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CROP PRODUCTION

Fitting nonlinear double sigmoid models to macauba fruit growth data using Bayesian inference

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ABSTRACT: Macauba (*Acrocomia aculeata*) is a palm tree native to the tropical regions of the Americas. It is used as a food and applied in the cosmetic and pharmaceutical industries. Macauba fruit is a source of raw material for oil and bioenergy production. Determining the ideal time to harvest macauba fruit is essential to ensure maximum quality and optimize the post-harvest and storage processes. The present study characterized the accumulation of total dry mass and mesocarp dry mass of macauba fruit over time. We used Bayesian inference to estimate the parameters of the nonlinear double logistic and double Gompertz regression models to find the model that best describes the data. Fruit samples were collected from one week after spathe opening and continuing weekly until natural fruit drop, at 62 weeks after anthesis, and dried in a forced ventilation oven. Statistical analyses were conducted in the R software, and the weighted resampling method was used. The Criterion of density Predictive Ordered (CPO) and deviance information criterion (DIC) were used to select the models. The double Gompertz model best fitted the data. The fruit mesocarp showed no difference in growth rate between the first and second stages of development. In the final stage, it reached values close to 9 g. The total dry mass showed faster growth in the first stage, with values close to 34 g in the final stage.
Key words: regression, double Gompertz, double logistic, mesocarp.

Ajuste de modelos não lineares duplo sigmoides no crescimento do fruto de macaúba por inferência bayesiana

RESUMO: A macaúba (*Acrocomia aculeata*) é uma palmeira nativa das regiões tropicais dos continentes americanos. É utilizada na alimentação, e aplicada na indústria cosmética e farmacêutica. Seus frutos são fontes de matéria-prima para a produção de petróleo e bioenergia. Determinar o momento ideal para a colheita dos frutos é fundamental para garantir a máxima qualidade, além de otimizar os processos de pós-colheita e armazenamento. Este estudo teve como objetivo caracterizar o acúmulo de massa seca total e do mesocarpo do fruto de macaúba ao longo do tempo utilizando inferência bayesiana na estimação dos parâmetros dos modelos de regressão não linear duplo Logístico e duplo Gompertz. Além de selecionar o modelo que melhor descreveu os dados. A coleta dos frutos foi realizada a partir de uma semana após a abertura da espata, continuando semanalmente até a queda natural dos frutos, às 62 semanas após a antese, sendo secos em estufa de ventilação forçada. As análises estatísticas foram conduzidas no software R. Foi utilizado o método numérico de reamostragem por importância ponderada. Na seleção dos modelos foram utilizados o critério da densidade preditiva ordenada (CPO) e o critério de informação da deviance (DIC). O modelo duplo Gompertz foi o que melhor se ajustou aos dados. O mesocarpo do fruto não apresentou diferença na velocidade de crescimento entre a primeira e segunda etapa de desenvolvimento. Na etapa final, apresentou valores próximos a 9g. A massa seca total mostrou crescimento mais acelerado na primeira etapa, com valores próximos a 34 g na etapa final.
Palavras-chave: regressão, duplo Gompertz, duplo logístico, mesocarpo.

INTRODUCTION

Macauba (*Acrocomia aculeata*) belongs to the family Arecaceae and is also known in Brazil as bocaiúva, bacaiuveira, bacaúva, and other names depending on the region where it is found. It is a species of palm tree native to the tropical regions of the American continent. It can be found in a variety of environments, from areas of the cerrado, which have a dry climate and acidic soil that is poor in nutrients, to the Pantanal, characterized by flooded soils. This demonstrates its adaptive capacity (CICONINI et al., 2013).

This plant provides several sources of raw material. Its leaves are used in animal feed, while the fruit can be consumed and has applications in both the

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cosmetic and pharmaceutical industries (DIAS et al., 2021; FONSECA et al., 2021). In addition, macauba fruit has a high oil content and is a source of raw material for oil and bioenergy production, including biodiesel, biogas, biooil, biokerosene, second-generation ethanol, and solid biofuels, contributing to the generation of electricity and steam (ALVES et al., 2022). The macauba plant plays an important role in the recovery of degraded lands and agroecological intercropping, in addition to acting as a carbon-retaining species (MOREIRA et al., 2020).

This plant blooms once a year, usually in December and January. The fruiting period lasts about 14 months, from the emergence of the inflorescence to the fruit harvest. In general, the harvest of macauba fruit is based on extraction, that is, they are collected when they fall to the ground (NUNES et al., 2015). One of the most exploited raw materials is the oil present in the fruit; however, the processing between harvesting and storage of the fruit must be quick to avoid losses in the quality of the oil. In this sense, it is important to determine the ideal time for harvesting, as this increases the useful time for its use, whether for consumption or as a source of raw material (EVARISTO et al., 2016).

Macauba fruit are drupes and consist of epicarp (peel), mesocarp (pulp), endocarp (integument) and seed (kernel). The mesocarp represents approximately 40% of the fruit and may vary according to the region where it is found. The mesocarp and kernel contain the largest amount of oils. The concentration of oils varies from 55% to 69% of the dry matter in the mesocarp, and from 55% to 58% in the kernel. Both are products of the fruit of greater economic interest (EVARISTO et al., 2016; EVARISTO et al., 2017; COLOMBO et al., 2018).

Fruits, in general, can present a growth pattern with simple, double, and, in some cases, triple sigmoidal behavior. In stone fruits, growth follows three stages of development, resulting in a growth pattern with double sigmoidal characteristics. There is accelerated growth, followed by a deceleration, followed again by an acceleration in growth. This occurs because the seed grows first, followed by the pulp (COOMBE, 1976).

Nonlinear regression models describe well data with a double sigmoidal pattern and have the advantage of having a direct interpretation of some of their parameters. The fit of these models has been widely used in studies on growth curves, as demonstrated by AZARIAS et al. (2024), FERNANDES et al. (2017), MARTINS FILHO et al. (2008), SILVA et al. (2020b) and SILVA et al. (2021) in research with weeds, coffee, bean cultivars, blackberry, and coconut, respectively.

One approach that has been widely used in fitting nonlinear models is Bayesian inference, which can be applied to both large and small data sets, as well as to more complex models (FERNANDES et al., 2022; KAPLAN, 2023). This approach is based on Bayes' theorem and considers the model parameters as random variables that have a probability distribution. Furthermore, it allows the researcher to incorporate their prior knowledge about the problem into the analysis, called prior $P(\theta)$. This prior knowledge is then updated based on the joint probability distribution, which represents the information contained in the data and is called the likelihood function $L(Y|\theta)$ (SILVA et al., 2023b; SILVA et al., 2020a; TURKMAN et al., 2019).

Using Bayes' theorem, the combination of likelihood and prior results in the posterior distribution $P(\theta|Y)$, which contains probabilistic information about the parameter θ From this distribution, it is possible to obtain information about the parameters, such as mean, mode or median, in addition to the highest posterior density credibility interval (HPD), in which there is a specific probability of containing the true value of the parameter (KAPLAN, 2023; LYNCH, 2021; REIS et al., 2009; REIS et al., 2008; SAVIAN et al., 2009).

To obtain information related to each of the parameters, the joint posterior distribution is integrated in relation to the other parameters, thus obtaining the marginal posterior distribution for each one. However, this process involves integrals that can be difficult to solve analytically, requiring the use of numerical approximation methods to obtain these marginal distributions. One example is the weighted importance resampling method, which has the advantage of generating independent samples, does not requiring the use of convergence evaluators, and is computationally faster (FERNANDES et al., 2022b; PEREIRA et al., 2022; LYNCH, 2021; KAPLAN, 2023).

The objective of this study was to analyze the increase in mesocarp dry mass and total dry mass of macauba fruit over time. We used Bayesian inference to estimate the parameters of the nonlinear double logistic and double Gompertz regression models to identify the model that best describes the data.

MATERIALS AND METHODS

Data were obtained from MONTOYA (2016). Macauba plants analyzed were over 10 years old, and at the reproductive stage, being native to the

municipality of Acaiaca, located in the southeastern region of the state of Minas Gerais, Brazil, with a humid subtropical climate. Thirty bunches from 24 different plants were selected. Fruit collection was carried out manually, starting one week after spathe opening and continuing weekly until natural fruit drop, at 62 weeks after anthesis (WAA). Data were obtained by the cross-sectional method, which involves destructive data collection. Fruit samples were placed in paper bags and dried in a forced ventilation oven at 65 ºC (model TE-394/3-TECNAL) to constant weight.

After drying, the whole fruits were weighed again to determine the mass of the dried fruits. Then, all the constituent parts of the fruit were separated; however, in this study, only the dry mass of the mesocarp (pulp) and total dry mass were used, which were weighed on an analytical balance (GEHAKA model BG 400), being carried out at 45 DAA. For data analysis, the nonlinear models were adjusted: Double Logistic (1) and Double Gompertz (2), which can be defined as:

$$y_i = \frac{a_1}{1+e^{k_1(b_1-x_i)}} + \frac{(a_2-a_1)}{1+e^{k_2(b_2-x_i)}} + \varepsilon_i \quad (1)$$

$$y_i = a_1 e^{-e^{k_1(b_1-x_i)}} + (a_2-a_1)e^{-e^{k_2(b_2-x_i)}} + \varepsilon_i \quad (2)$$

Where i = 1, 2, ..., n, y_i is the dependent variable, indicating the dry mass of the mesocarp or total dry mass of the fruit; x_i is the independent variable, indicating weeks after anthesis of the fruit; a_1 represents the maximum expected dry mass of the mesocarp or total dry mass of the fruit for the first growth stage; a_2 is the final dry mass of the mesocarp or total dry mass of the fruit; b_1 and b_2 represent the abscissa of the inflection point in the respective stages 1 and 2; k_1 and k_2 represent the growth index in stages 1 and 2; respectively, the higher they are, the faster the fruit reaches its maximum growth. Where $\varepsilon_i \sim N(0,\sigma^2)$.

Once $\varepsilon_i \sim N(0,\sigma^2)$, then the likelihood can be represented by:

$$L(Y|\theta) = (2\pi\sigma^2)^{\frac{-n}{2}} exp\left\{-\frac{(Y-\mu)'(Y-\mu)}{2\sigma^2}\right\}$$

Where μ varies according to the fitted model.

Based on information from the literature and specialists on the growth of macauba fruit, a priori distributions, and hyperparameters were set for the two models, also considering the interpretation of the parameters. As previously mentioned, parameters a_1 and a_2, represent the maximum growth or accumulation of dry mass by the fruit at the end of each stage and can only assume positive values. Therefore, the gamma distribution was used. For b_1 and b_2, which indicate the abscissa of the inflection point for each of the sigmoids, and must be positive in both models, the gamma distribution was also considered. For k_1 and k_2, which represent the growth index and can assume both positive and negative values, the normal distribution was considered.

Using Bayes' theorem and assuming independent priors, the joint posterior distribution P(θ|Y) for the vector of each model parameter is given by the product of the priors P(θ) and the likelihood L(Y|θ).

$P(\theta|Y) = L(Y|\theta)P(a_1)P(b_1)P(k_1)P(a_2)P(b_2)P(k_2)$

Where

$a_1 \sim Gamma(\alpha_{a_1}; \gamma_{a_1})$, $b_1 \sim Gamma(\alpha_{b_1}; \gamma_{b_1})$, $k_1 \sim Normal(\mu_{k_1}; \sigma^2_{k_1})$, $a_2 \sim Gamma(\alpha_{a_2}; \gamma_{a_2})$, $b_2 \sim Gamma(\alpha_{b_2}; \gamma_2)$, $k_2 \sim Normal(\mu_{k_2}; \sigma^2_{k_2})$ are the prior distributions for the parameters, following the parameter space given in models 1 and 2.

$\alpha_{a_1}, \gamma_{a_1}, \alpha_{b_1}, \gamma_{b_1}, \mu_{k_1}, \sigma^2_{k_1}, \alpha_{a_2}, \gamma_{a_2}, \alpha_{b_2}, \gamma_{b_2}, \mu_{k_2}, \sigma^2_{k_2}$ are the hyperparameters of the prior distributions, not necessarily the same for the two models.

To obtain the joint posterior distribution, we used the weighted importance resampling technique proposed by SMITH & GELFAND (1992). This methodology consists of generating samples from a distribution whose shape is unknown, using another distribution in the same domain or parametric space as the distribution of interest that can be easily sampled. Based on the importance sampling theory (RUBIN, 1987), multiply and divide the expression f(·) by g(·), and obtain:

$$\frac{f(\theta)}{g(\theta)} \times g(\theta)$$

The samples are generated from g(θ), which is considered the candidate distribution, known and defined in the same parameter space as distribution f(θ) and weighted by calculated weights obtained based on the $\frac{f(\theta)}{g(\theta)}$ ratio. With this process, an approximate sample of f(θ) is obtained.

This research adopted the methodology proposed by FERNANDES et al. (2022b), which considers the candidate distribution g(θ) as an approximation of the likelihood, denoted by Q(θ). According to the authors, this approach aims to simplify the calculations of the weights for resampling. Considering a sample of size M from Q(θ). By assuming a normal distribution for the model and as $M \to \infty$, it follows that Q(θ) converges in distribution to L(Y|θ). Thus, the weights are calculated as follows:

$$w_i = \frac{f(\theta_i)}{g(\theta_i)} = \frac{P(\theta_i|Y)}{Q(\theta_i)} = \frac{L(Y|\theta_i)P(\theta_i)}{Q(\theta_i)} \simeq \frac{L(Y|\theta_i)P(\theta_i)}{L(Y|\theta_i)} = P(\theta_i)$$

and the normalization is given by:

$$q_j = \frac{w_j}{\sum_t^M w_j} = \frac{P(\theta_i)}{\sum_t^M P(\theta_t)} = P(\theta_i)$$

As M→ ∞, the denominator of the above expression tends to 1. Thus, the weights are obtained directly from the a priori distribution.

The criterion of predictive ordered (CPO) of GELFAND & DEY (1994) was used to compare the models. The CPO is given by the derivative of the posterior predictive density. Given an observation y_i, considering all the others $Y_{(-1)}$, where $Y_{(-1)}$ is the vector of observations except the i-th. The CPO measure can be written as:

$$CPO_i(l) = f(y_i|Y_{(-i)}, l) = \int f(y_i|Y_{(-i)}, \theta, l)P(\theta|Y_{(-i)}, l)\, d\theta$$

Where $P(\theta|Y_{(-i)})$ is the posterior density for parameters $\boldsymbol{\theta}$ related to the data $\mathbf{Y}_{(-i)}$ and l is an index indicating the model.

Using numerical methods and considering that $\boldsymbol{\theta}^{(j)}$, with j = 1, 2, ..., M as the generated samples, the above equation can be approximated by:

$$\hat{c}_l = \hat{f}(y_i|Y_{(-i)}, l) = \frac{1}{m}\sum_{j=1}^{m} f(y_i|Y_{(-i)}, \theta^{(j)}, l)$$

The choice of model l is made such that $C_l = \prod_{i=1}^{n} C_i(l)$ is maximum. The highest value of C_l (on average) indicates the best model. Another way to use the CPO is to choose the model l such that $\sum_{i=1}^{n} lnc_i(l)$ is maximum (ANDRADE FILHO et al., 2010; TURKMAN et al., 2019).

The deviance information criterion (DIC) is a measure of comparison and adequacy of models based on the posterior distribution of the deviance and includes a penalty for model complexity. It is used in the selection of Bayesian models and the posterior distribution of the parameters is obtained by the samples of the marginal distribution of the parameters generated by weighted resampling. The deviance of the model is defined as follows:

$D(\theta) = -2\log L(y|\theta)$

Where $L(y|\theta)$ is the likelihood function for the observed data y given the vector of unknown parameters θ.

The effective number of parameters of the model, given by p_D, is determined by

$$p_D = \bar{D}(\theta) - D(\bar{\theta})$$

Where $D(\bar{\theta}) = -2logL(y|\bar{\theta})$ and $\bar{\theta}$ is the a posteriori mean of the parameters. The DIC criterion is calculated by:

$$DIC = \bar{D}(\theta) + p_D = 2\bar{D}(\theta) - D(\bar{\theta})$$

Lower DIC values indicate better model fit (KAPLAN, 2023). The intervals (HPD) were also obtained, with a credibility level of 95%, and the results were used to check for significance and overlap of the parameters. The estimation of the model parameters, the statistical tests, the graphs, as well as the entire computational part of this research, were performed using the open-access statistical software R (R DEVELOPMENT CORE TEAM, 2024).

RESULTS AND DISCUSSION

The marginal distributions of each parameter were obtained through weighted importance resampling, which generates independent samples. A total of 10.000 samples of Q($\boldsymbol{\theta}$) were considered for each model. This approach eliminates the need for diagnostics to assess convergence, such as those of GEWEKE (1992) and RAFTERY & LEWIS (1992), which are commonly used in Markov Chain Monte Carlo (MCMC), using the Gibbs sampler and Metropolis-Hastings algorithms (FERNANDES et al., 2022b; REIS et al., 2011; PEREIRA et al., 2022; STRAUB & PAPAIOANNOU, 2015).

Table 1 presents the estimates obtained for the a posteriori mean, as well as the 95% HPD interval, for all parameters of the fitted double sigmoidal models, considering the data related to the dry mass of the mesocarp and total dry mass of the macauba fruit. It is observed that all parameters were statistically different from zero, as verified by the HPD intervals that did not contain zero. These results suggest that the double Logistic and double Gompertz models are suitable to describe the analyzed data.

Nonlinear models were also fitted using Bayesian inference by ZARZAR et al. (2022) in shrimp growth, PEREIRA et al. (2022) in fitting the height growth of coffee plants, PEREIRA et al. (2009) in predicting mineralized nitrogen, and SILVA et al. (2022) in the dynamics of decomposition of organic residues in the soil.

Parameters k_1 and k_2 represent the growth rate in stages 1 and 2, respectively. The higher these values, the faster the fruit reaches its maximum growth. In table 1, the estimates of these parameters, considering the dry mass of the mesocarp, presented overlap in their HPD intervals, both for the double Gompertz model and the double Logistic model, indicating that there is no difference in the growth acceleration of the macauba fruit mesocarp between the first and second stages. This is different from what occurs with peach fruit, in which one of the growth stages is more accelerated than the other, according to FERNANDES et al. (2022a).

Parameter a_2, which represents the dry mass of the fruit mesocarp in the final growth stage,

Table 1 - Mean and HPD interval for the parameters of the double logistic and double Gompertz models, in describing the dry mass of the mesocarp and total dry mass of the macauba fruit.

Model	--------Parameter-------	----------Mean----------	----------HPD$_{95\%}$----------	
			----------LL----------	----------UL----------
Dry mass of the mesocarp				
double logistic	a_1	3.3058	3.1464	3.4606
	b_1	5.3627	4.3618	6.3798
	k_1	0.4881	0.2452	0.7241
	a_2	9.6592	9.3194	10.0153
	b_2	46.1342	45.4137	46.7858
	k_2	0.3184	0.2755	0.3612
double Gompertz	a_1	3.3742	3.2208	3.5498
	b_1	4.4701	3.2355	5.5531
	k_1	0.3685	0.2299	0.5092
	a_2	9.9345	9.4350	10.4487
	b_2	44.4869	43.7450	45.2051
	k_2	0.2093	0.1700	0.2462
Total dry mass				
double logistic	a_1	16.4819	15.5169	17.3418
	b_1	8.9787	8.6961	9.2735
	k_1	0.4409	0.4013	0.4845
	a_2	36.1029	31.9232	40.1280
	b_2	50.1049	46.1136	54.3683
	k_2	0.0975	0.0735	0.1210
double Gompertz	a_1	18.2803	17.7824	18.7881
	b_1	7.7101	7.4377	7.9954
	k_1	0.2514	0.2355	0.2683
	a_2	39.4148	34.5765	44.1248
	b_2	49.5980	45.6897	53.3973
	k_2	0.0617	0.0481	0.0763

showed values close to 9 g in both models (Table 1). These results are similar to those reported by EVARISTO et al. (2017) in fruits harvested in Sete Lagoas, state of Minas Gerais. Similar values were found by MACHADO et al. (2015), who evaluated the biometric characteristics of macauba fruit from two different ecotypes, observing a mesocarp mass of 10.49 g in the total ecotype. However, the sclerocarpa ecotype presented a higher mesocarp dry mass, 31.05 g.

The estimates of parameters b_1 and b_2 indicate the ages at which the fruit mesocarp reaches the maximum growth rate in each of the stages. In table 1, considering the Logistic model, the fruit mesocarp reached the maximum growth rate at approximately 5 WAA for the first stage (b_1), and 46 WAA for the second stage (b_2). The Gompertz model estimates this occurrence a little earlier, with b_1 at 4 SAA, and b_2 at 44 SAA.

Parameter a_2, which represents the total dry mass of the fruit in the final growth stage, presented values close to 34 g in both models. EVARISTO et al. (2017) reported an average of 25.23 g for fruit analyzed from three different regions. These values also differ from those observed by MACHADO et al. (2015), who found a total dry mass of 66.76 g for the sclerocarpa ecotype, and 19.67 g for the totai ecotype.

Estimates of parameters b_1 and b_2 indicate the ages (weeks after anthesis - WAA) at which the fruit reaches the maximum growth rate in each of the stages. The total dry mass of the fruit reached the maximum growth rate between 7 and 9 SAA for the first stage (b_1), and between 45 and 54 WAA for the second stage (b_2), which occurred after the growth of the mesocarp dry mass.

Regarding the parameters k_1 and k_2, which did not present overlapping HPD intervals, k_1 was greater than k_2. This indicates that, concerning the total dry mass of the fruit, growth is faster in the first stage in macauba fruit. This same behavior was also verified by FERNANDES et al. (2022b) in coffee

fruit, with faster growth in the first stage. This is different from what was observed with blackberries, which present a faster second stage growth than the first in some cultivars (SILVA et al., 2020b).

The densities of the marginal distributions for the parameters of these models are illustrated in figure 1. For most parameters, the variance of the prior distribution is greater than that of the posterior distribution, indicating a gain in posterior precision. However, for parameters b_1 and b_2, there was not much gain in precision, which may have occurred because the priors used were more informative.

Table 2 presents the estimates for CPO and DIC considering the fit of the two models. The double Gompertz model presented the highest CPO value and lowest DIC, indicating that it best describes the data, indicating asymmetry in relation to the inflection point. The double Gompertz model also presented good fits in the description of the growth of blackberry fruit (SILVA et al., 2020b) and in the growth of peach fruit (FERNANDES et al., 2022a).

The inflection point (IP) indicates when growth is maximum. Based on the means of the posterior marginal distribution of the parameters, the IP calculations of the double Gompertz model were made for the first and second growth stages, since this model presented the best fit according to the selection criteria adopted. SILVA et al. (2023a) also fitted nonlinear double sigmoid models in blackberry fruit and used the calculations to determine the IP coordinates, as described by SILVA et al. (2021).

Figure 2 shows the growth rate of the macauba fruit mesocarp, obtained from the first derivative of the double Gompertz model. This figure highlights the double sigmoidal growth characteristic of the mesocarp, characterized by a first stage of rapid growth, followed by a stage with slow growth, and finally, by another stage of rapid growth, as described by COOMBE (1976). The PI coordinates were calculated according to the expressions derived by SILVA et al. (2021). The IP occurred at the peak of the curve in both growth stages, i.e., 4 WAA and 44 WAA, for stages 1 and 2, respectively.

The densities of the marginal distributions for the model parameters, considering the total dry mass of the fruit, are shown in figure 3. In both models, for parameters k_1, k_2 and a_1, the variance of the prior distribution is greater than the variance of the posterior distribution, indicating a gain in posterior precision. In contrast, for parameters a_2 and b_2, the variance of the posterior is greater than that of the prior, suggesting a reduction in precision. This may occur due to the choice of less informative priors, as in the case of a_1 in the Gompertz model, where the prior shows a not very satisfactory initial precision.

The parameter k is generally in an interval between 0 and 1 (AZARIAS et al., 2023; AZARIAS et al., 2024). With the inclusion of data information,

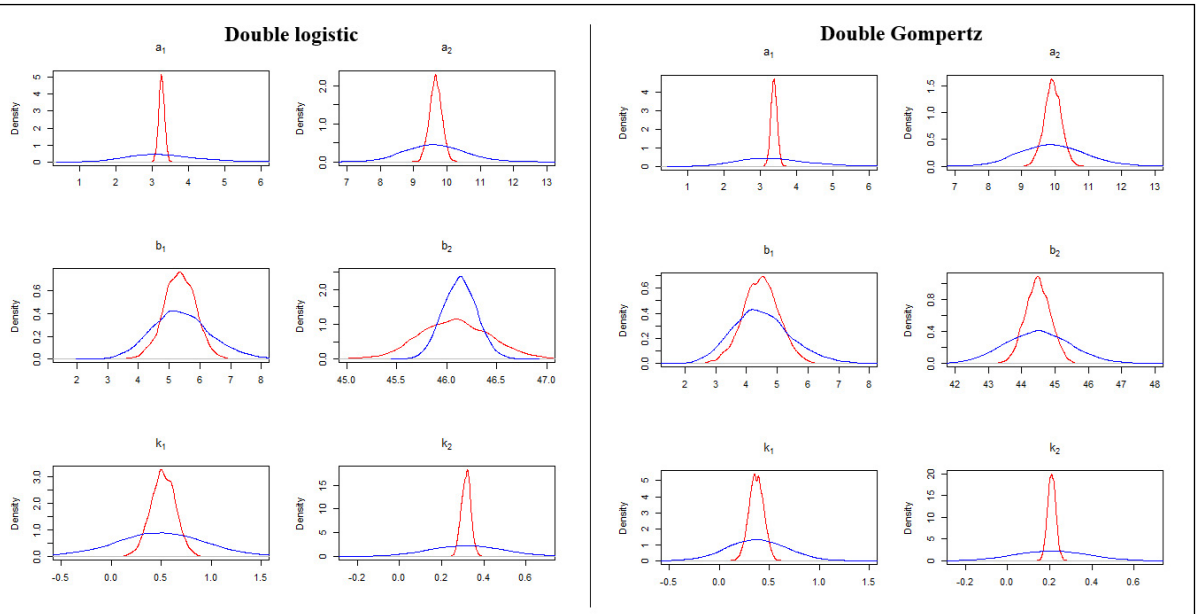


Figure 1 - Prior density (in blue) and posterior density (in red) of the double logistic and double Gompertz models for the dry mass of the macauba fruit mesocarp.

Table 2 - Estimates for the CPO and DIC considering the fit of the double logistic and double Gompertz models for the dry mass of the macauba fruit mesocarp.

Model	-------CPO-------	--------DIC--------
double logistic	5.9459	267.7572
double Gompertz	9.0502	222.7404

there was an increase in precision, as can be observed in figures 1 and 3. Considering the mesocarp dry mass, the posterior distribution for the parameters k_1 and k_2 is within the credibility intervals k_1 = (0.2299 e 0.5092) e k_2 = (0.1700 e 0.2462). For total dry mass, the credibility intervals are k_1 = (0.2355, 0.2683) and k_2 = (0.0481, 0.0763).

Table 3 presents the estimates for CPO and DIC, considering the fits of the two models. The double Gompertz model again presented the highest CPO value and the lowest value for DIC, indicating that this model describes the data better and that, for the total dry mass, asymmetry also occurs in relation to the inflection point.

Based on the means of the marginal posterior distribution of the parameters, the IP of the double Gompertz model was also calculated for the first and second growth stages, considering the total

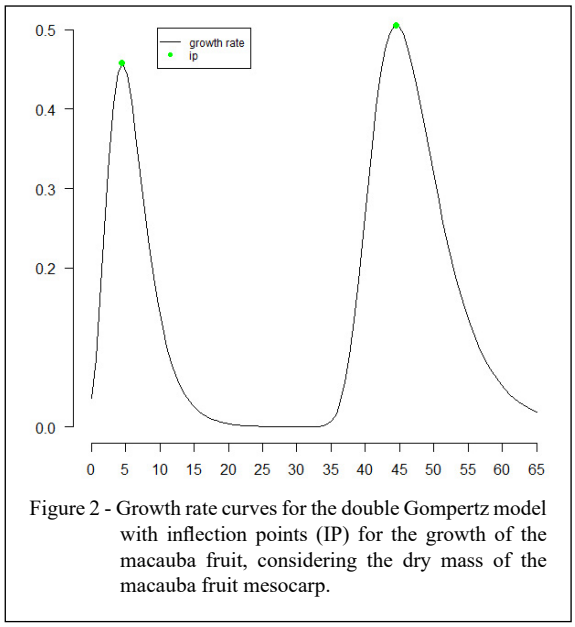


Figure 2 - Growth rate curves for the double Gompertz model with inflection points (IP) for the growth of the macauba fruit, considering the dry mass of the macauba fruit mesocarp.

dry mass of the macauba fruit. Figure 4 shows the double sigmoidal growth character of the total dry mass of the macauba fruit. The IP occurred at the peak of the curve in both growth stages, that is, at 8 WAA and 49 WAA for stages 1 and 2, respectively. In addition, k_1 is greater than k_2, indicating a more

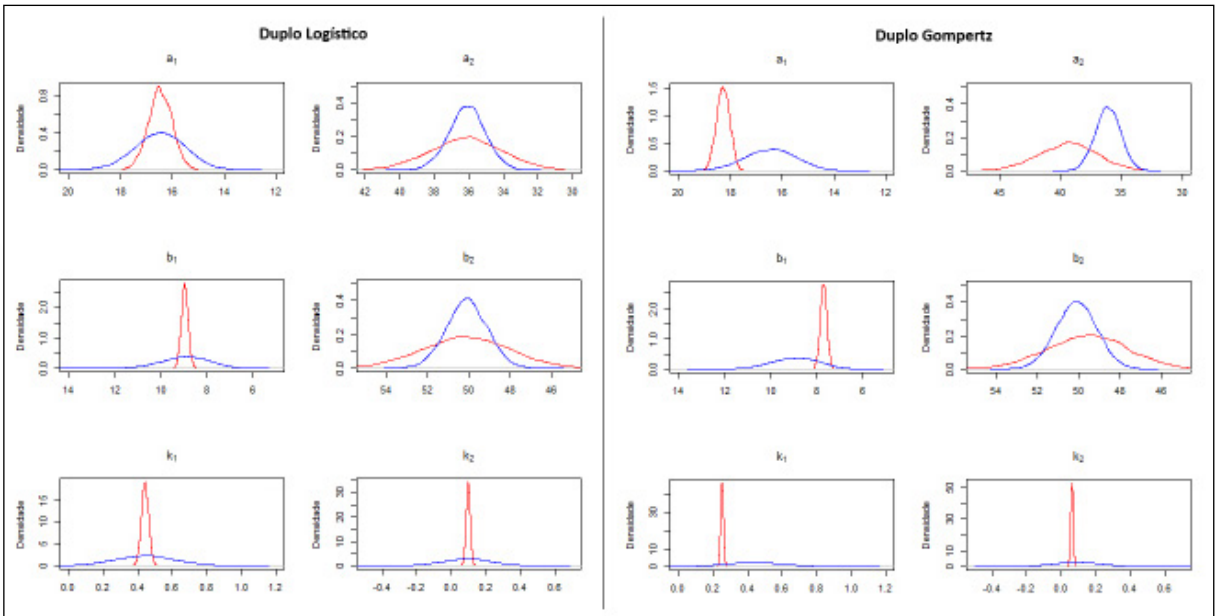


Figure 3 - Prior density (in blue) and posterior density (in red) of the double logistic and double Gompertz models for the total dry mass of the macauba fruit.

Table 3 - Estimates for the CPO and DIC considering the fit of the double logistic and double Gompertz models for the total dry mass of the macauba fruit.

Model	--------CPO-------	--------DIC--------
double logistic	1.9537	509.3359
double Gompertz	2.8179	339.3501

pronounced growth in the first stage compared to the second stage.

CONCLUSION

The Bayesian theory, with the weighted importance of resampling, was efficient in estimating the parameters, presenting estimates consistent with the literature. The macauba fruit mesocarp did not present any difference in growth rate between the first and second stages of development, while the total dry mass showed faster growth in the first stage.

In the final stage of growth, the fruit mesocarp presented values close to 9 g. The maximum accumulation of dry mass by the mesocarp occurred at 4 WAA and 44 WAA for stages 1 and 2, respectively. The total dry mass of the fruit in the final growth stage presented values close to 34 g, with maximum accumulation observed at 8 WAA and 49 WAA for stages 1 and 2, respectively.

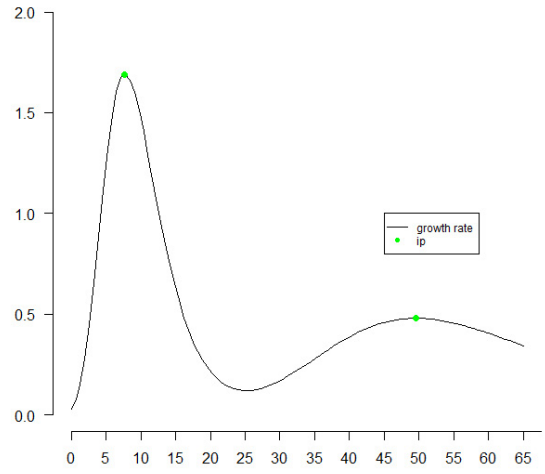


Figure 4 - Growth rate curves for the double Gompertz model with inflection points (IP) for the growth of the macauba fruit, considering the total dry mass.

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DECLARATION OF CONFLICT OF INTEREST

We have no conflict of interest to declare.

AUTHORS' CONTRIBUTIONS

The authors contributed equally to the manuscript.

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