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Research Article

The Bifurcation and Control of a Single-Species Fish Population Logistic Model with the Invasion of Alien Species

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The objective of this paper is to study systematically the bifurcation and control of a single-species fish population logistic model with the invasion of alien species based on the theory of singular system and bifurcation. It regards *Spartina anglica* as an invasive species, which invades the fisheries and aquaculture. Firstly, the stabilities of equilibria in this model are discussed. Moreover, the sufficient conditions for existence of the trans-critical bifurcation and the singularity induced bifurcation are obtained. Secondly, the state feedback controller is designed to eliminate the unexpected singularity induced bifurcation by combining harvested effort with the purification capacity. It obviously inhibits the switch of population and makes the system stable. Finally, the numerical simulation is proposed to show the practical significance of the bifurcation and control from the biological point of view.

1. Introduction

Singular system, also known as differential-algebraic system, is relatively accurate model description of some problems in the practical applications. The theory of singular system has been widely applied to various fields [1–4]. Both the Leontief dynamic input-output model and Hopfield neural network model are differential algebraic-systems [5, 6] and so are dynamic model of multiple robot body coordination operation and power system model with nonlinear load. Moreover, the study of the various bifurcations in the power system has produced great significance to reveal the mechanism of voltage instability [7–9]. It is known through [10–12] that the bifurcation theory of singular system already has had a solid theoretical foundation. Recently, Zhang and other scholars studied several kinds of biological dynamical systems [13–18] by utilizing the theory of singular system, which generates a profound impact on the better description of development situation for biological resources and the more accurate grasp of the real law for exploitation of biological resources.

This paper is a study on the aquatic plants *Spartina anglica*. According to the situation of our country, the introduction of aquatic species abroad accelerates so that the concern over the security of aquatic invasive species has also grown with the rapid development of aquaculture industry and the need of propagation and release for resource. *Spartina anglica*, which is one of the aquatic plants introduced abroad by the water conservancy department, is used to promote the sedimentation and reclamation. Now, it has become a big disaster. A few years ago, the shoals of Ling-kun Island were overrun with the *Spartina anglica* which occupied the tens of thousands of acres of tidal flats so that the fish and shellfish of the tidal flats vanished. It took a lot of manpower and material resources to burn and cut it [19]. In consequence, the further study on the influence of alien species on the singular ecological economic system has become an irresistible trend. In order to protect the ecological environment and guarantee the economic development simultaneously, many biological economic models are established by some mathematical researchers, and some valuable results are obtained. For example, the problems of the optimal harvesting in the biological economic model are studied, respectively, in [20–23], which provide the theory basis for reasonable development of biological resources. The existence and stability of equilibrium solution and positive periodic solution in the biological economy model are discussed, respectively, in

[24, 25]. However, the singular biological economic system which further considers the purification capacity and harvested effort in the case of invasive species, according to the author's knowledge, has never been studied systematically.

The innovation of this paper considers purification capacity for the *Spartina anglica*, which is first proposed by us in this field. Since *Spartina anglica* can promote the sedimentation and reclamation, increasing interest has been stimulated to study it. On one hand decreasing the density of *Spartina anglica* can facilitate survival of fish and other aquatic organisms; on the other hand it cannot be too low to play a favorable part in promoting the sedimentation and reclamation. The character of soil can be improved so that the environment can be protected. In consequence, it is essential to control the density of *Spartina anglica*. Besides, the purification capacity plays an important role in the economically sustainable harvesting capacity of the fish population, which at least provides space and adequate nutrients for the growth of fish. In fact, due to the appearance of the purification capacity, some complex dynamic characteristics will arise, such as bifurcation, impulse, and other characteristics, which will affect the stability of the system. Specially, the model in this paper exhibits two bifurcation phenomena when the economic revenue is zero. One is trans-critical bifurcation which changes the stability of the system, and the other is singular induced bifurcation which indicates that zero economic revenue brings impulse, and from the biological aspect, the impulse means a rapid expansion of the population. There are both theoretical significance and practical value to deeply study the problems of bifurcation and impulse, which are restrained with the purification capacity for invasion of alien species and can further reveal the instability mechanism of the whole biological economic system. Therefore, the proposed model is significantly different from the existing models in the field of research which use differential algebraic equations approach and needs separate investigation.

2. Modeling

Introducing a density restriction factor $(1 - N/K)$ to Malthus model leads to the famous Logistic equation on ecology:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right), \tag{1}$$

where $N(t)$ is the density or quantity of population at time t, r is the intrinsic growth rate of the population, and K is the environmental capacity.

To be more reasonable, we introduce alien species to this model. Assuming that $y(t)$ is the density of *Spartina anglica* at time t, the original water nutrients, water, oxygen, and living space are used due to the invasion of alien species, and we get the following model:

$$\begin{aligned} \frac{dx(t)}{dt} &= rx(t)\left(1 - \frac{x(t) + y(t)}{K}\right) - E(t)\,x(t), \\ \frac{dy(t)}{dt} &= \beta y(t) - hE_1(t), \\ \frac{dE_1(t)}{dt} &= \theta y(t) - wE_1(t) - \alpha x(t), \end{aligned} \tag{2}$$

where $x(t)$, $E_1(t)$, $E(t)$ are, respectively, the density of fish population, the purification capability for *Spartina anglica,* and the harvested effort to fish at time t. β is the intrinsic growth rate of the *Spartina anglica*. $-hE_1(t)$ is the quantity of purification to *Spartina anglica*. $\theta y(t)$ is the growing degree of *Spartina anglica*. w is the cost of a unit of purification effort. $-\alpha x(t)$ represents that the purification capability is stimulated by the reduction of fish to a certain extent. The constants mentioned above are supposed to be positive.

Gordon founded the open or public fishery economic theory in 1954 [26]. Sustainable Economic Profit = Sustainable Total Revenue − Sustainable Total Cost, when the harvested effort $E(t)$ is given. Therefore, when the harvested effort $E(t)$ switches with time t, and we get the following algebraic equation:

$$E(t)\left(x(t)\,p - c\right) = m(t), \tag{3}$$

where p is the market price of the captive population. c is the cost of a unit of capture effort. $m(t)$ is the net economic revenue. Based on (2) and (3), the following singular biological economic system is established:

$$\begin{aligned} \dot{x}(t) &= rx(t)\left(1 - \frac{x(t) + y(t)}{K}\right) - E(t)\,x(t), \\ \dot{y}(t) &= \beta y(t) - hE_1(t), \\ \dot{E}_1(t) &= \theta y(t) - wE_1(t) - \alpha x(t), \\ 0 &= E(t)\left(x(t)\,p - c\right) - m(t). \end{aligned} \tag{4}$$

Equation (4) also can be expressed as the following matrix form:

$$A(t)\,\dot{X}(t) = G\left(x(t), y(t), E_1(t), E(t)\right), \tag{5}$$

where

$$\begin{aligned} X(t) &= \left(x(t), y(t), E_1(t), E(t)\right), \\ A(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ G\left(x(t), y(t), u(t), E(t)\right) &= \begin{bmatrix} rx(t)\left(1 - \dfrac{x(t) + y(t)}{K}\right) - E(t)\,x(t) \\ \beta y(t) - hE_1(t) \\ \theta y(t) - wE_1(t) - \alpha x(t) \\ E(t)\left(x(t)\,p - c\right) - m(t) \end{bmatrix}. \end{aligned} \tag{6}$$

The objective of this paper is to study the dynamic behavior of the system (4) and propose the singular state feedback controller to stabilize the bifurcation. The structures of the reminder of this paper are arranged as follows. In Section 3, the stability of equilibria is discussed. In Section 4, the dynamic behaviors around equilibria are studied. In Section 5, state feedback controller is designed. Finally, a real-world example is given to discuss the practical significance of the bifurcation and control from biological perspective.

3. Equilibria and Stability

When $m = 0$, (4) can be written as

$$\begin{aligned} \dot{x}(t) &= rx(t)\left(1 - \frac{x(t)+y(t)}{K}\right) - E(t)x(t), \\ \dot{y}(t) &= \beta y(t) - hE_1(t), \\ \dot{E}_1(t) &= \theta y(t) - wE_1(t) - \alpha x(t), \\ 0 &= E(t)\left(x(t)p - c\right). \end{aligned} \tag{7}$$

It is clear that (7) has three equilibria:

$$\begin{aligned} &P_1(0,0,0,0), \\ &P_2\left(\frac{K(h\theta-\beta\omega)}{M}, \frac{hK\alpha}{M}, \frac{K\alpha\beta}{M}, 0\right), \\ &P_3\left(\frac{c}{p}, \frac{ch\alpha}{p(h\theta-\beta\omega)}, \frac{c\alpha\beta}{p(h\theta-\beta\omega)}, r - \frac{crM}{Kp(h\theta-\beta\omega)}\right), \end{aligned} \tag{8}$$

where $M = h\alpha + h\theta - \beta\omega$. Next, we consider the stability in the neighborhood of each equilibrium.

Theorem 1. *(1) Equation (7) is unstable at the equilibrium* $P_1(0,0,0,0)$.

(2) If $\omega - \beta > 0$ *and* $(c - (Kp(h\theta-\beta\omega)/(h\alpha+h\theta-\beta\omega))) \neq 0$, *then (7) is stable at the equilibrium*

$$P_2\left(\frac{K(h\theta-\beta\omega)}{M}, \frac{hK\alpha}{M}, \frac{K\alpha\beta}{M}, 0\right). \tag{9}$$

Proof. The Jacobian matrix of (7) at P_1 is

$$\begin{aligned} J_{P_1} &= \left|\begin{matrix} -\frac{rx}{K} + r\left(1 - \frac{x+y}{K}\right) - E & -\frac{rx}{K} & 0 & -x \\ 0 & \beta & -h & 0 \\ -\alpha & \theta & -\omega & 0 \\ Ep & 0 & 0 & xp - c \end{matrix}\right|_{P_1} \\ &= \begin{vmatrix} r & 0 & 0 & 0 \\ 0 & \beta & -h & 0 \\ -\alpha & \theta & -\omega & 0 \\ 0 & 0 & 0 & -c \end{vmatrix}. \end{aligned} \tag{10}$$

Then the characteristic polynomial is

$$\begin{aligned} \det\left(\lambda A - J_{P_1}\right) &= c(-r + \lambda) \\ &\quad \times \left(h\theta - \beta\lambda + \lambda^2 - \beta\omega + \lambda\omega\right) = 0. \end{aligned} \tag{11}$$

The characteristic roots are

$$\begin{aligned} \lambda_1 &= r > 0, \\ \lambda_2 &= \frac{1}{2}\left(\beta - \omega - \sqrt{\beta^2 - 4h\theta + 2\beta\omega + \omega^2}\right), \\ \lambda_3 &= \frac{1}{2}\left(\beta - \omega + \sqrt{\beta^2 - 4h\theta + 2\beta\omega + \omega^2}\right). \end{aligned} \tag{12}$$

It can be judged that P_1 is an unstable equilibrium.

Similarly, for $P_2(K(h\theta-\beta\omega)/M, hK\alpha/M, K\alpha\beta/M, 0)$, in order to guarantee the fish population, *Spartina anglica* and purification capability all exist, and the following inequalities need to be satisfied:

$$\begin{aligned} &\frac{K(h\theta-\beta\omega)}{M} > 0, \quad \frac{hK\alpha}{M} > 0, \quad \frac{K\alpha\beta}{M} > 0, \\ &\text{that is } h\theta - \beta\omega > 0, \end{aligned} \tag{13}$$

where

$$M = h\alpha + h\theta - \beta\omega = h\alpha + (h\theta - \beta\omega) > 0. \tag{14}$$

In order to study the stability of P_2, we firstly obtain the Jacobian matrix of (7) at P_2 :

$$J_{P_2} = \begin{vmatrix} \frac{-hr\theta + r\beta\omega}{h(\alpha+\theta) - \beta\omega} & \frac{-hr\theta + r\beta\omega}{h(\alpha+\theta) - \beta\omega} & 0 & \frac{-hK\theta + K\beta\omega}{h(\alpha+\theta) - \beta\omega} \\ 0 & \beta & -h & 0 \\ -\alpha & \theta & -\omega & 0 \\ 0 & 0 & 0 & -c + \frac{Kp(h\theta-\beta\omega)}{h\alpha + h\theta - \beta\omega} \end{vmatrix}. \tag{15}$$

Based on $\det(\lambda A - J_{P_2}) = 0$, then

$$\begin{aligned} &\left(c - \frac{Kp(h\theta-\beta\omega)}{M}\right) \\ &\times \left(-h\left(-\theta\lambda - \frac{hr\alpha\theta}{M} - \frac{hr\theta^2}{M} + \frac{r\alpha\beta\omega}{M} + \frac{r\beta\theta\omega}{M}\right)\right. \\ &\left. + (-\beta + \lambda)(\lambda + \omega)\left(\lambda - \frac{-hr\theta + r\beta\omega}{M}\right)\right) = 0, \end{aligned} \tag{16}$$

where $M = h\alpha + h\theta - \beta\omega$.

It is a cubic equation of λ. When $(c - (Kp(h\theta-\beta\omega)/(h\alpha + h\theta - \beta\omega))) \neq 0$, it can be simplified as follows:

$$\lambda^3 + \Delta_1\lambda^2 + \Delta_2\lambda + \Delta_3 = 0, \tag{17}$$

where

$$\begin{aligned} \Delta_1 &= -\beta + \omega + \frac{r(h\theta-\beta\omega)}{h(\alpha+\theta) - \beta\omega}, \\ \Delta_2 &= \frac{(h\theta-\beta\omega)(h(\alpha+\theta) - \beta\omega + r(-\beta+\omega))}{h(\alpha+\theta) - \beta\omega}, \\ \Delta_3 &= r(h\theta - \beta\omega). \end{aligned} \tag{18}$$

Since $\omega - \beta > 0, h\theta - \beta\omega > 0, \operatorname{Re}\lambda_1 < 0, \operatorname{Re}\lambda_2 < 0, \operatorname{Re}\lambda_3 < 0$.

When $\Delta_1 > 0, \Delta_2 > 0, \Delta_3 > 0, \Delta_1 \times \Delta_2 - \Delta_3 > 0$. Then based on the *Routh-Hurwitz* [27] theorem, (7) is stable at P_2. □

It is explained through this theorem that the system keeps stable under certain condition that fish population, *Spartina anglica*, and purification capacity all exist while harvested effort does not exist. Since the economic interests are obtained through the harvest effort, we always pay more attention to the dynamic character of the positive equilibrium. Straightforward computation shows that P_3 is the singular point [28] of (7), and the dynamics is relatively complex. The discussion will be shown in Section 5.

4. Trans-Critical Bifurcation

The topological structure of (4) will be switched when the biological phenomenon of economic balance appears. We consider the situation in the neighborhood of $P_1(0,0,0,0)$.

Theorem 2. *If $m = 0$, $(h\theta - \beta\omega) \neq 0$, $\alpha \neq (-h\theta + \beta\omega)$, then (4) undergoes the trans-critical bifurcation at $P_1(0,0,0,0)$, and $r = 0$ is the bifurcation value.*

Proof. For (4), let

$$f(x,y,E_1,E,m) = \begin{vmatrix} rx(t)\left(1 - \dfrac{x(t)+y(t)}{K}\right) - E(t)\,x(t) \\ \beta y(t) - hE_1(t) \\ \theta y(t) - \omega E_1(t) - \alpha x(t) \end{vmatrix},$$
$$g(x,y,E_1,E,m) = E(t)\,(x(t)\,p - c) - m(t). \tag{19}$$

We choose r as the bifurcation parameter. If $m = 0$, then $\det(D_E g)|_{p_1} = -c \neq 0$ ($D_E = \partial/\partial E$, similarly hereinafter). Applying the implicit function theorem, in certain neighborhood of $P_1(0,0,0,0)$, (4) can be simplified to the ordinary differential equation $\dot{X} = f_R(X,r)$, where $X = (x,y,E_1)^T$, $f_R(X,r) = (f_{R1}(X,r), f_{R2}(X,r))^T$, which are adequately smooth two-dimensional vector functions. $f_R(0,r) = \vec{0}$ and the matrix

$$\begin{aligned} D_X f_R(0,0) &= D_X f - D_E f\,(D_E g)^{-1} D_X g \\ &= \begin{vmatrix} -E & 0 & 0 \\ 0 & \beta & -h \\ -\alpha & \theta & -\omega \end{vmatrix} - \frac{1}{(xp-c)} \begin{vmatrix} -x \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} Ep & 0 & 0 \end{vmatrix} \Bigg|_{P_1} \\ &= \begin{vmatrix} 0 & 0 & 0 \\ 0 & \beta & -h \\ -\alpha & \theta & -\omega \end{vmatrix}. \end{aligned} \tag{20}$$

It is clear that the characteristic roots of the matrix above are

$$\begin{aligned} \lambda_1 &= 0, \\ \lambda_2 &= \frac{1}{2}\left(\beta - \omega - \sqrt{\beta^2 - 4h\theta + 2\beta\omega + \omega^2}\right), \\ \lambda_3 &= \frac{1}{2}\left(\beta - \omega + \sqrt{\beta^2 - 4h\theta + 2\beta\omega + \omega^2}\right). \end{aligned} \tag{21}$$

There are one simple zero characteristic root and two nonzero characteristic roots. Through computation, the right eigenvector of the zero characteristic root is

$$\Psi = \begin{vmatrix} -\dfrac{-h\theta + \beta\omega}{\alpha\beta} \\ \dfrac{h}{\beta} \\ 1 \end{vmatrix}, \tag{22}$$

and the left eigenvector is $\Phi = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix}$. Consider

$$\begin{aligned} &\Phi\left(\left(\frac{\partial^2 f_R}{\partial r \partial X}\right)(\Psi)\right) \\ &\quad = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} -\dfrac{-h\theta + \beta\omega}{\alpha\beta} \\ \dfrac{h}{\beta} \\ 1 \end{vmatrix} \\ &\quad = -\frac{-h\theta + \beta\omega}{\alpha\beta} \neq 0, \\ &\Phi\left(\left(\frac{\partial^2 f_R}{\partial X^2}\right)(\Psi,\Psi)\right) \\ &\quad = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} \Psi^T & \Lambda_1 & \Psi \\ \Psi^T & \Lambda_2 & \Psi \\ \Psi^T & \Lambda_3 & \Psi \end{vmatrix} \\ &\quad = \frac{hr(-h\theta + \beta\omega)}{K\alpha\beta^2} - \frac{r(h\theta - \beta\omega)}{K\alpha\beta} \\ &\qquad \times \left(\frac{2(h\theta - \beta\omega)}{\alpha\beta} + \frac{h}{\beta}\right) \\ &\quad = \frac{2hr(-h\theta + \beta\omega)}{K\alpha\beta^2}\left(1 - \frac{(-h\theta + \beta\omega)}{\alpha}\right) \neq 0, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \Lambda_1 &= \begin{vmatrix} -\dfrac{2r}{K} & -\dfrac{r}{K} & 0 \\ -\dfrac{r}{K} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \\ \Lambda_2 &= \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \\ \Lambda_3 &= \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}. \end{aligned} \tag{24}$$

Based on the definition of trans-critical bifurcation in [29], (4) undergoes the trans-critical bifurcation at $P_1(0,0,0,0)$. □

Obviously, the trans-critical bifurcation changes the stability of the system. In fact, the phenomenon of trans-critical bifurcation means that, for some reason, such as, the shortage of nutrition, water, and oxygen with the invasion of *Spartina anglica*, fish population loses the natural fertility. The ecological interpretation of trans-critical bifurcation is that the phenomenon of economic overfishing [26] appears so that the extinction of population will arise in the biological economy system at the above depicted moment. The phenomenon admonishes people simultaneously that although *Spartina anglica* can promote the sedimentation

and reclamation and improve the soil, its overproduction can also lead to a breakdown in local ecosystem. That means the density of the fish population, *Spartina anglica*, harvested effort, and purification capacity are all zero. In consequence, it is necessary to guarantee the fish species reproduce normally to maintain the sustainable survival of population when introducing the alien species to promote silting reclamation and protect the environment.

5. The Singularity Induced Bifurcation

We pay more attention to local dynamic characteristics near the positive equilibrium of the system in the actual situation of biological economics. Therefore, we obtain the following theorem.

Theorem 3. *If* $(h\theta-\beta\omega)>0$, $(r-(cr(h\alpha+h\theta-\beta\omega)/Kp(h\theta-\beta\omega)))>0$, *then (4) will show the phenomenon of singularity induced bifurcation at*

$$P_3\left(\frac{c}{p},\frac{ch\alpha}{p(h\theta-\beta\omega)},\frac{c\alpha\beta}{p(h\theta-\beta\omega)},r-\frac{crM}{Kp(h\theta-\beta\omega)}\right), \tag{25}$$

and m *is a bifurcation parameter so that the system loses stability, where* $M=h\alpha+h\theta-\beta\omega$.

Proof. In order to guarantee that each component exists at P_3, the following inequalities need to be satisfied:

$$\begin{aligned}&\frac{c}{p}>0,\quad \frac{ch\alpha}{p(h\theta-\beta\omega)}>0,\\ &\frac{c\alpha\beta}{p(h\theta-\beta\omega)}>0,\quad r-\frac{crM}{Kp(h\theta-\beta\omega)}>0,\end{aligned} \tag{26}$$

which means $(h\theta-\beta\omega)>0$, $r-(crM/Kp(h\theta-\beta\omega))>0$. Consider m as the bifurcation parameter, $X=(x,y,E_1)^T$, $P_3=(x^*,y^*,E_1^*,E^*)$, and the singular surface

$$S:=\left\{(X,E,m)\in R^{n+m+p}:g(X,E,m)=0,\Delta(X,E,m)=0\right\}, \tag{27}$$

where $\Delta(X,E,m):=\det[D_Eg(X,E,m)]$.

When $m=0$, P_3 passes through the singular surface S so that

$$D_Eg\big|_{P_3}=(xp-c)\big|_{P_3}=0. \tag{28}$$

Therefore D_Eg has a zero characteristic root.

$$\begin{aligned}&\operatorname{trace}\left[D_Ef\operatorname{adj}(D_Eg)D_Xg\right]\big|_{P_3}\\ &\quad=\operatorname{trace}\left[D_Ef\operatorname{adj}(D_Eg)\left(D_xg\ \ D_yg\ \ D_{E_1}g\right)\right]\big|_{P_3},\\ &\operatorname{trace}\begin{vmatrix}-xEp & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{vmatrix}_{P_3}\\ &\quad=-\left(cr-\frac{c^2rM}{Kp(h\theta-\beta\omega)}\right)=-cE^*\neq 0.\end{aligned} \tag{29}$$

Then $\dim\ker(D_Eg|_{P_3})=1$ means that the dimension of kernel is one, and

$$\begin{aligned}&\det\left(D_Eg\left(x(m),y(m),E_1(m),E(m),m\right)\right)\neq 0\\ &\qquad\text{when } m\neq 0,\end{aligned} \tag{30}$$

where $(x(m),y(m),E_1(m),E(m))$ represents the equilibrium curve of the initial value at $P_3(x(0),y(0),E_1(0),E(0))$.

Through computation, we get

$$\begin{aligned}\begin{vmatrix}D_Xf & D_Ef\\ D_Xg & D_Eg\end{vmatrix}_{P_3}&=\begin{vmatrix}-r-\frac{cr}{Kp}+\frac{cr(h\alpha+h\theta-\beta\omega)}{Kp(h\theta-\beta\omega)}+r\left(1-\frac{(c/p)+(ch\alpha/p(h\theta-\beta\omega))}{K}\right) & -\frac{cr}{Kp} & 0 & -\frac{c}{p}\\ 0 & \beta & -h & 0\\ -\alpha & \theta & -\omega & 0\\ p\left(r-\frac{cr(h\alpha+h\theta-\beta\omega)}{Kp(h\theta-\beta\omega)}\right) & 0 & 0 & 0\end{vmatrix}\\ &=-p\left(-\frac{ch\theta}{p}+\frac{c\beta\omega}{p}\right)\left(r-\frac{cr(h\alpha+h\theta-\beta\omega)}{Kp(h\theta-\beta\omega)}\right)=c(h\theta-\beta\omega)E^*\neq 0\\ \begin{vmatrix}D_Xf & D_Ef & D_mf\\ D_Xg & D_Eg & D_mg\\ D_X\Delta & D_E\Delta & D_m\Delta\end{vmatrix}_{P_3}&=\begin{vmatrix}-r-\frac{cr}{Kp}+\frac{cr(h\alpha+h\theta-\beta\omega)}{Kp(h\theta-\beta\omega)}+r\left(1-\frac{(c/p)+(ch\alpha/p(h\theta-\beta\omega))}{K}\right) & -\frac{cr}{Kp} & 0 & -\frac{c}{p} & 0\\ 0 & \beta & -h & 0 & 0\\ -\alpha & \theta & -\omega & 0 & 0\\ p\left(r-\frac{cr(h\alpha+h\theta-\beta\omega)}{Kp(h\theta-\beta\omega)}\right) & 0 & 0 & 0 & -1\\ p & 0 & 0 & 0 & 0\end{vmatrix}\\ &=ch\theta-c\beta\omega\neq 0,\end{aligned} \tag{31}$$

where $\Delta(X, E, m) = xp - c$, and

$$\begin{aligned} B &= -\operatorname{trace}\left[D_E f \operatorname{adj}\left(D_E g\right) D_X g\right]\big|_{P_3} \\ &= -\operatorname{trace}\begin{vmatrix} -xEp & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}_{P_3} \\ &= \left(cr - \frac{c^2 r\left(h\alpha + h\theta - \beta\omega\right)}{Kp\left(h\theta - \beta\omega\right)}\right), \\ C &= D_u\Delta - \left|D_X\Delta \;\; D_E\Delta\right| \begin{vmatrix} D_X f & D_E f \\ D_X g & D_E g \end{vmatrix}^{-1} \begin{vmatrix} D_m f \\ D_m g \end{vmatrix} \\ &= -\left|p \;\; 0 \;\; 0 \;\; 0\right| \begin{vmatrix} D_X f & D_E f \\ D_X g & D_E g \end{vmatrix}_{P_3}^{-1} \begin{vmatrix} 0 \\ 0 \\ 0 \\ -1 \end{vmatrix} \\ &= \frac{1}{r - \left(cr\left(h\alpha + h\theta - \beta\omega\right)/Kp\left(h\theta - \beta\omega\right)\right)}. \end{aligned} \tag{32}$$

Clearly, $B/C = c(r - (cr(h\alpha + h\theta - \beta\omega)/Kp(h\theta - \beta\omega)))^2 = cE^{*2} > 0$. Based on theorem 3 in [28], we know that one characteristic root of (4) moves from C^- to C^+ and along the real axis diverges to infinity when m goes from negative to positive. And

$$\begin{aligned} \operatorname{dimker}\left(D_E g\big|_{P_3}\right) &= 1, \\ \operatorname{trace}\left[D_E f \operatorname{adj}\left(D_E g\right) D_X g\right]\big|_{P_3} &\neq 0, \\ \begin{vmatrix} D_X f & D_E f \\ D_X g & D_E g \end{vmatrix}_{P_3} &\neq 0, \\ \det\left(D_E g\left(x\left(m\right), y\left(m\right), E_1\left(m\right), E\left(m\right), m\right)\right) &\neq 0, \end{aligned} \tag{33}$$

where $(x(m), y(m), E_1(m), E(m))$ represents the equilibrium curve of the initial value $P_3(x(0), y(0), E_1(0), E(0))$. □

Based on the theorem 1.1 in [30], the other characteristic root of (4) in the neighborhood of $m = 0$ is continuous, which means that it will not affect the switch of stability near the equilibrium. And through computation we know that this characteristic root has negative real part. In consequence, the system switches from being stable to being unstable as m moves from being negative to being positive through zero.

Singularity induced bifurcation is a kind of bifurcation which arises in singular system, and it never occurs in an ordinary differential equation. The singularity induced bifurcation that the system (4) shows indicates that the real parts of some eigenvalues for the linear matrix $(D_X f - D_E f(D_E g)^{-1} D_X g)$ of the system (4) will change as bifurcation parameter and lead to the changes of system stability when $D_E g$ is singular. In addition, singularity induced bifurcation also has a great impact on the topological structure near the equilibrium point of the system, which leads to the emergence of pulse and even the collapse of the whole system [31].

From the ecological prospective, singularity induced bifurcation and the system instability will cause the following effects on the ecological system. The collapse which singularity induced bifurcation leads to will be embodied by a surge increase of a population density within a short period in the ecological system and loss of the stability of the system. On one hand, the fierce competitions for the limited resources and survival occur within a population, which results that the young population in the inferior position would die in competition due to shortage of resources. There is no benefit to the sustainable development of this population. On the other hand, surge of a population may affect various food chains and lead to competition of limited living resource among populations. Some other populations in inferior position will face the extinction due to the shortage of resources, which is not beneficial to the maintenance of the population diversity in the ecological system.

Accordingly, in this paper, the singularity induced bifurcation may cause an impulsive phenomenon of the single-species ecosystem especially with the invasion of alien species which can occupy the environmental and biological resources. On the other hand, the purification capacity plays an important role in the economically sustainable harvesting capacity of the fish population, which at least provides space and adequate nutrients for the growth of fish. The singularity induced bifurcation means that if fishers purify the *Spartina anglica*, the phenomenon of economic overfishing [26] appears when the total income is equal to the total cost. The rapid increase of the density of *Spartina anglica* and the rapid decline of fish population can lead to the fact that one characteristic root of this system will tend to infinite and impulse phenomenon arises. It means the quantities of fish population and alien species switch in an instant even beyond the capacity of the environment. Surge increase of the population consumes a large number of nutrients in water so that nutrient concentration decreases. The decline of nutrient concentrations, which in turn affects breeding of populations, leads to mortality of a large number of populations and causes ecological imbalance. The existence of a singularity induced bifurcation phenomenon is verified at the equilibrium of the model system with zero economic revenue. It should be pointed out that the trapping behavior and purifying behavior cannot be unlimitedly accessed to the populations in the ecological system and they need to be controlled in a certain range. Otherwise the population resources would lose due to the excessive capture and purification, which affects the relevant populations in the food chain and the sedimentation and reclamation by *Spartina anglica*. It is not conducive to the sustainable development of the population diversity in the ecological system.

To solve the above problem, we will introduce a state feedback controller to cure the ecological unbalance by some artificial methods in the next part.

6. Feedback Control of the Singularity Induced Bifurcation

From a long-term perspective, we should take some appropriate strategies to control harvested effort and increase purification capability so that gains keep in a proper range to eliminate singularity induced bifurcation.

Let

$$u(t) = k_1\left(E(t) - E^*\right) + k_2\left(E_1(t) - E_1^*\right), \tag{34}$$

where k_1, k_2 are feedback gains; then the closed-loop system is

$$\begin{aligned}
\dot{x}(t) &= rx\left(1 - \frac{x+y}{K}\right) - Ex,\\
\dot{y}(t) &= \beta y - hE_1,\\
\dot{E}_1(t) &= \theta y - wE_1 - \alpha x + k_2\left(E_1 - E_1^*\right),\\
0 &= E\left(xp - c\right) + k_1\left(E - E^*\right).
\end{aligned} \tag{35}$$

It is easy to verify that (35) is locally controllable.

Theorem 4. *Let*

$$u(t) = k_1\left(E(t) - E^*\right) + k_2\left(E_1(t) - E_1^*\right), \tag{36}$$

where k_1, k_2 are feedback gains and they satisfy

$$\begin{aligned}
k_1 &> \frac{cr\left(2cr\beta + hKp\theta\right)\left(Kp\left(h\theta - \beta\omega\right) - c\left(h\left(\alpha+\theta\right) - \beta\omega\right)\right)}{\left(2c^2r^2\beta + K^2p^2\beta\left(-\beta^2 + h\theta\right) + cKpr\left(-2\beta^2 + h\left(\alpha+\theta\right)\right)\right)\left(h\theta - \beta\omega\right)}\\
k_2 &< \min\left\{\frac{c\left(-hr\alpha + 2r\beta\omega\right) + Kp\left(\beta^3 - 2h\beta\theta + h\theta\omega\right)}{2cr\beta + hKp\theta},\right.\\
&\qquad \frac{Kp\left(\beta^4 - h\beta^2\theta - h^2\theta^2 - \beta^3\omega + 2h\beta\theta\omega\right) + cr\left(2\beta^3 + h\left(-\alpha\beta - 2\beta\theta + \alpha\omega\right)\right)}{chr\alpha - Kp\beta\left(\beta^2 - 2h\theta\right)},\\
&\qquad \left.\frac{2c^2hr^2\alpha - K^2p^2\left(\beta^2 - h\theta\right)\left(-h\theta + \beta\omega\right) + cKpr\left(2h\beta\theta + h\alpha\omega - 2\beta^2\omega\right)}{Kp\left(cr\left(h\alpha - 2\beta^2\right) + Kp\beta\left(-\beta^2 + h\theta\right)\right)}\right\};
\end{aligned} \tag{37}$$

then (35) is stable at $P_3(x^*, y^*, E_1^*, E^*)$, where $P_3(x^*, y^*, E_1^*, E^*) = (c/p, ch\alpha/p(h\theta - \beta\omega), c\alpha\beta/p(h\theta - \beta\omega), r - (cr(h\alpha + h\theta - \beta\omega)/Kp(h\theta - \beta\omega)))$.

Proof. The Jacobian matrix of (35) at P_3 is

$$J_{p3} = \begin{vmatrix}
-r - \frac{cr}{Kp} + \frac{cr\left(h\alpha + h\theta - \beta\omega\right)}{Kp\left(h\theta - \beta\omega\right)} + r\left(1 - \frac{c/p + \left(ch\alpha/p\left(h\theta - \beta\omega\right)\right)}{K}\right) & -\frac{cr}{Kp} & 0 & -\frac{c}{p}\\
0 & \beta & -h & 0\\
-\alpha & \theta & k_2 - \omega & 0\\
p\left(r - \frac{cr\left(h\alpha + h\theta - \beta\omega\right)}{Kp\left(h\theta - \beta\omega\right)}\right) & 0 & 0 & k_1
\end{vmatrix}. \tag{38}$$

Then the characteristic polynomial is $\lambda^3 + \Delta_1'\lambda^2 + \Delta_2'\lambda + \Delta_3' = 0$, where

$$\begin{aligned}
\Delta_1' &= -\frac{c}{k_1}E^* + \frac{cr}{Kp} - \beta + \omega - k_2,\\
\Delta_2' &= \frac{c\beta}{k_1}E^* - \frac{c\omega}{k_1}E^* + h\theta - \beta\omega\\
&\quad + \frac{c}{k_1}E^*k_2 - \frac{cr\left(\beta - \omega + k_2\right)}{Kp} + \beta k_2,\\
\Delta_3' &= -\frac{ch\theta}{k_1}E^* + \frac{c\beta\omega}{k_1}E^* - \frac{c\beta}{k_1}E^*k_2\\
&\quad + \frac{cr\left(h\left(\alpha+\theta\right) - \beta\omega + \beta k_2\right)}{Kp},\\
E^* &= \left(r - \frac{cr\left(h\alpha + h\theta - \beta\omega\right)}{Kp\left(h\theta - \beta\omega\right)}\right).
\end{aligned} \tag{39}$$

Its characteristic roots are λ_1, λ_2, λ_3. When $\Delta_1' > 0$, $\Delta_2' > 0$, $\Delta_3' > 0$, $\Delta_1' \times \Delta_2' - \Delta_3' > 0$. Then based on the *Routh-Hurwitz*

[27] theorem, (35) is stable at $P_3(x^*, y^*, E_1^*, E^*)$. Thus, k_1, k_2 satisfy

$$\begin{aligned}
&-\frac{c}{k_1}E^* + \frac{r}{K}x^* - \beta + \omega - k_2 > 0,\\
&\frac{c\beta}{k_1}E^* - \frac{c\omega}{k_1}E^* + h\theta - \beta\omega\\
&\quad + \frac{c}{k_1}E^*k_2 - \frac{r(\beta - \omega + k_2)}{K}x^* + \beta k_2 > 0,\\
&-\frac{ch\theta}{k_1}E^* + \frac{c\beta\omega}{k_1}E^* - \frac{c\beta}{k_1}E^*k_2\\
&\quad + \frac{r(h(\alpha+\theta) - \beta\omega + \beta k_2)}{K}x^* > 0,
\end{aligned} \tag{40}$$

where $x^* = c/p$. Simplify the inequalities above, and we have

$$\begin{aligned}
k_1 &> \frac{cr(2cr\beta + hKp\theta)(Kp(h\theta - \beta\omega) - c(h(\alpha+\theta) - \beta\omega))}{(2c^2r^2\beta + K^2p^2\beta(-\beta^2 + h\theta) + cKpr(-2\beta^2 + h(\alpha+\theta)))(h\theta - \beta\omega)},\\
k_2 &< \min\left\{\frac{c(-hr\alpha + 2r\beta\omega) + Kp(\beta^3 - 2h\beta\theta + h\theta\omega)}{2cr\beta + hKp\theta},\right.\\
&\qquad \frac{Kp(\beta^4 - h\beta^2\theta - h^2\theta^2 - \beta^3\omega + 2h\beta\theta\omega) + cr(2\beta^3 + h(-\alpha\beta - 2\beta\theta + \alpha\omega))}{chr\alpha - Kp\beta(\beta^2 - 2h\theta)},\\
&\qquad \left.\frac{2c^2hr^2\alpha - K^2p^2(\beta^2 - h\theta)(-h\theta + \beta\omega) + cKpr(2h\beta\theta + h\alpha\omega - 2\beta^2\omega)}{Kp(cr(h\alpha - 2\beta^2) + Kp\beta(-\beta^2 + h\theta))}\right\}.
\end{aligned} \tag{41}$$

At this point, we obtain $\operatorname{Re}\lambda_1 < 0$, $\operatorname{Re}\lambda_2 < 0$, $\operatorname{Re}\lambda_3 < 0$. Thus (35) is stable at P_3. □

Notice. In practice, it is not enough to only control harvested effort. It is necessary to control the population density of alien species and release certain density of fish population. This paper studies the problem of alien species invasion. Controlling harvested effort and purification capability and taking k_1, k_2 appropriate values can eliminate the singularity induced bifurcation of the biological economic system. In other words, the biological economic system can be stabilized by increasing purification capability, controlling harvesting effort, and other strategies in the case of zero income temporarily.

7. Application Instance

Spartina anglica has played a positive role in protecting dam to shape earth and lessening wave at the beginning of the planting at the coast of eastern Fujian Province. However, some other characteristics of *Spartina anglica* are highlighted at later period, such as strong fertility, developed root system, seeds fluttering around in the wind to spread rapidly, and the lack of natural enemies. At the present time, the tidal flats of 20 coastal towns are encroached in 4 counties (districts or cities) of Jiaocheng, Fuan, and Xiapu. The polluted area has been 110 thousand acres, which accounts for more than half of tidal flats of fish-farming. *Spartina anglica*, which is described as the cannibal grass, has already been overrun at the coast of Shandong province in China, especially in the Yellow River delta of the national conservation area at the Yellow River estuary of Dongying City. *Spartina anglica* today mainly aggregates in shellfish beds in south of estuary of fairy ditch, Xianhe Town, Hekou District, and Dongying City. It tends to overgrow at the speed of 6 times per year on the beach of 10 km^2 approximately, which is a serious threat to marine life. *Spartina anglica* invades the coastal tidal flats of Dongying City, and the cover is more and more extensive so that the damaged area has been 86616 km^2. The visible area of *Spartina anglica* is about 3333 to 4000 km^2, and the seed drifting area has been over 6666 km^2. In addition, the scattered and small pieces of *Spartina anglica* are also found in Binzhou City, Weifang City, and Yantai City [32].

Based on the relevant data in Chinese alien marine species basic information database [32], we properly process the data (nondimensional transformation, equal ratio simplification, approximation, and so on) according to the parameters in the model of this paper. In sequence, the values of parameters are as follows:

$$\begin{aligned}
&r = 1, \quad K = 4, \quad \beta = 1.5, \quad h = 3, \quad \theta = 2, \quad \omega = 2,\\
&\alpha = 1, \quad p = 0.8, \quad c = 0.4, \quad m = 0.
\end{aligned} \tag{42}$$

Equation (4) can be written as

$$\begin{aligned}
\dot{x}(t) &= x(t)\left(1 - \frac{x(t) + y(t)}{4}\right) - E(t)x(t),\\
\dot{y}(t) &= 1.5y(t) - 3E_1(t),\\
\dot{E}_1(t) &= 2y(t) - 2E_1(t) - x(t),\\
0 &= E(t)(0.8x(t) - 0.4).
\end{aligned} \tag{43}$$

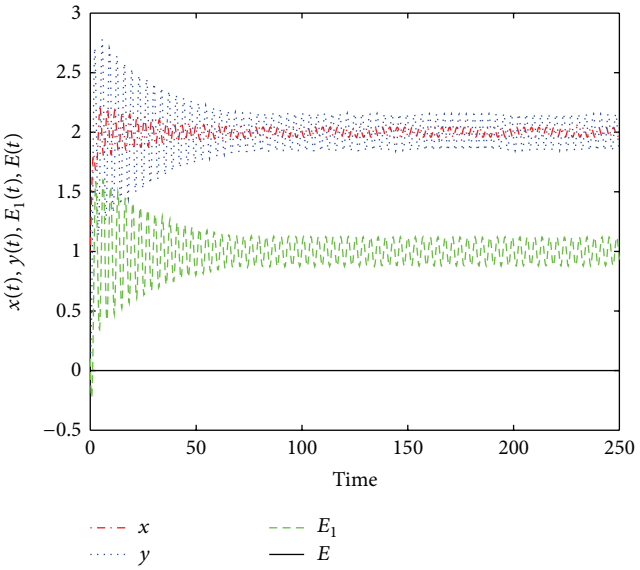


Figure 1: The dynamic responses of (45) near equilibrium before control.

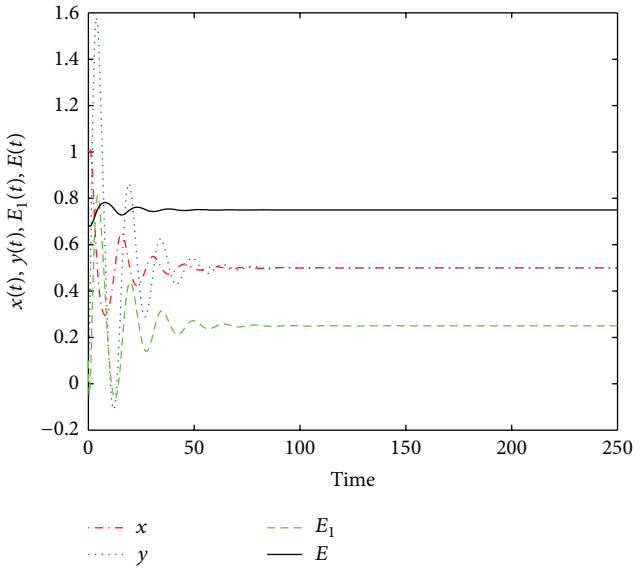


Figure 2: The dynamic responses of (45) near equilibrium ($k_1 = 4$, $k_2 = -1.75$).

And by evaluating, the positive equilibrium P_3 is $(0.5, 0.5, 0.25, 0.75)$. Based on Theorem 3, (43) undergoes singularity induced bifurcation near P_3. According to the discussion in Section 6, let

$$u(t) = k_1 (E(t) - 0.75) + k_2 (E_1(t) - 0.25), \tag{44}$$

where k_1, k_2 are feedback gains to be determined.

Then the closed-loop system is

$$\begin{aligned} \dot{x}(t) &= x(t)\left(1 - \frac{x(t) + y(t)}{4}\right) - E(t)\,x(t), \\ \dot{y}(t) &= 1.5y(t) - 3E_1(t), \\ \dot{E}_1(t) &= 2y(t) - 2E_1(t) - x(t) + k_2 (E_1(t) - 0.25), \\ 0 &= E(t)(0.8x(t) - 0.4) + k_1 (E(t) - 0.75). \end{aligned} \tag{45}$$

Based on Theorem 4, when k_1, k_2 satisfy

$$\begin{aligned} k_1 &> 0.31 \\ k_2 &< \min\{-0.35, -1.09, -1.71\}, \end{aligned} \tag{46}$$

(45) is stable at P_3, and the singularity induced bifurcation of (43) is eliminated. The dynamic responses of (45) before and after control are illustrated in Figures 1 and 2.

Figure 1 shows that the purification capacity fluctuates as the density of the *Spartina anglica* fluctuates. At the same time the density of the fish also fluctuates which is less choppy than the above two terms. The abundance of *Spartina anglica*, the small number of fish, and the sustainable purification capacity together make the harvesting capacity nonexistent before controlling of the purification capacity and the harvesting capacity. In comparison with Figure 1, Figure 2 reflects the effectiveness of the feedback controller. It is noted that the four variations are sustainable at expected levels after ephemeral fluctuation in Figure 2. Furthermore, the density of fish and *Spartina anglica* are at the same level 0.5 and the harvesting capacity is much higher than the purification capacity, which ensures the economic revenue to be existent. It effectively shows the purpose of feedback control and demonstrates convincingly the theory proposed in this paper.

8. Conclusions

There exists broad recognition that the fisheries and aquaculture are under stress.

The invasion of *Spartina anglica* has been one of the most intimidating and dangerous factors to threaten the fisheries and aquaculture. Clearly, new management approaches or options must be considered to stem the damage and ensure that the fisheries and aquaculture ecosystems, as well as their unique features, are protected and restored. Current management of fish resources is insufficient. In this regard, purification capacity is increasingly proposed as major tools to relieve the stress on environmental resources and ecosystems. On the basis of this requirement and current trends, using differential algebraic systems theory, we analyze the dynamical behavior of a single-species fish population logistic model by treating the invasion of *Spartina anglica* as a regulatory mechanism.

This paper has mainly investigated the problems of bifurcation and control for a singular species fish population with logistic growth by incorporating invasion of alien species and further gotten the following conclusions. At the zero equilibrium, the system will raise trans-critical bifurcation so that the fish population is extinct. At this moment, the local ecological cycle of the system will breakdown and the respective component will not exist. It may be concluded that the ecosystem will collapse owing to the massive harvested

effort of fish and disregarding of the control of *Spartina anglica* for a long time. It is observed that the alien species *Spartina anglica* is the activator for the extinction. At the positive equilibrium, the system will raise singularity induced bifurcation. The quantity of fish population and alien species *Spartina anglica* switch at the instant, even beyond the capacity of the environment, eventually lead to ecological disaster and ecological imbalance. It is evident from the obtained results that the alien species can cause a stable equilibrium to become unstable, and even an impulsive phenomenon may occur when the economic revenue goes through its critical value. It is also clear that when the purification capacity is not adequate, both the fish and alien species populations reach immeasurable change around the equilibrium. As the density of the alien species increases, the fish population dies out. Since the environmental resources are finite, the alien species dies out eventually. It is obvious that both the purification capacity and the harvested effort are necessary to be considered.In consequence, the *Spartina anglica*, which is introduced for the purpose of improving soil and protecting environment, will lead to ecological destruction with the improper control of the density. Since this paper studies the alien species *Spartina anglica*, it is inevitable to introduce the purification capacity and controlling; moreover both too high and too low density of *Spartina anglica* cannot attain the aim of the introduction. On the other hand, the purification capacity plays an important role in the economically sustainable harvested effort of the fish population which at least provides space and adequate nutrients for the growth of fish.

It should be noted that the proposal of purification makes the work studied in this paper have some new and positive features. The feedback controller designed in this paper eliminates the singularity induced bifurcation and inhibits the switch of population to make the system stable by controlling purification capability and harvested effort. It is deemed important to undertake this type of study for the purpose of investigating the impact of purification so that sustainability of the ecosystem may be resumed through achieving the commercial purpose of the fishery. Hence, the results of our study are not only useful for assessing the biological, social, and economic impacts of existing resources but also provide appropriate measures to maintain long-term sustainability of the resource.

In practice, the biological economic system can be stable by formulating some strategies according to the purification capacity and harvested efforts of fishers, such as taxation, license fees, lease of properties rights, seasonal harvesting, marine protected areas, and issuing purification subsidy. Therefore, not only ecological resources can be protected to promote silting reclamation, but also economics can be developed sustainably. The study has produced far-reaching influence to maintain the sustainable and favorable cycle of the ecosystem and improve the environment of water and soil and gain economic interest in the long term. Therefore, the theory proposed in this paper is significantly different from the existing models in the field of research which use differential algebraic equations and needs separate investigation.

The entire study of the paper is primarily based on a deterministic framework. On the other hand, it will be more realistic if it is possible to incorporate ecological fluctuations and uncertainty in the model system. Thus, a future research problem would be to consider the system with interval parameter functions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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