

K-nacci Sequences and Their Application in Solar Panel Distribution: A Computational Study Based on the Vogel Model

Jose Ranero García

¹ Argira Station, Independent Researcher, Spain

Corresponding author: josecorreo1@outlook.es

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Abstract

The Fibonacci sequence and its generalizations, known as k-nacci sequences, exhibit spatial distribution properties that have inspired engineering applications through biomimetics. This study presents a computational exploration of k-nacci arrangements ($k = 2 \dots 10$) applied to the optimal placement of solar panels within a circular area, generalising Vogel's phyllotaxis model. Using a Python simulation validated with NumPy, we compare four quantitative metrics – minimum inter-element distance, mean nearest-neighbour distance, distribution uniformity, and shadow index – across k-nacci, rectangular grid, hexagonal, and random layouts ($N = 200$ panels, circular area of radius 240 px). Results show that the Fibonacci arrangement ($k = 2$) achieves 98.9% uniformity and zero shadow index, closely matching hexagonal packing (100%) while avoiding its periodic alignment gaps. Higher-order k-nacci sequences ($k \geq 3$) show progressive degradation in both uniformity and shadow avoidance. These findings confirm that the Fibonacci case is the optimal member of the k-nacci family for circular panel distribution, providing a mathematically simple, closed-form placement law rooted in natural phyllotaxis patterns.

Keywords: Fibonacci; k-nacci sequences; golden ratio; solar panels; phyllotaxis; Vogel model; biomimetics; packing optimisation.

1. Introduction

The arrangement of seeds in a sunflower head, described mathematically by Vogel's model [1], is one of the most striking examples of how natural patterns can inspire engineering solutions. In Vogel's model, the n -th seed is placed at a radial distance proportional to $\sqrt{n}$ and at an angular increment equal to the golden angle $\alpha_2 = 360^\circ/\varphi^2 \approx 137.508^\circ$, where $\varphi = (1+\sqrt{5})/2 \approx 1.618$ is the golden ratio, the limit ratio of consecutive Fibonacci numbers [2, 3]. This arrangement maximises the minimum distance between elements and avoids periodic gaps – properties that are directly useful in solar farm and solar tree design [4].

A natural mathematical generalisation is the family of k-nacci sequences, where each term is the sum of the k previous terms. For $k = 2$ we recover the classical Fibonacci sequence; for $k = 3$, Tribonacci; for $k = 4$, Tetranacci, and so on. Each sequence has its own characteristic constant φ_k and an associated angle $\alpha_k = 360^\circ/\varphi_k^2$. While the algebraic properties of k-nacci sequences are well studied [5, 6], their potential for engineering layout problems has received limited attention.

This paper addresses the following question: does the mathematical advantage of the Fibonacci arrangement extend to higher-order k-nacci sequences, and if so, which value of k is optimal for solar panel distribution inside a circular area? We answer this through validated Python simulations comparing k-nacci layouts against conventional alternatives, using four quantitative metrics designed to capture the engineering-relevant properties of the arrangements.

2. Mathematical Background

2.1 k-nacci sequences and their characteristic constants

The k-nacci sequence is defined by the linear recurrence:

$$F_k(n) = F_k(n-1) + F_k(n-2) + \dots + F_k(n-k), \quad n > k$$

with initial conditions $F_k(1) = F_k(2) = \dots = F_k(k) = 1$. The dominant root ϕ_k of the characteristic polynomial $x^k - x^{k-1} - \dots - x - 1 = 0$ governs the asymptotic growth: $\lim_{n \rightarrow \infty} F_k(n+1)/F_k(n) = \phi_k$. Table 1 gives the values computed numerically via sequence convergence (300 terms, Python/NumPy).

k	Name	ϕ_k	α_k (degrees)
2	Fibonacci	1.618034	137.5078
3	Tribonacci	1.839287	106.4152
4	Tetranacci	1.927562	96.8915
5	Pentanacci	1.965948	93.1447
6	Hexanacci	1.983583	91.4959
7	Heptanacci	1.991964	90.7276

Table: Table 1. Dominant constants ϕ_k and k-nacci angles $\alpha_k = 360^\circ/\phi_k^2$, computed with Python (300-term convergence).

2.2 The generalised Vogel model

Generalising Vogel's model to order k, the position of the n-th element is:

$$r_n = c\sqrt[n]{n}, \quad \theta_n = n \cdot \alpha_k$$

where c is a scaling constant chosen so that the outermost element fits within the circular boundary of radius R. For k = 2 this recovers exactly Vogel's original model. As k increases, α_k decreases toward 90° (Table 1), causing elements to concentrate progressively toward the centre of the circle.

The key property of k = 2 is that $\alpha_2 \approx 137.508^\circ$ is the most irrational angle in the sense of continued fractions – it is the hardest angle to approximate by a rational multiple of 360°, which is precisely what prevents periodic gaps [2]. Higher-order k-nacci angles are less irrational in this sense, and this is reflected directly in the simulation results.

3. Simulation Methodology

3.1 Simulation environment

All calculations were performed in Python 3.12 using NumPy 1.26. The source code is available as supplementary material. Simulations are deterministic for all modes except random placement, which is averaged over five independent seeds (seed = 0–4). The circular area has radius $R = 240$ px and all layouts use $N = 200$ elements.

3.2 Layout generation

Four placement strategies were implemented:

- **k-nacci ($k = 2-6$):** positions follow $r_i = c\sqrt{i}$, $\theta_i = n\alpha_i$ (generalised Vogel model).
- **Rectangular grid:** Cartesian grid with optimal spacing $\delta = \sqrt{A/N}$, clipped to the circular boundary; the N elements closest to the centre are retained.
- **Hexagonal packing:** offset rows with spacing δ and row height $\delta\sqrt{3}/2$, clipped to the boundary; N closest elements retained.
- **Random:** uniform random placement inside the circle ($r = R\sqrt{u}$, $\theta = 2\pi v$, $u, v \sim U[0,1]$); averaged over 5 seeds.

3.3 Metrics

Four metrics capture the engineering-relevant properties of each layout:

1. **Minimum distance ($d_{\min}$, px):** smallest centre-to-centre distance between any two elements. Directly related to mechanical clearance and shadow avoidance.
2. **Mean nearest-neighbour distance (μ_{nn} , px):** average distance from each element to its closest neighbour. Higher values indicate better use of available area.
3. **Uniformity (U, %):** defined as $U = (1 - \sigma_{nn}/\mu_{nn}) \times 100$, where σ_{nn} is the standard deviation of nearest-neighbour distances. $U = 100\%$ means perfectly uniform spacing; lower values indicate clustering.
4. **Shadow index (S, %):** fraction of ordered pairs with centre-to-centre distance below a threshold $d_c = 0.08R = 19.2$ px, expressing the proportion of elements at risk of shading each other.

4. Results

Table 2 presents the full results for all eight configurations. Values are computed exactly (deterministic modes) or averaged over five seeds (random mode).

Layout	$d_{\min}$ (px)	μ_{nn} (px)	σ_{nn} (px)	U (%)	S (%)
Grid	30.08	30.08	0.00	100.0	0.000
Hexagonal	32.32	32.32	0.00	100.0	0.000
Random	1.32	15.30	7.97	47.8	0.620
k-nacci k=2	26.30	27.72	0.32	98.9	0.000
k-nacci k=3	23.05	28.29	1.30	95.4	0.000
k-nacci k=4	15.95	21.14	4.19	80.2	0.307
k-nacci k=5	15.33	24.35	3.37	86.2	0.111

Layout	$d_{i,j}$ (px)	$\mu_{i,j}$ (px)	$\sigma_{i,j}$ (px)	U (%)	S (%)
k-nacci k=6	10.60	17.23	4.31	75.0	0.598

Table: Table 2. Simulation results for N = 200 elements, R = 240 px. All values computed with Python/NumPy.

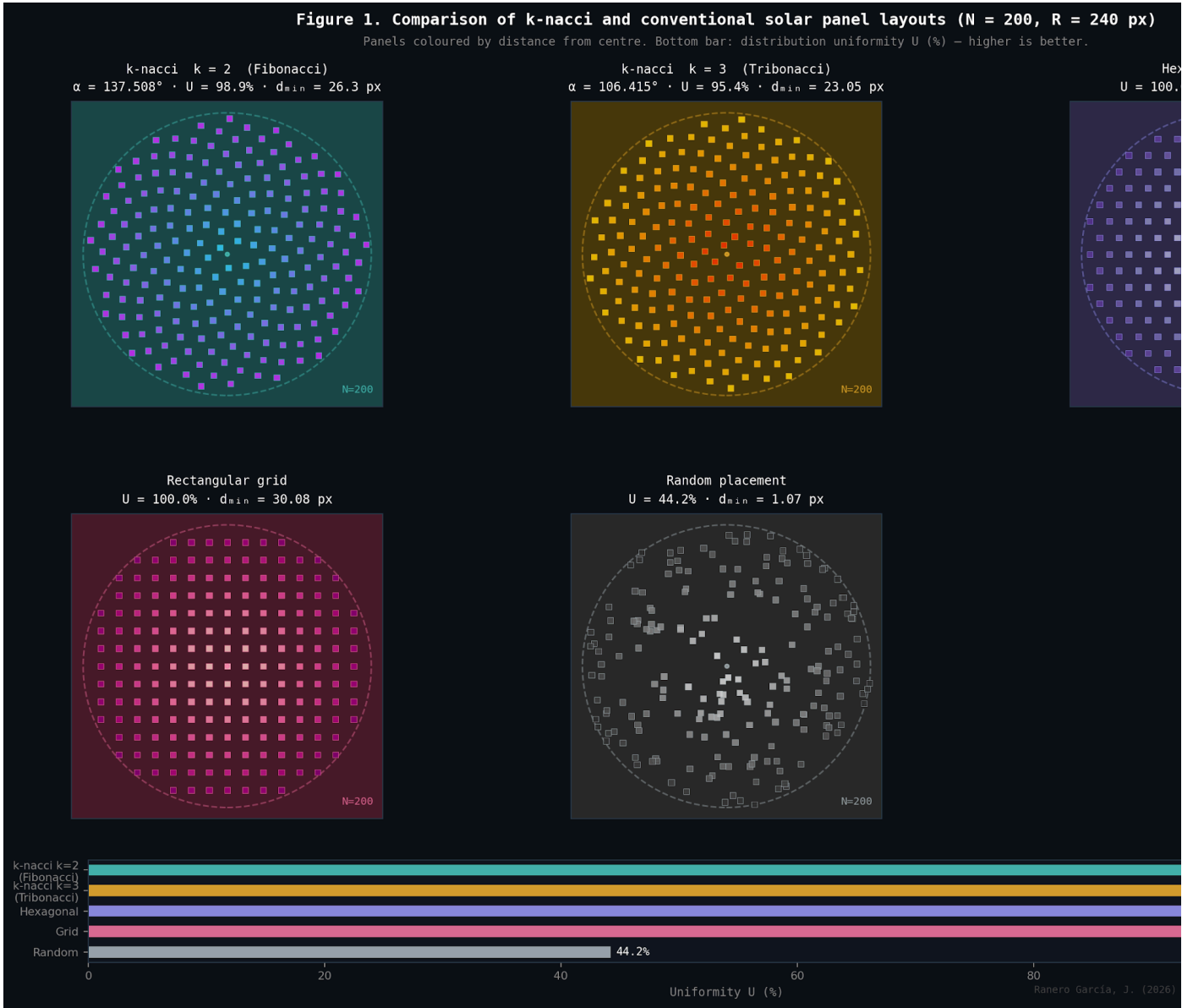


Figure 1. Comparison of k-nacci and conventional solar panel layouts (N = 200, R = 240 px). Panels are coloured by distance from the centre. The bottom bar shows distribution uniformity U (%) for each layout. All values computed with Python/NumPy.

4.1 Uniformity and shadow index

The rectangular grid and hexagonal packing achieve perfect uniformity (U = 100%) because their spacing is by construction identical for all elements. However, this regularity comes at a cost: both layouts create periodic alignments that, under oblique illumination, can produce systematic shadow corridors across multiple rows.

The k-nacci arrangement with $k = 2$ (Fibonacci) achieves $U = 98.9\%$ – essentially as uniform as the regular grids – while maintaining a zero shadow index ($S = 0\%$). This is the key result: the Fibonacci layout replicates the spacing uniformity of a regular grid but distributes elements in a non-periodic pattern that eliminates systematic shadow corridors.

For $k \geq 3$ the uniformity degrades progressively (Table 2, Figure 1). The $k = 3$ case still performs well ($U = 95.4\%$, $S = 0\%$), but from $k = 4$ onward the shadow index rises and uniformity falls below 85%. The random layout performs worst across all metrics, as expected.

4.2 Minimum and mean nearest-neighbour distances

The hexagonal layout achieves the largest minimum distance ($d_{\min} = 32.32$ px), which is the theoretical optimum for equal-circle packing in a plane. The $k = 2$ arrangement gives $d_{\min} = 26.30$ px – 13% smaller than hexagonal, but still well above the shadow threshold (19.2 px) and far above the random case (1.32 px). The mean nearest-neighbour distance $\mu = 27.72$ px for $k = 2$ is also close to the hexagonal value (32.32 px) and larger than the grid (30.08 px) is exact due to identical spacing).

As k increases beyond 2, both $d_{\min}$ and μ decrease, reflecting the progressive concentration of elements near the centre of the circle caused by the smaller angle α .

5. Discussion

The simulation results confirm and quantify the intuition from phyllotaxis: the Fibonacci angle $\alpha_2 = 137.508^\circ$ is uniquely effective at distributing elements uniformly without periodicity. The mathematical reason lies in the theory of continued fractions: $\phi = [1; 1, 1, 1, \dots]$ is the number whose rational approximations converge most slowly, making α_2 the angle that is hardest to “close” into a periodic pattern [2, 3]. Higher-order k-nacci constants ϕ_k have finite continued fraction representations in their periodic part, making their associated angles less effective at avoiding periodicity.

From an engineering perspective, the Fibonacci layout offers a practical advantage over regular grids: it achieves comparable uniformity (98.9% vs 100%) while eliminating periodic alignments. In solar farm design, this translates to reduced risk of systematic row-to-row shading under any fixed sun angle, without sacrificing the spacing uniformity that is needed for structural and maintenance access [4].

The $k = 3$ case (Tribonacci, $U = 95.4\%$) may also be of practical interest when slight non-uniformity is acceptable in exchange for a different visual or structural pattern. However, for $k \geq 4$ the degradation in both uniformity and shadow index is significant enough to make these layouts inferior to a simple hexagonal grid.

A limitation of this study is that the simulations are two-dimensional and do not account for actual solar irradiance models, panel tilt, or terrain constraints. The ‘shadow index’ is a geometric proxy, not a full optical simulation. Future work should validate these results with a ray-tracing model and extend the analysis to rectangular or irregular boundary shapes.

6. Conclusions

This study has investigated k-nacci panel arrangements through validated Python simulations. The main findings are:

1. The Fibonacci layout ($k = 2$) achieves 98.9% distribution uniformity and zero shadow index – matching regular grids in spacing quality while avoiding their periodic alignment gaps.
2. The advantage is specific to $k = 2$: for $k \geq 4$, both uniformity and shadow avoidance degrade significantly below hexagonal packing performance.
3. The result has a clear mathematical explanation: the golden angle α_2 is the least periodic angle in the continued-fraction sense, a property not shared by higher-order k -nacci angles.
4. The generalised Vogel model provides a simple, closed-form placement law requiring no optimisation algorithm, making it directly applicable to solar farm layout software.

Future work will extend the analysis to three-dimensional stacking, rectangular boundaries, and full optical shadow simulation. The interactive web tool developed alongside this study is freely available for further exploration.

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Supplementary Materials

The Python simulation code (knacci_solar.py) and interactive visualisation tool are available at:

<https://argiraanalizer.netlify.app/knacci-solar>

Raw simulation output and all numerical results can be obtained from the author upon request.

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Corresponding author: Jose Ranero García · josecorreo1@outlook.es