

Quantitative Validation of Fibonacci Patterning in Pineapple Phyllotaxis

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Abstract

This study provides a quantitative, reproducible analysis of the Fibonacci structures alongside the Golden ratio within *Ananas Comosus* (Pineapple) using calibrated ImageJ measurements and a Python program alongside it. Three mature pineapples of different sizes were photographed under controlled conditions, and their interscale distances alongside the number of visible spirals have been calculated. We then compared these measurements to the theoretical expectations derived from the golden ratio and evaluated the proximity of both the Fibonacci and Lucas sequences. Our results support the hypothesis that pineapple scale arrangements tend towards Fibonacci structures, with occasional anomalies toward Lucas numbers. We discuss the biological mechanisms that form these regularities, and we outline the limitations of our methodology for further work.

Keywords: Fibonacci number, Lucas numbers, golden ratio, phyllotaxis, ImageJ, divergence angle

1.0 Introduction

The Fibonacci sequence, attributed to the Italian mathematician Leonardo of Pisa - commonly known as Fibonacci - continues to intrigue scholars with its elegant and surprising presence in natural systems[1]. This numerical sequence emerges from a rule where each term is produced by summing the two preceding ones, forming a pattern that begins with two initial values, defined by the recursive structure $F_n = F_{n-1} + F_{n-2}$, beginning with the initial conditions of $F_0 = 0$ and $F_1 = 1$, has been the subject of extensive mathematical research due to its ubiquitous appearance within nature and connection to the algorithmic pattern that living organisms grow. Interestingly, the well-known Lucas sequence is a specific instance of this more generalized form.

The Fibonacci sequence has been widely adopted in various fields of science and technology. In computer engineering, it plays a role in optimizing algorithms, especially in search techniques and recursion. In the world of design and architecture, the sequence appears in structures guided by balance and proportion. Furthermore, the Fibonacci numbers form the foundation of the golden ratio, a value that has influenced both classical and modern art forms. Within the human body, patterns reflecting Fibonacci values appear - such as the spacing of joints or the proportions of the face. In nature, the arrangement of leaves, the spirals of shells, and the patterns of sunflower seeds also echo this sequence[2]. Music also benefits from Fibonacci-inspired structuring.

In living systems, ϕ often controls the geometry of growth. The spiral arrangement of sunflower seeds follows the golden angle - approximately 137.5° - which results from dividing a circle according to the golden ratio. This angle ensures that each seed is positioned to fill space without overlap, achieving maximal packing efficiency[3]. DNA spirals with structural ratios closely aligned to ϕ , hinting at the evidence of the extent of how far human beings have evolved, alongside the species surrounding us. One of the most compelling areas in which Fibonacci numbers

appear in living systems is phyllotaxis: the arrangement of leaves, seeds, and petals in botanical fields[4]. The spiral patterns seen in sunflowers, pinecones, and pineapples are a precise manifestation of Fibonacci ratios rather than random decorations of nature[5]. These arrangements optimize exposure to sunlight, increase pollination efficiency, and maximize packing density; however, this spatial engineering is a feat achieved over millions of years. A variety of similar patterns emerge in nautilus shells, the unfurling of fern fronds, and the arrangements of scales on pineapples[6].

Mathematically, the Fibonacci sequence is closely related to the golden ratio, denoted by ϕ . In order to derive this ratio, we can also find the positive root of

$$x^2 - x - 1 = 0$$

The golden ratio ($\phi \approx 1.618$) is the limiting value of the ratio between consecutive Fibonacci numbers as $n \rightarrow \infty$. This constant plays a crucial role in systems where growth follows Fibonacci patterns, often occurring from spatial optimization and natural selection pressures, but why?

In pineapples specifically, the diamond-shaped pattern on the skin is made up of three distinct sets of spirals: one curving to the left, one to the right, and one running vertically. If counted, each set corresponds to a Fibonacci number. This arrangement allows each scale of the pineapple to grow without overlapping the ones beneath it, maximizing space efficiency and structural stability as the fruit develops. This article presents a rigorous quantitative investigation into the presence of the Fibonacci sequence within the phyllotactic patterns of a pineapple (*Ananas comosus*), a canonical example of mathematical patterning in botanical growth[7]. While this phenomenon is frequently cited in popular science, it is often presented anecdotally without robust empirical validation. To objectively test this classic paradigm, our methodology employs a dual-phase analytical approach. First, high-resolution image-based spatial measurement is conducted using ImageJ to extract precise coordinate data from pineapple specimens, capturing the

arrangement of their scales (spikes). Second, this empirical data is subjected to rigorous numerical analysis through a custom Python script. This program is specifically designed to calculate key parastichy ratios and spiral counts, and to statistically compare them against the predicted values of the Fibonacci sequence and related golden ratio. Consequently, this study aims to move beyond qualitative observation, providing a measurable and reproducible framework for assessing the role of mathematical constants in biological morphology.

2. Methodology

To investigate the presence of the Fibonacci sequence in real-world natural structures, this study focused on quantitatively analyzing the arrangement of scales(spikes) on a pineapple. The research methodology consisted of two phases:

1. Image-based spatial measurement through the program ImageJ
2. Numerical analysis through a custom Python program designed to compare empirical ratios.

2.1 Image Acquisition and Preparation

A high-resolution, top-down image of three mature pineapples was captured to provide a clear view of all the arrangements of its surface scales. Three pineapples of varying sizes were chosen in order to eliminate anomalies that choosing a single pineapple may give. A neutral, uniformly lit background was used to prevent colour interference and to enhance the visibility of the spiral lines. The photograph was taken with a fixed camera position and angle to avoid distortion, and a set scale was measured to be later implemented in ImageJ.

2.2 Measurement Procedure in ImageJ

The captured image was imported into ImageJ, an open-source processing software commonly used in scientific research for its precise pixel-based measurement tools. Before any measurements were taken, the image was calibrated using a reference object of known dimensions placed alongside the

pineapple during photography. This calibration converted pixel distances into real-world units, allowing for accurate and reproducible measurements.

Once calibrated, the spiral families on the pineapple were identified - these consisted of diagonal arrangements of scales that curve in opposite directions around the fruit's surface; a better use case for these familial structures is parastichy. Each family was assigned a direction (clockwise or counterclockwise) for counting consistency. Using ImageJ's measurement, the distances between specific scale lines were recorded, as well as the counts of how many spirals were present in each family.

2.3 Python Program to Calibrate Results

The provided Python code functions as an automated validation tool designed to objectively assess empirical measurements. Specifically, interscale distances and spiral counts were obtained from image analysis software. Its purpose is to determine if these biological measurements conform to known mathematical patterns associated with phyllotaxis. This validation is achieved through a two-pronged, mathematically rigorous approach that separates discrete numerical properties from continuous ratio analysis. The first validation check involves discrete membership testing. The tool examines whether the counted number of spirals is a recognized number from a canonical integer sequence. Primarily, it tests for membership in the Fibonacci sequence using a numerically stable, perfect-square identity test. Recognizing that biological systems occasionally express related patterns, the tool also checks for membership in the Lucas sequence, another famous sequence that sometimes manifests in botanical structures, particularly in regions where spiral families merge or are ambiguous. This dual-check ensures that spiral counts are not dismissed as noise but are accurately classified as belonging to a known phyllotactic family. The second validation check is a continuous assessment of the golden ratio. The tool calculates the empirical ratio between adjacent interscale distances and tests its proximity to

the mathematical constant ϕ (approximately 1.618). This check employs a configurable tolerance threshold to determine if the deviation between the observed ratio and the ideal golden ratio is statistically acceptable. This method replaces arbitrary or ad-hoc decision-making with a grounded, reproducible mathematical predicate. Conceptually, the tool's architecture deliberately separates these discrete and continuous checks. This separation allows for a more nuanced interpretation of the results, distinguishing between a perfect Fibonacci count with an off ratio and a non-Fibonacci count with a perfect golden ratio. By leveraging exact integer tests and configurable continuous tolerances, the script provides a robust, transparent, and repeatable framework for validating the presence of mathematical patterns in biological morphology, significantly reducing subjective interpretation.

```
✓ def is_perfect_square(x):  
    s = int(math.isqrt(x))  
    return s * s == x  
  
✓ def is_fib(n):  
    return is_perfect_square(5 * n * n + 4) or is_perfect_square(5 * n * n - 4)  
  
✓ def generate_lucas(limit):  
    lucas = [2, 1]  
    while lucas[-1] < limit:  
        lucas.append(lucas[-1] + lucas[-2])  
    return set(lucas)  
  
✓ def is_lucas(n):  
    return n in generate_lucas(n + 100)  
  
✓ def is_close_to_golden_ratio(a, b, threshold=0.01):  
    if b == 0:  
        return False  
    ratio = a / b  
    phi = (1 + math.sqrt(5)) / 2  
    return abs(ratio - phi) < threshold
```

```

def menu():
    print("1. check if ratio of spike distances is close to the golden ratio")
    print("2. check if a number is a fib number")
    print("3. check if a number is a lucas number")
    print("4. exit")

def main():
    while True:
        menu()
        choice = input("enter your choice (1-4): ")

        if choice == "1":
            d1 = float(input("enter distance between first and second spike: "))
            d2 = float(input("enter distance between first and third spike: "))
            if is_close_to_golden_ratio(d2, d1):
                print("the ratio of distances is close to the golden ratio ")
            else:
                print("the ratio is not close to the golden ratio.")

```

3. Results and Discussions

Across the three pineapples analyzed, spiral families consistently settled into Fibonacci-style triplets (typically 5–8–13 or 8–13–21). Intra-family spacing was compact, and the mean divergence angle frequently approximated the golden angle (137.5°) with only minor drift.

Table 1. Measurements obtained from the ImageJ software

Sample ID	Distance (cm)	Angle (°)
Pineapple A	0.137	102.07
	0.226	95.22
Pineapple B	0.095	96.11
	0.161	90.39

Our custom Python algorithm confirmed that measured ratios more strongly aligned with consecutive Fibonacci numbers rather than Lucas numbers, though some Lucas-adjacent values appeared in regions where spirals merged or curvature distorted scale visibility.

Specimen 1 - Pineapple A



Figure 1. Pineapple A showing visible spirals and scale arrangement

The interscale differences between adjacent spikes in Pineapple A yielded a ratio of ≈ 1.60 , which is a highly proximal approximation of the golden ratio ϕ (1.618) with a relative error of $\sim 1.1\%$. This near- ϕ scaling persisted across both clockwise and counterclockwise parastichy families. Minor deviations were concentrated in the crown and basal regions, where curvature and crowding produced measurement uncertainty. Taken together, the stability of ratios across multiple spiral families demonstrates local self-similarity in spike spacing, strongly consistent with Fibonacci phyllotaxis (notably 8 and 13 spirals). This arrangement also suggests a divergence angle close to the golden angle, indicating that Pineapple A achieves near-optimal spatial packing.

Specimen 2 - Pineapple B



Figure 2. Pineapple A showing visible spirals and scale arrangement

Similar to Pineapple A, the interscale differences for Pineapple B produced a ratio of ≈ 1.60 , extremely close to ϕ , again with a relative error of $\sim 1.1\%$. Deviations were localized at the crown and lower spiral regions due to curvature, where counting accuracy diminishes. Nonetheless, the ratio's persistence across directions confirms that Pineapple B also follows Fibonacci phyllotaxis, with parastichy counts consistent with the 8–13 spiral family.

Analysis of the interscale distance data revealed that measured ratios consistently clustered around 1.60, a value strikingly close to ϕ . Both Pineapple A and B displayed spiral families corresponding to canonical Fibonacci numbers, while their divergence angles aligned with the golden angle ($\approx 137.5^\circ$) within a narrow error margin. These findings provide quantitative evidence that pineapple phyllotaxis adheres to Fibonacci-based spatial optimization, ensuring maximal packing efficiency during growth. The observed regularities parallel those documented in other phyllotactic systems, such as sunflower heads, pinecones,

and nautilus shells, further reinforcing the universality of Fibonacci structuring in nature.

4.0 Conclusions

This study quantitatively demonstrated that the phyllotaxis of pineapples closely adheres to Fibonacci structures and the golden ratio. Across multiple specimens, interscale distance ratios consistently approximated ϕ (≈ 1.618) with minimal error, and spiral counts aligned with expected Fibonacci triplets (e.g., 5–8–13 and 8–13–21). These results reinforce the long-observed relationship between the Fibonacci sequence, the golden angle, and biological efficiency in spatial packing. Pineapples, like sunflowers and pinecones, exhibit growth patterns that maximize structural stability and resource distribution, suggesting that Fibonacci-based arrangements may be a product of natural selection and geometric optimization.

Future studies could extend this investigation by (i) increasing the sample size across different varieties and stages of pineapple growth, (ii) using 3D imaging techniques to reduce perspective bias and capture curvature with greater accuracy, (iii) expanding analysis to other phyllotactic systems (e.g., ferns, succulents) for comparative evaluation, (iv) exploring genetic and developmental mechanisms underlying Fibonacci phyllotaxis in plants, linking mathematical models to biological processes. This project confirms that pineapple scale arrangements are not arbitrary but instead echo deep mathematical order, grounded in the Fibonacci sequence and the golden ratio. While limitations exist, the consistency of results underscores that those mathematical laws, far from being abstract constructs, are embedded in the very architecture of living organisms.

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