 in Rochester, NY, USA. All leaves were taken from the same tree for each leaf type. The three leaf types represent a span of large broadleaf optical properties. For example, to the naked eye, the maple leaves were mat in color and flexible, the sweetgum leaves also were flexible, but with a slightly glossy epidermis layer, and the oak leaves had yet a more glossy epidermis layer, and were more rigid due to a thicker cuticle. The largest leaves were selected, thin sun leaves with darker pigmentation. Although not a true representation of leaves higher in the canopy, these larger leaves were selected in order to reduce error due to field-of-view (FOV) and mechanical pointing of the goniometer. Fig. 1 displays images representing the typical size, shape, and appearance of leaves measured.

Measurements were made at 49 different locations across the hemisphere, every 30° in azimuth, and 15° in zenith, from 0 – 60° . GRIT-T collects measurements in azimuth planes, and nadir measurements were repeated in each azimuth plane to ensure consistency during the collection. We chose the density of measurements to ensure measurement scans were completed in a minimal amount of time while still capturing the light directional structure. Minimizing the measurement scan time (~ 12 min) was a key consideration, as there was notable leaf drying due to heat from the quartz-tungsten-halogen light source.

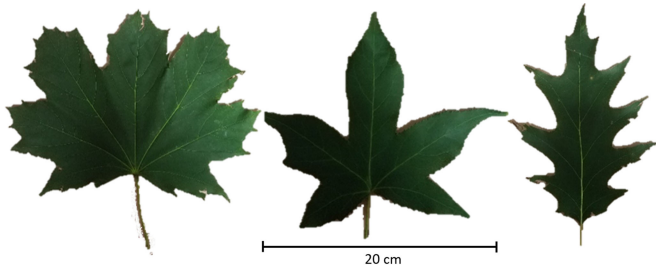


Fig. 1. Typical size, shape, and appearance of the three leaf types measured; from left-to-right: maple, sweetgum, and oak leaf.

All measurements were made approximately 0.5 m from the sample. Data were acquired with a 3° fore-optic, resulting in a 2.6-cm FOV at nadir and a projected 5.2 cm at the 60° zenith angle. A sample holder, consisting of two hinged plates with a 7.6-cm diameter hole, served to flatten the leaf, center the sample, and ensure adequate sample size, with margin to ensure that the target filled the sensor FOV. We used a new leaf for every measurement collection, one for each of the four illumination angles, in an attempt to reduce artifacts from excessive leaf drying [12]. By measuring a new leaf for every illumination angle (four leaves total for each leaf type), we account for some leaf-to-leaf variability even though we did not have enough samples to complete a statistical population analysis.

We measured reflectance at four source elevation angles, specifically zenith angles of 10° , 25° , 40° , and 55° . The GRIT-T laboratory has an overhead 300-W ARRI-300 studio halogen lamp for reflectance measurements, located about 1.5 m from sample placement. The studio lamp has a Fresnel lens that illuminates approximately a square meter on the laboratory table. This light source delivers adequate source directionality, when operated at the 1.5-m distance. We converted reflectance measurements to measured BCRF via the established method used for GRIT-T and other spectroradiometers, i.e., dividing the radiance at a measurement location by the diffuse signal from a white reference, a Spectralon panel [22] and scaling by the calibration coefficient of the Spectralon panel. For measurements, we placed the leaves in the sample holder made of aluminum, coated in flat black “high heat” paint, laid on top of black optical felt, and centered under the sensor at nadir. The desired outcome was to capture the leaf structure in its entirety, including veins, small undulations, and other surface properties. This was feasible with the FOV of the ASD 3° spectrometer fore-optic.

We identified outliers in the data, so these could be eliminated for analysis and future model fitting. This was possible because there were very few outliers, and data collections showed a clear separation when these did exist. Identified outliers belonged to a one interval interquartile range (IQR) below the 25th percentile. The IQR is the difference between the 75th and 25th percentiles, effectively providing a measurement of the data spread [23]. In Fig. 2, we outline the method by which we identified outliers.

III. MEASUREMENT RESULTS

Fig. 3 shows a high-level view of the BCRF measurements with overlaid spectral plots at every GRIT-T measurement

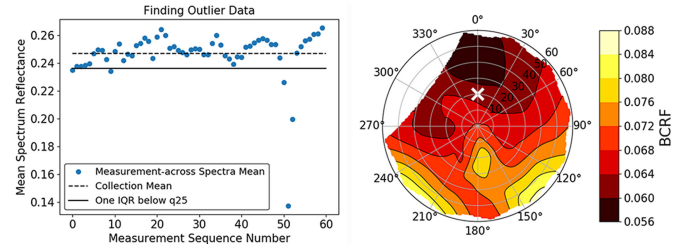


Fig. 2. Example plots for identifying outliers. A scatter plot of the mean signal across all wavelengths for each measurement location (left-hand side); the scatter plot displays the data mean as a dotted line, and the one IQR as a solid line. An interpolated BCRF polar plot from measurements at 550 nm with the identified outliers omitted (right-hand side).

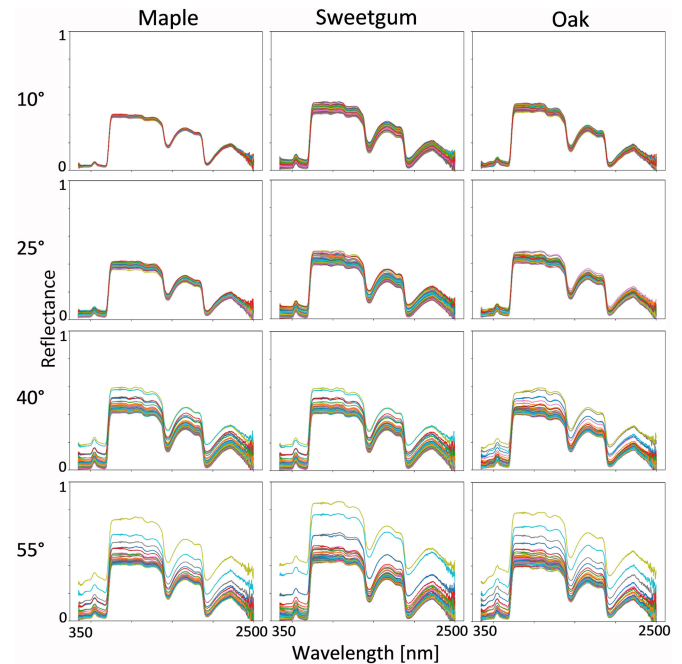


Fig. 3. BCRF spectral plots of each leaf type (maple, sweetgum, and oak; left-to-right) and at each illumination zenith angle (10° , 25° , 40° , and 55° ; top-to-bottom). Each plot contains a spectral reflectance curve (350–2500 nm) for each of the 49 measurement locations over the hemisphere collected by GRIT-T.

location, for each leaf type and illumination angle. Although the spatial structure of the BCRF at individual wavelengths cannot be deciphered from these plots, it does present an overall idea of the spectral shape, and magnitude of the specular component. The maple leaf is obviously the most diffuse of the three leaf types, with the smallest vertical separation in the spectral lines. From these plots, the sweetgum and oak leaves look fairly similar, with the sweetgum displaying a slightly larger specular component.

The reflectance curves in Fig. 3 exhibit some obvious noise at both extreme ends of the measurement, which is expected due to the detector quantum efficiencies. The number of data samples at each measurement location is configurable within the GRIT-T system, which are then averaged for a final measurement. A modest data sample of 60 was chosen due to leaf drying concerns, although a greater sample number would have resulted in less noise. A signal-to-noise analysis on the computed

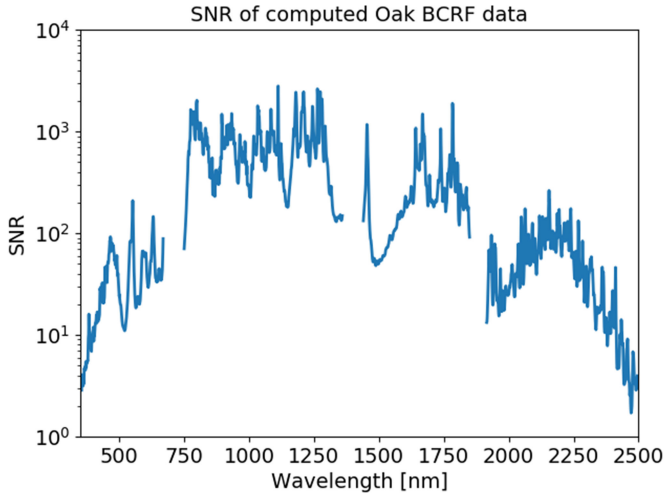


Fig. 4. Signal-to-noise computed for the sweetgum BCRF data point measurement with 55° illumination, opposite the specular peak on the principal plane (0° azimuth and 60° zenith). Lowest signal-to-noise occurs at both extremes of the spectral measurement.

BCRF was completed by using a 10-nm moving window, from which both the mean and standard deviation were computed. The assumption here is that the signal should be constant over this 10-nm window. This is a solid assumption, except for parts of the spectrum that exhibit extreme slopes due to the expected structure of vegetation spectra. The red edge (670–750 nm) and water absorption features (1360–1440 nm; 850–1915 nm) were excluded. We computed the standard deviation via (1), and the signal-to-noise is then the mean divided by the standard deviation $\frac{\bar{S}}{\sigma_s}$

$$\sigma_s = \sqrt{\sum_{i=1}^N \frac{[S_i - \bar{S}]^2}{(N-1)}} \quad (1)$$

where σ_s is the standard deviation, N are the number of data points, S_i is the i th data point, and $\bar{S}$ is the data mean. We completed the analysis on the expected lowest signal-level data points, namely on the principal plane opposite the specular peak for the greatest illumination angle. Fig. 4 includes the results of this analysis, which clearly shows that the ranges below 500 nm and above 2300 nm have the lowest signal to noise. Analysis of a number of different spectra yielded similar results for all three leaf types.

The first step taken in analyzing the accuracy of the leaf measurements was to compare the directional-hemispherical reflectance (DHR; total reflected radiance divided by directional incidence radiance) to previously published results. We calculated the DHR according to Schaepman-Strub *et al.* [2], by integrating the estimated BRDF over the hemisphere, where $\text{BRDF} = \frac{\text{BRF}}{\pi}$

$$\text{DHR}(\theta_s, \phi_s, \lambda) = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} f_r(\theta_s, \phi_s; \theta_r, \phi_r) \cos(\theta_r) \sin(\theta_r) d\theta_r d\phi_r \quad (2)$$

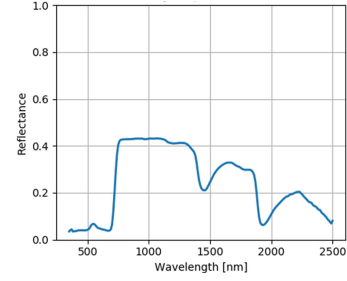


Fig. 5. Calculated DHR from GRIT-T measurements for Norway maple with the source at 20° zenith. The DHR results are very similar and can be compared to reflectance values previously published for Norway maple leaves by Hovi *et al.* [24].

where $f_r(\theta_s, \phi_s; \theta_r, \phi_r)$ is the BRDF, θ_s and ϕ_s are the illumination source zenith and azimuth angles, respectively, whereas θ_r and ϕ_r are the received zenith and azimuth angles, respectively. We created a grid of cubic interpolated points (polar-projected in azimuth and zenith) in order to obtain the required values for integration and used nearest neighbor interpolation to extend values beyond those measured. This method yielded comparable results to previously published measurements of the estimated BRDF [4], [8], [15], [24], as seen in Fig. 5.

The water absorption bands centered near 1400 and 1900 nm are less pronounced than those published by Hovi *et al.* [24], due to leaves being picked later in the season and the measurement method. The measurements take about 12 min, and there is noticeable leaf drying during the measurement process, whereas previously published integrating sphere measurements can be made in a fraction of this time.

A qualitative comparison of measurements, against those published in the OPTICLEAF database [3], is warranted to evaluate the consistency in leaf measurements using different instruments. The comparison is limited, however, since none of the leaf species in this study are found in that database. In addition, the database only spans the spectrum between 400–880 nm, whereas this study spanned the spectrum from 350–2500 nm. During the investigation, we determined that measurements in the database from 2003 are most likely BRDF, instead of BCRF as labeled, exhibiting values about three times lower than expected. We chose two of the leaf species that encompass the range of measurements found in this study, namely walnut (*Juglans regia*) and cherry laurel (*Prunus laurocerasus*) for qualitative comparison, as seen in Figs. 6 and 7.

In general, the BCRF patterns and amplitudes from our measurements are similar to those contained in the OPTICLEAF database. Variations in the BCRF pattern were attributed to the relatively large sensor FOV of GRIT-T, which captured various leaf structures, whereas the OPTICLEAF database was created with the spectrophoto-goniometer from the Institut National de la Recherche Agronomique, which has a viewing area of 6 mm², focused on areas free of major veins on the leaf [4]. Similar BCRF patterns between the two measurement campaigns include a generally Lambertian structure outside of the specular peak in the NIR, specular broadening in the NIR, and a greater forward scatter specular angle for leaves that exhibit

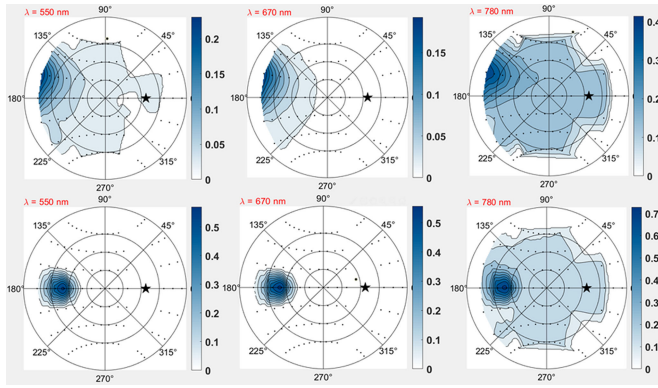


Fig. 6. Leaf BCRF data from the OPTICLEAF database [3]. The top row shows data for a walnut (*Juglans regia*) leaf. The bottom row represents data for a cherry laurel (*Prunus laurocerasus*) leaf. All the plots displayed are for an illumination source at a zenith angle of 45° for wavelengths of 550, 670, and 780 nm, from left-to-right. The black star represents the illumination direction, whereas the black dots are measurement locations. The axial rings are in 20° increments spanning to 90°.

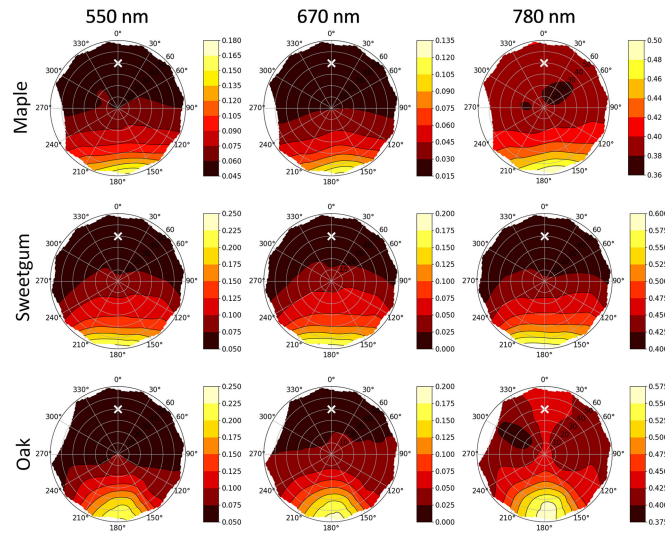


Fig. 7. Leaf BCRF data from GRIT-T measurements. The top row displays data collected from the maple, middle row from the sweetgum, and bottom row from the oak leaf. All the plots displayed are for an illumination source at 40° and at 550, 670, and 780 nm wavelengths, from left-to-right. The white X represents the illumination direction and blank areas are due to either no measurements, or where outliers were eliminated. The axial rings are in 10° increments spanning to 60°.

more diffuse characteristics. Bousquet *et al.* [4] noted all of these features. Likewise, Walter-Shea *et al.* [16] and Wooley [15] recognized, respectively, the dominance of the localized specular peak in high chlorophyll absorption wavelengths and the greater forward scatter angle for rougher leaf surfaces. In both measurement campaigns (this study and the OPTICLEAF database), the specular components at 550 and 670 nm are nearly identical, with the peak broadening in the NIR at 780 nm. The OPTICLEAF plots exhibit no backscatter, and similarly, GRIT-T measurements also show little backscatter. At 780 nm, the GRIT-T measurements agree with an increase in the diffuse reflectance; however, there is some structure beginning to form,

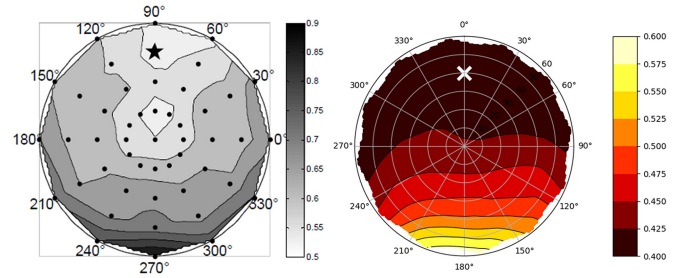


Fig. 8. Left plot is the BCRF of a beech leaf (*Fagus sylvatica* L.), measured by Biliouris *et al.* [9]. The light source is at 45°, as depicted by the black star, and measurement locations are marked by black dots at 15° zenith and 30° azimuth steps. The right plot shows the GRIT-T-measured BCRF of the sweetgum leaf, with the illumination source at 40°, marked by the white X. Note: Both plots are for 850 nm (Figure adapted with permission of the author, copyright holder).

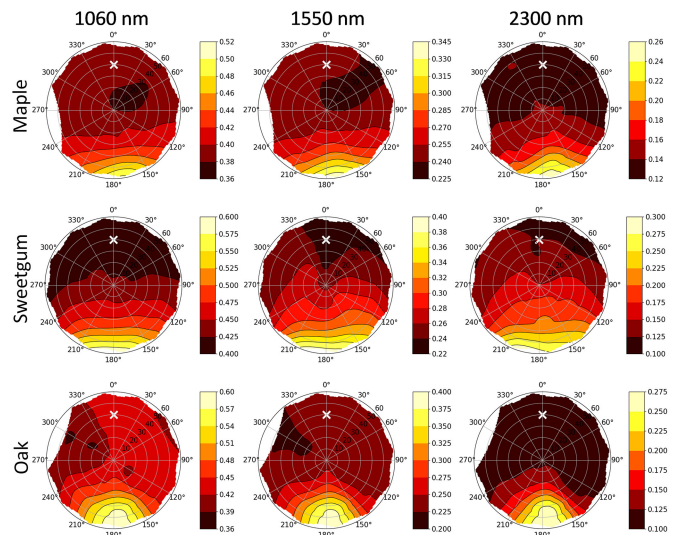


Fig. 9. Top row displays data collected from the maple, middle row from the sweetgum, and bottom row from the oak leaf in three of the SWIR bands. All the plots displayed are for an illumination source at zenith angle 40° and at wavelengths 1060, 1550, and 2300 nm, from left-to-right. The white X represents the illumination direction and blank areas are due to either no measurements, or where outliers were eliminated. The axial rings are in 10° increments spanning to 60°. Note that the scales for each plot are different to highlight the BCRF structure.

which may be due to increased backscatter compared to the other wavelengths shown in Fig. 7.

The only SWIR leaf directional scattering measurements known to the authors were performed by Biliouris *et al.* [9], whereas the only published BCRF plot is of a beech leaf at 850 nm in the NIR, with a 45° illumination source, as seen in Fig. 8. The closest leaf type to the beech leaf in this study arguably is the sweetgum leaf. A comparison of the two BCRF plots in Fig. 8 shows some stark contrasts. The BCRF measured by Biliouris *et al.* [9] shows divergent contours away from the specular peak, and a much higher amplitude for this peak. The difference in the two patterns possibly are due to leaf type, instrumentation, or collection method; more research is needed to draw a definitive conclusion. However, further investigation into measurements in the SWIR bands, as seen in Fig. 9, displays an overall consistency in the BCRF patterns. Note that the patterns

seen in Fig. 9 are very similar to those seen in the NIR. The consistency of the BCRF patterns from NIR to SWIR regions suggests that the observations and assumptions of the stable specular peak, with slight deterioration in the NIR observed by Bousquet *et al.* [4], continue into the SWIR. Also note that the BCRF structure is relatively constant for each wavelength shown in the SWIR, but the underlying amplitude fluctuations depend on the wavelength.

IV. MODELING METHODS

BRDF models to fit the data were necessary in order to generalize reflective properties for all illumination and observation angles. A model allows for further analysis of the leaves' scattering properties and provides insight into physical properties. To this end, we investigated both empirically and physically based models, including the RPV model [11], the canopy vegetation model known as the Ross-Li [25]–[27], the Cook and Torrance microfacet model [7], [28], and the Smith GGX microfacet model [29]–[31]. When attempting to fit the measured data to the RPV and Ross-Li models, no reasonable solutions resulted. Although the GRIT-T measurements could be fit to the RPV model, extrapolating to illumination and view angles beyond the measurements proved to be overly problematic. We found the Ross-Li model did not adequately describe individual facets. Ross-Li works well for high shadowing scenarios in which the retroreflectance “hot spot” effect is the prominent feature, specifically in the case of passive imagery of a forest canopy [25]–[27]. The “hot spot” occurs where the illumination and view directions are close in angle, resulting in limited shadowing for a highly variable scene. The Cook and Torrance model fit the BCRF data with reasonable accuracy and generalized well beyond measurements. However, a major limitation of many of these models was their inability to characterize transmission measurements. The Smith GGX model is the only one of those investigated that has been applied to transmission, which is an objective for future work. Moreover, the Smith GGX model, unlike the Cook and Torrance approach, accounts for the projected area of the individual microfacets, thereby resulting in improved accuracy [32]. Therefore, we used the Smith GGX BRDF model, outlined by Heitz and Walter [30]–[32], for the specular component in this study. The associated symbols for the model development are shown in Table I.

The leaf measurements were fit to BRDF models consisting of a summation of a specular and diffuse component, previously used by Bousquet *et al.* [4]

$$\rho(\omega_o, \omega_i) = \rho_{\text{diff}}(\omega_o, \omega_i) + \rho_{\text{spec}}(\omega_o, \omega_i). \quad (3)$$

The diffuse component then is constant over all incident and observation vectors and is defined as

$$\rho_{\text{diff}}(\omega_o, \omega_i) = \frac{k_L}{\pi} \quad (4)$$

with k_L as the Lambertian coefficient. Even though not explicitly shown in the equations here, the BRDF is also a function of wavelength. As the BRDF models in these references can be modified for specific applications, we present the specific formulation that we used for clarity. The master BRDF formula used

TABLE I
TABLE OF SYMBOLS

Symbol	Description
θ_i	illumination source zenith angle
θ_o	received zenith angle
ϕ_o	received azimuth angle
$\rho(\omega_o, \omega_i)$	BRDF
$\rho_{\text{diff}}(\omega_o, \omega_i)$	diffuse component of BRDF
$\rho_{\text{spec}}(\omega_o, \omega_i)$	specular component of BRDF
k_L	Lambertian coefficient
ω_o	view vector
ω_i	illumination vector
$F(\omega_o, \omega_h)$	Fresnel term
$G_2(\omega_o, \omega_i, \omega_m)$	masking-shadowing term
$D(\omega_m)$	microfacet distribution term
ω_g	macro surface normal
ω_h	half-vector, vector halfway between the incident and outgoing vector
ω_m	micro-surface normal
$\Lambda(\omega_i), \Lambda(\omega_o)$	masking-shadowing associated function, also known as Smith Lambda function
$a(\omega_o)$	masking-shadowing associated function, weighted slope of the outgoing direction
α	Roughness parameter
X^+	step function, equaling one when greater than zero and zero otherwise
g	Fresnel associated function
c	Fresnel associated function
n_i	real part of index of refraction of incoming medium (air)
n_t	real part of index of refraction of incident medium (leaf)

and commonly implemented in computer graphics applications is

$$\rho_{\text{spec}}(\omega_o, \omega_i) = \frac{F(\omega_i, \omega_h) G_2(\omega_o, \omega_i, \omega_m) D(\omega_m)}{4 |\omega_g \cdot \omega_o| |\omega_g \cdot \omega_i|}. \quad (5)$$

The BRDF, $\rho_{\text{spec}}(\omega_o, \omega_i)$, is a function of the observation or view vector ω_o and the illumination vector ω_i . The equation consists of the Fresnel term $F(\omega_i, \omega_h)$, the masking-shadowing term $G_2(\omega_o, \omega_i, \omega_m)$, and the microfacet distribution term $D(\omega_m)$. The terms in the denominator are a result of the Jacobian that relates the micro- to macrosurface and the fundamental constraint that the projected area of visible microsurfaces is equal to the projected area of the geometric surface, as explained by Heitz [32]. The remaining vectors appearing in the formula are: ω_g , which is the macrosurface normal, ω_h , which is the half-vector (vector halfway between the incident and outgoing vector), and ω_m , which is the microsurface normal. When using Fresnel facets, $\omega_h = \omega_m$, but the distinction between the two is sometimes useful in context. For instance, we use ω_h when referring to the vector halfway between the incident and outgoing vector, and ω_m when referring to the microsurface normal vectors.

Because the illumination azimuth angle for the purposes of model fitting is always zero, the incident vector (vector pointing toward the source) is

$$\omega_i = \begin{bmatrix} \sin \theta_i \\ 0 \\ \cos \theta_i \end{bmatrix} \quad (6)$$

with θ_i as the illumination zenith angle. The outgoing vector (vector pointing toward the receiver) then is

$$\omega_o = \begin{bmatrix} \sin \theta_o \cos \phi_o \\ \sin \theta_o \sin \phi_o \\ \cos \theta_o \end{bmatrix} \quad (7)$$

with ϕ_o being the azimuth observation angle and θ_o being the zenith observation angle. θ_o and ϕ_o parameterized the vectors over the hemisphere. This process created an array of size $[3, N_{\phi_o}, M_{\theta_o}]$, with N_{ϕ_o} being the number of ϕ_o samples and M_{θ_o} being the number of θ_o samples. To simplify mathematical operations, the size of the vector ω_i was the same size as the ω_o array. To determine the half-vector, which is also the normal to the microfacets that reflect the light, the model equates ω_m and ω_h according to

$$\omega_m = \omega_h = \frac{\omega_o + \omega_i}{\|\omega_o + \omega_i\|}. \quad (8)$$

The Smith shadowing and masking function described by Heitz *et al.* [31] takes the form

$$G_2 = \frac{1}{1 + \Lambda(\omega_i) + \Lambda(\omega_o)} \quad (9)$$

where

$$\Lambda(\omega_o) = \frac{-1 + \sqrt{1 + \frac{1}{a(\omega_o)^2}}}{2} \quad (10)$$

$$a(\omega_o) = \frac{1}{\alpha \tan \theta_o} \quad (11)$$

where α is the roughness, one of the fundamental fitting parameters. The subscript “2” of the Smith masking function G_2 identifies it as a function incorporating both the shadowing of nonilluminated microfacets and masking of nonvisible microfacets. The GGX distribution of microfacets obeys

$$D = \frac{X^+(\omega_m \cdot \omega_g)}{\pi \alpha^2 \cos^4 \theta_m \left(1 + \frac{\tan^2 \theta_m}{\alpha^2}\right)^2}. \quad (12)$$

χ^+ is a step function equaling one when $\omega_m \cdot \omega_g$ is greater than zero and zero otherwise. Note that $\omega_m \cdot \omega_g = \cos \theta_m$, which is the z component of the ω_m unit vector. As shown in Walter *et al.* [30], the Fresnel factor has the form

$$F(\omega_i, \omega_m) = \frac{1}{2} \frac{(g - c)^2}{(g + c)^2} \left(1 + \frac{(c(g + c) - 1)^2}{(c(g - c) + 1)^2}\right) \quad (13)$$

$$g = \sqrt{\frac{n_t^2}{n_i^2} - 1} + c^2 \quad (14)$$

$$c = |\omega_i \cdot \omega_m| \quad (15)$$

where the variables g and c are used to simplify the Fresnel equation. Note that for air, $n_i \approx 1$. From the aforementioned equations, the fundamental fitting parameters that determine the BRDF are the roughness parameter α , index of refraction n_t , and the Lambert coefficient k_L .

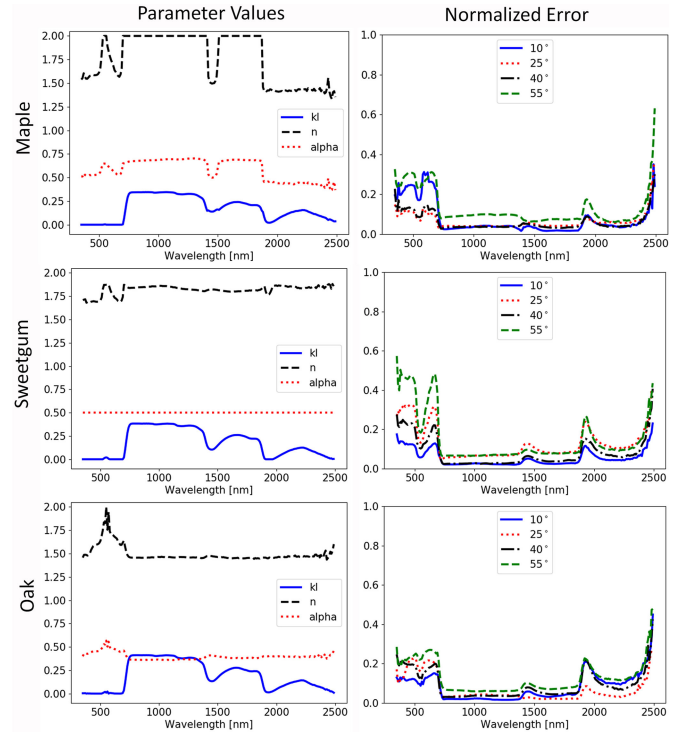


Fig. 10. Left plots are the fit parameters found from least squares minimization, whereas the right plots show the RMS normalized error. The three rows show results for the three leaf types: maple, sweetgum, and oak, from top-to-bottom.

V. MODELING RESULTS

Using the Levenberg–Marquardt nonlinear least squares minimization, we fit the BRDF models to our data with the python package LMFIT [33]. The first step in the fitting procedure was to concatenate all illumination direction measurements for each leaf type. For the same illumination angles, we also concatenated model values at the equivalent locations, followed by least squares minimization according to

$$\Psi^2(\lambda) = \sum_{\theta_i, \theta_o, \phi_o} (\rho(\omega_o, \omega_i)_{\text{measured}} - \rho(\omega_o, \omega_i)_{\text{modeled}})^2. \quad (16)$$

The plots in Fig. 10 display the BRDF parameters and normalized RMS error (NRMSE) that resulted from the minimization. We calculated the NRMSE according to

$$\text{NRMSE} = \frac{\left(\frac{1}{n} \sum_{i=1}^n \sqrt{(y_i - \hat{y}_i)^2}\right)}{\bar{y}} \quad (17)$$

where n is the total number of data points, y_i are the measurements, $\hat{y}_i$ are the modeled data points, and $\bar{y}$ is the mean of the measurements. The NRMSE then is the mean of the squared difference, divided by the mean of the measurements. The inversion parameters are listed in Table II. Slight variations in these initial values led to little change in the minimization result. Moreover, a multiregression algorithm that seeks stability over the wavelengths also resulted in almost identical final values. In this latter approach, if the values from the previous wavelength regression

TABLE II
INITIAL PARAMETER VALUES FOR INVERSION

	k_L	n_t	α
Initial value	0.3	1.5	0.5
Lower bound	0	1	0.2
Upper bound	1	2	0.8

differed by a set threshold, the algorithm would regress multiple times, starting from different initial values. While related to leaf biophysical properties, the resulting parameters from the minimization do not represent absolute physical measurements of these properties, partly due to correlations between variables in the model. Further research is needed to link microfacet model parameters directly to leaf physiology.

In general, leaves that are more diffusely scattering tend to have a higher index of refraction and a higher roughness parameter. An increase in the roughness parameter corresponds to a decrease in the specular component at large illumination zenith angles, whereas increasing the index of refraction increases the specular component at small illumination incident angles [4]. The diffuse component follows the expected reflectance of vegetation. Oak has the most defined specular component of the three leaves, with an index of refraction at about 1.5 and roughness at 0.4. The spike in NRMSE in the visible region, associated with roughness and index of refraction, is likely due to the regressions having difficulty in locating a minimum, when the prominent component is the specular portion. Regression for the sweetgum leaf data resulted in a constant roughness parameter and slight variations in the index of refraction. The index of refraction and roughness are noticeably higher than the oak leaf, due to the more forward scattering nature of the leaf. The maple leaves are the most diffuse of the three leaf types, resulting in the highest index of refraction and roughness values obtained during the minimization. Moreover, these values are seen to be greater at wavelengths that exhibit a higher diffuse component. We found that the model would not converge for the maple leaf unless the index of refraction was constrained. To minimize inaccuracy and improve convergence, we constrained the index of refraction by setting a maximum value. The resulting NRMSE corroborates the choice of this parameter constraint. The highly diffuse nature of the maple leaf over the measured wavelengths likely contributed to this limiting behavior of the optimization; therefore, the specular component of the solution did not contribute significantly to the result. The NRMSE is still quite low for the maple leaf, since the diffuse portion of the leaves is easier to fit than the specular component. Overall, the NRMSE may appear somewhat high for the leaves, but this, in part, reflects the fact that the regression model used data from four different leaves of the same species, one for each illumination angle. In an attempt to reduce the impacts of leaf drying, we measured BRDF for each leaf type using a different leaf at each illumination angle. A large portion of the NRMSE therefore likely stems from the differences between the four leaves of the same species. Another observation is that the error is larger for wavelengths with a smaller diffuse component. We

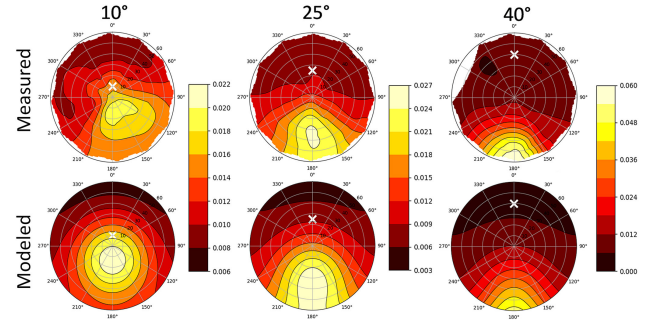


Fig. 11. BRDF of the oak leaf at the 670-nm wavelength, estimated from measurements (top row) and regression modeling results (bottom row) at 10°, 25°, and 40° illumination angles, from left-to-right. The white X denotes the illumination angle position.

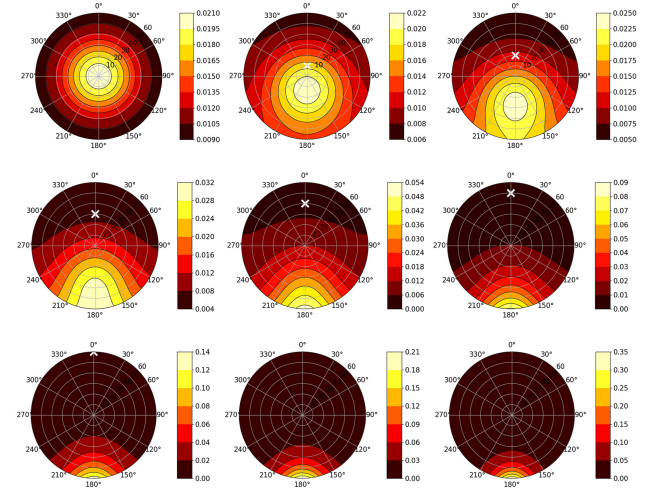


Fig. 12. BRDF model results for the oak leaf at the 670-nm wavelength at 10° illumination angle increments, starting at an illumination zenith angle of 0° in the top-left corner to 80° in the bottom-right corner. The model yields a reasonable BRDF pattern, showing a forward scattering lobe that increases with increasing illumination zenith angle. Moreover, the BRDF model captures patterns noted by Bousquet *et al.* [4], i.e., a generally Lambertian structure outside of the specular peak, and a greater forward scatter angle for leaves that exhibit more diffuse characteristics.

expect this increased error, due to natural asymmetries in leaf components, since the measured specular component may be slightly off-center axis in these cases. The error then stems from the fact that the model can only place a symmetrical specular component on the primary axis. The high NRMSE values, less than 450 nm and greater than 2300 nm, likely stem from noise, due to lower responsivity of the silicon and InGaAs photodiodes in the FieldSpec 4 spectroradiometers at ranges beyond these wavelengths [34]. Fig. 11 shows an example of a fit from the model regression. The model generalizes well to illumination angles beyond those measured, as seen in Fig. 12.

We computed the DHR, according to (2), using only the modeled specular component, $\rho_{\text{spec}}(\omega_o, \omega_i)$, in order to complete a specular fraction analysis of the modeled results. Then, we calculated the specular fraction (fractional contribution of

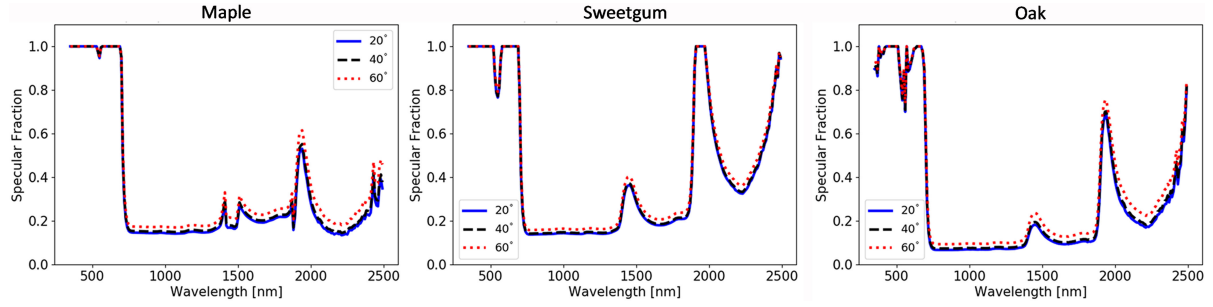


Fig. 13. Left to right: Specular fraction plots of maple, sweetgum, and oak leaves for illumination zenith angles of 20°, 40°, and 60°. Note that the higher illumination angles present slightly higher specular fractions.

specular reflectance to total DHR) according to

$$\text{Specular Fraction} = \frac{\text{DHR}_{\text{spec}}}{\text{DHR}_{\text{spec}} + k_l} \quad (18)$$

where DHR_{spec} is the DHR of the specular component, and $k_l = \pi \rho_{\text{diff}}(\omega_o, \omega_i)$ is the diffuse parameter found through regression corresponding to the diffuse DHR component. Note that the diffuse term found in the regression may not capture the diffuse DHR in its entirety, mainly due to correlations with the roughness and index of refraction parameters. Although the absolute specular fraction is not captured, the wavelengths in which the specular component dominates are revealed. Fig. 13 displays the specular fractions of the three leaf types for three illumination angles, 20°, 40°, and 60°. The plots clearly show that the specular component dominates in the visible, as well as having a significant contribution to some of the water absorption bands, where the diffuse component is minimized.

Besides the fraction of the specular component, we can also study the constancy of the specular component. As seen in our previous plots, and as suggested by Bousquet *et al.* [4], modeling leaves with a constant specular component across wavelengths may be a reasonable assumption. In order to assess the increased error of this assumption, we fixed the specular component across wavelengths, using the index of refraction and roughness parameters found at 670 nm, where there is almost no diffuse component due to chlorophyll absorption. We then optimized the model, allowing only the diffuse component to vary, with the specular component being held constant across wavelength. We calculated the NRMSE for the measurement paradigm according to the method previously discussed in this section. We then determined the NRMSE difference due to the fixed specular assumption by subtracting the mean NRMSE (across illumination angles) at each wavelength when all three parameters were allowed to fluctuate from the mean NRMSE at each wavelength holding the specular component constant. The NRMSE difference therefore indicates the increased error when making the constant specular assumption. We plot the mean difference for the three leaf types in Fig. 14. The oak leaf had the greatest increased error of 0.23 at 380 nm, though the vertical axis in the figure was limited for better visualization. Between approximately 750 and 1800 nm, the NRMSE difference is near zero for all three species indicating the validity of this assumption within this wavelength range. Except for the maple

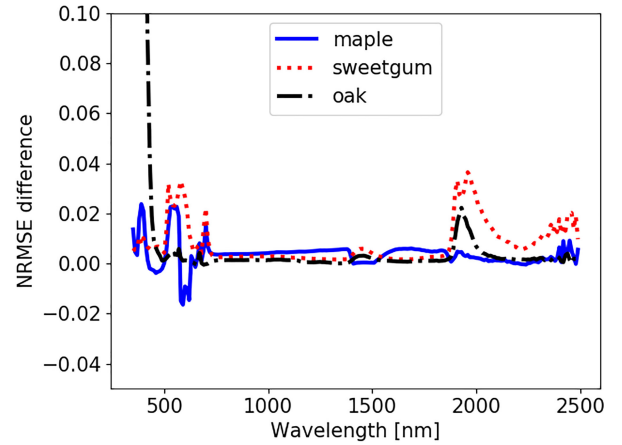


Fig. 14. Mean NRMSE difference between a fixed specular component model and a model in which all three parameters fluctuate.

leaf at a couple narrow spectral regions (between 450 and 500 nm and 580 and 620 nm) the mean NRMSE for the constant specular component was greater for all wavelengths and all three leaf types. The negative NRMSE difference for the maple leaf at a few spectral bands was partially attributed to local minima despite precautions taken as previously discussed. The highly diffuse nature of the maple leaf, which is not perfectly modeled, may increase difficulties in convergence due to local minima. A few isolated spectral ranges show substantial increased error due to the assumption, for example, below 450 nm for all three leaf types, between 500 and 600 nm for the sweetgum and oak, between 1800 and 2000 nm for the sweetgum and oak, and above 2300 nm for the sweetgum leaf. In general, assuming a constant specular component across wavelengths leads to increased error, at least in some wavelength ranges, and depending upon the leaf type and the wavelength, this error can be significant. We calculated the mean increased NRMSE across all wavelengths and found an increase of 0.4% for the maple leaf and 0.8% for both the sweetgum and oak leaves.

In an attempt to capture the overall accuracy of the Smith GGX modeled results, we computed a mean NRMSE from 450–2300 nm to exclude low signal-to-noise spectra, as seen in Fig. 4. Using the Cook and Torrance microfacet model, as presented by Bousquet *et al.* [4], we also fit the leaf measurements and computed the mean NRMSE in the same manner.

TABLE III
MEAN NRMSE VALUES FOR THE THREE LEAF TYPES AND TWO MICROFACET
MODEL METHODS

	Smith GGX	Cook and Torrance
Maple	.07	.085
Sweetgum	.087	.095
Oak	.078	.093
Total Mean	.079	.091

The Smith GGX model consistently resulted in lower NRMSE, as seen in Table III. The increased accuracy of the Smith GGX model resulted from its accounting for the projected area of the individual microfacets, thereby resulting in improved accuracy. This increased accuracy is most noticeable for more diffuse materials, which are more accurately described in the physical model of the shadowing function. The Smith GGX model also produces realistic backscattering for very rough surfaces, as demonstrated by Heitz [32].

VI. CONCLUSION

We measured leaf directional reflectance for Norway maple (*Acer platanoides*), American sweetgum (*Liquidambar styraciflua*), and Northern red oak (*Quercus rubra*). The Smith GGX microfacet model yielded a highly accurate fit to these measurements, with a mean NRMSE of 7.91%, as compared to the Cook and Torrance model, which produced a mean NRMSE of 9.1% (NRMSE mean between 450 and 2300 nm). The models allow the generation of BRDF for any illumination, view zenith, and view azimuth angle for insertion into remote sensing radiative transfer models. We also extracted relative leaf physical parameters of index of refraction, physical roughness, and the diffuse component for the three species. Generalized across the spectra, the oak leaf regression resulted in the lowest roughness and index of refraction, followed by the sweetgum, whereas the maple leaf resulted in the highest roughness and index of refraction parameters. We based our analysis on a separable BRDF model, which incorporated a sum of diffuse and specular components. Using a specular fraction analysis, we determined the ratio of the specular reflectance compared to the total reflectance from the leaves. Overall, the specular component dominates in the visible part of the spectrum and also contributes significantly in some of the water absorption bands, where the diffuse component contributes minimally. We also evaluated the accuracy of assuming a constant specular component for all wavelengths. In general, the assumption introduces slightly more error from 750–1800 nm, but can introduce significant error in a few spectral bands outside of this region, depending on the wavelength and leaf type. Future work will determine the role that the specular component plays in the signal seen by remote sensing systems. To improve leaf-level models, future work also may evaluate the link between the regression parameters associated with leaf measurements and leaf physiology. Finally, improvements to statistical analysis of leaf variability and error analysis will require the creation of larger data sets of measurements of the same leaf type using

the same measurement method. In conclusion, contributions of this work to the scientific body include novel high spectral resolution leaf BCRF measurements from the visible through SWIR, revealing differentiating features in the BRDF between leaf species, an analysis of specular fraction contributions at different wavelengths, and a highly accurate method to model leaf BRDF for insertion into radiative transfer models for future remote sensing studies. The increased understanding gained in this study of light-vegetation interactions will certainly open new opportunities to further develop canopy-level BRDF models, increase accuracy of remote sensing simulations, and further our understanding of vegetation remote sensing data.

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